Course on Introduction to Medical Imaging and Analysis Softwares
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Module 02
Lecture 09: Active Contours for Segmentation

Welcome to today's lecture which is on another segmentation method and this particular one is called as Active Contours now you might be having a curious note as to why is there the term contour and then there is an activity over there because of its being called as an active. Now there is also another name for this one called as a snakes method might have heard about that one if you are doing some sort of other advanced techniques as well.

Now incidentally what comes out is that this is a quite interesting technique because the contour over here is actually something which can flow along the different perspectives on the image itself and then it comes down to a convergence and from this particular attribute of the contour itself it gets its name called as an active counter.

Now without much of a like leaving you into this whole dilemma about trying to sort of imagine as to what this might be, let me give you an example.

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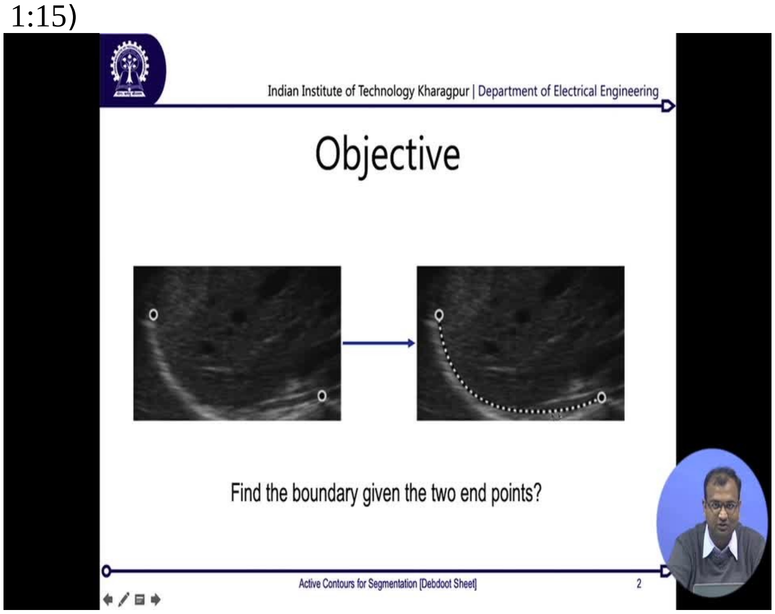

So say that you are given down with this objective where you have an image and you are just marking down two nodes over there, two points over there and say that there is some sort of a contour which is supposed to follow and this is supposed to follow this particular line and then these are the two ends of the contour. So there is the contour is not going to exceed beyond those lines or even exist anything beyond that but between them.

Now in general you will basically have a straight line if you are just given down two points but here the question comes down is can you feed down a model which is somewhat flowing along this particular curve over here which is at bright line. Now this is basically small snapshot of the ultrasound of your liver as we had seen in the earlier examples on texture where you had seen that cross section of a liver ultrasound as well. So this is just a diaphragm over there and you just have to mark down the boundary over here, so this is the problem.

Now the question is well formulated as find the boundary given that there are just 2 end points and then finally you should be able to get this sort of a curve line over there.

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Now, in terms of an active contour how it is defined is that say that there are basically two hinge points which are called as r0 and r1 which are the two edges over there, now these two points will not be moving in any way, they will always remain static and concentrated over there.

Now all the points between them so if there is some sort of a contour or image that initially you just had one straight line over there and there are you can have certain nodes over there in these multiple points. Now the idea is something like that that if you have a complex curve over here so you can basically define this into a summation of piecewise linear things. So basically a curve can be represented as small segments of small line segments which are just cumulative one beyond that other.

So over here there will be there can be one line segment linking r0 to rs1, another line segment linking rs1 to rs2 and like this such that you approximate almost this curve over here. And these points rs1, rs2, rs3, rs4 initially they might locate somewhere over here and then eventually they will be dragged in a way such that they can be located onto this particular curve over here which is an attribute of the image, that is what all of this business about active contours is to do about.

So you need to remember one thing in mind that since it is called as an active and that whole thing comes down from the fact that this contour is mobile, so the contour can move around over there and since it has to move and convert at a point of time so this necessarily has to be an iterative process otherwise it will not be converging. So all of this what we are going to do is to come down with an explanation of what to convergent how to convergence since basically this has to move so you will have to find out how to move then how do you find out some sort of a force which will basically be drifting this contour in a particular direction otherwise this might not come down to a convergence point over there. And also on top of that you need to understand what is an energy over there which is going to be minimum in case the contour is at the particular point, okay.

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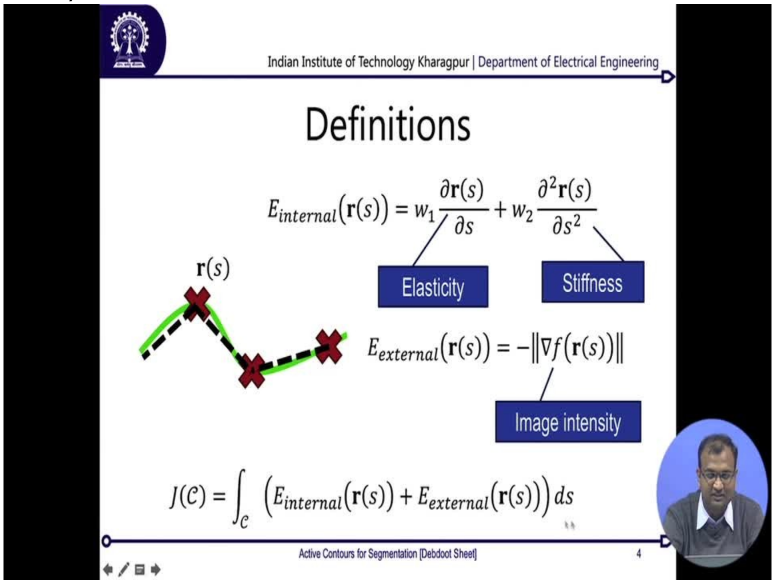

So let us start with the basic definitions, now consider that there is a curve over here which is what you would like to approximate that is the actual final state of the curve and then assume that there are just three points given out on this curve, okay. this very straight forward curve fitting problem and let any of this points be called as r(s), now if you look over here this r is a vector so this r is basically an xy coordinate couple just a coordinate representation. So every point basically is a representation of x, y in this coordinate space.

And s is basically a number which is which it denotes which particular point you are taking along this contour. So over here s is equal to 1, this is equal to 2, this is s equal to 3 and so on and so forth, so basically s varies in the order of number of points you have on that contour. And these points are basically hinge points. So you can have infinite number of points basically between them, but these hinge points are what will be guiding as to how the contour can flow across on that image.

Now from there you can obviously have a piecewise linear approximation between these points and this dotted black line is the contour which is an sort of very good estimate or an approximate estimate of the actual final contour where it should be lying down over there. Now if that is the situation, at a point r(s) we compute something called as an internal energy, okay so this internal energy is basically a weighted summation of the first order derivative and second order derivative along this particular curve.

Now look at this one that we are taking a derivative along the points on the curve, along this parameter s, okay. It is not an image derivative or anything it is not a derivate of the grey scale intensity of the image which obviously be used in a different point but this is basically del r del s is a vector of del x at that position r(s) del x with respect to del s i cap and del y with respect to del s j cap this is how this whole thing this first term will broken down into. The second term would appropriately be broken down in terms of a second order derivative as well and then you need to take down the weighted summation of that.

Now this first term over here is called as elasticity of the contour, the second term is also is known as stiffness of the contour. And imagine that you have a rubber band in your hand and this rubber band is supposed to go down, so imagine that there is a plane surface on which you have a few pins and you have a rubber band and you have basically stapled the two ends of the rubber band over there and then you have to pull this rubber band such that it goes across these pins over there. Now when it goes across these pins you would see that the first term elasticity it basically gives you what is the total energy which is due to the elastic force being applied on the rubber band over there at any point.

And then the second term will basically give you the range of stiffness which is like it is basically the derivative of the force which is being applied on that rubber band over there and if like you reach an elastic limit and it is at a verge of breaking then this force as such starts becoming 0 that is where it will come down to. Now from there, there is another force on every point which is called as E external and this is basically the first derivative of the intensity of the image itself, so this f over here is basically intensity at this particular point on that image.

So as your points keep on moving as say this point it keeps on moving from somewhere here till here so the intensity over there will also be moving, so appropriately your gradient of the intensity which is nabla of s will also be moving so and you basically taking down the absolute value of that gradient over there.

So this part comes down from the gradient of an image, the standard gradient of an image which you can compute with a sobel operator or a prewitt operator over there. So this part is called as the external energy, now the objective over here is basically that say you have a curve which is denoted as C and we define this as some sort of a cross function in terms of this curve, then this cross function of the curve is basically the integral of the internal energy and the external energy over the whole length of the curve, okay.

So basically the whole curve is divided into multiple of these s s over there, so you are just going to take the total summation over there and then integrate it along the curve length and you will get down an energy true position over there. Now this is how the energy or the state of a particular curve at any point of time is denoted as.

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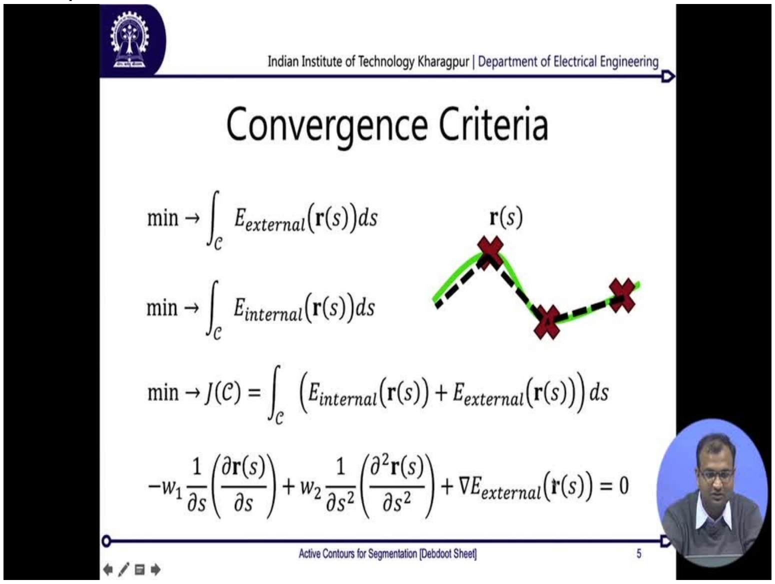

Now from there we come down to the next part which is a convergence criteria. Now as I said that initially what you are given is basically an image on which you are just given 2 points and now you have to start with some sort of an arbitrary curve in between these is two points and then all the points in between these 2 points on the curve will basically be moving and coming down to a point.

So initially it starts with something like this that you are given down two points and then you draw a third point over here and this is your estimate of the curve, okay. Now given this fact you start by minimizing the external energy and the internal energy along the length of the curve which in turn will lead to minimization of this whole cross function because if you can minimize internal and minimize external you will end up minimizing this whole thing together over there.

Now as this whole term reaches a minimization you would see that finally it converges on to this criteria over here. Now eventually you know that if this whole integral has a minimum value, then since these are energy values over here so they would finally have an energy which is equal to 0, okay. Now if you equate this minimization criteria so wherever you have a minimization you basically have the derivative of this is equal to 0. So by solving that first derivative criteria you would get down that in order to achieve convergence you need to have this sort of a criterion established over there.

Which means that the first derivative of the elasticity function which was my del r(s) del s as in the previous slides so it is basically first derivative of this one and the second derivative of my stiffness function together with the summation of the gradient over there everything together has to be 0 such that I am able to achieve this criteria, okay. Now this particular equation over here which is my condition at convergence actually acts as the major driver in order to design something called as a solver.

Now solvers are basically when you have a mathematical model in which over time you can iterate certain thing and then come down to a convergence one. Now look into this one, what you initially start with is basically 2 points and 1 point interpolated somewhere in between, okay. Now in the first iteration this is where your energy criteria is found out now based on that you will be getting some update rules over there as well, so we will be coming down to what those update rules are.

Now once you get an update rule, then you have a second position where you can move this intermediate point, okay based on that you will again get some update rule, based on that you will again be moving you keep on moving until you achieve this particular criteria or this particular criteria any one of them, the point when your atom minimization position you just stop over there, okay.

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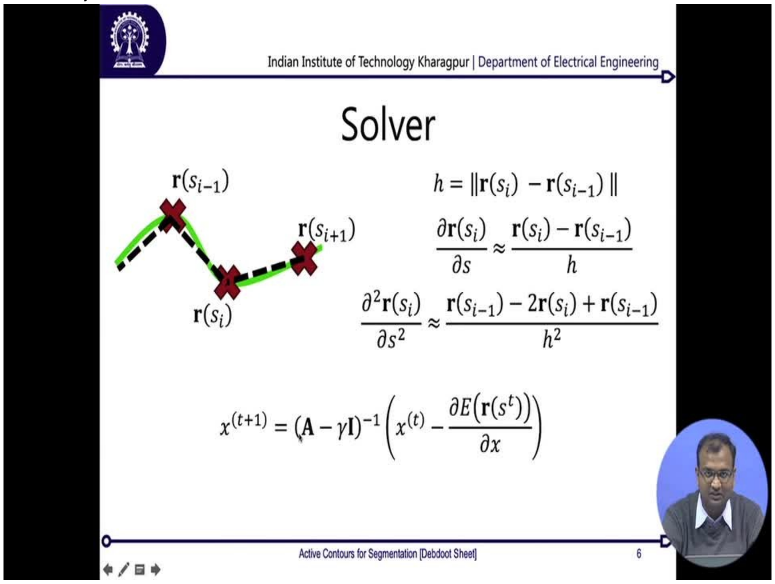

So let us look at how this solver is going to work, so imagine that this is a condition of the contour at so this is what it will be at the final spot but you can imagine it to be any intermediate point as well, okay. Now between these two points s minus s of i minus 1 and s of i you can find out what is the distance between them, so this is just basically the Euclidean distance between them because these are these exist on a rectangular Cartesian coordinate space, okay.

Based on that first you need to find out what is a first derivative of this r(si) for each of these points. Now you can compute this numerically with the estimation going down by the (fo) by the backward difference rule over here, okay you can employ the forward difference rule, you can apply employ the central difference theorem any of them accordingly over there.

Now, from that one you can also compute numerically the second order derivative along the curve contour as well, right? It is not such a complicated task. Now once you have all of these 3 computed next what you need to compute is the gradient which is for your external energy. Now gradient for an external energy is a very straight forward computation because you just take the derivative of the image along x direction derivative along y direction and then take their summation to get the total gradient energy over there so that is a straight forward computation.

Now once you have all of them and you solve it out completely what you would get down is that the final update rule looks something like this that say you start with a so for any point s okay, any of this si's over here say it has a value of x in a superscript within bracket t which is at a particular iteration over there. So t equal to 0 is the starting point, t equal to say 10 or 15 whichever is the final position that is your final end point, okay.

Now over here what will happen is that you start with an initial estimation so say t0 is your starting point now at this t0 you have your value of xt t equal to 0, then over there based on that you also have your r of st for t equal to 0 so you can find out this del of e of rst del x. So this part over here is basically finding out the total energy total energy which was computed and taking its derivative with respect to the x coordinate itself, so this is the x derivative over there, okay.

Over here you get another term called as the matrix a. Now this matrix a is a much larger and complicated matrix and where this comes down and this i is basically the identity matrix, now this a comes down by basically writing down a linear algebra equivalent of the earlier minimization energy functional form. So if you write it down expand it out completely you can find out the details in the actual text where it suggested down for more of it.

Now for most of this solvers this part over here is just a matrix which is kept constant, so there is a constant over a complete image this does not vary over there. And this gamma over here is basically an update coefficient which you would need. So now what you need to so is now that you have this matrix you need to subtract this gamma times of i which is basically a diagonal subtraction over there and then take its inverse and then basically multiply that inverse with this particular form over here. Now interestingly what would come out is that this matrix in its inverted form multiplied by this whole thing over here is going to give you a scalar value and that is your updated version of x, okay. This a matrix is also called as the state matrix of this update rule equation over here.

Now similarly, you can also update the other part of it which your y coordinate because you need to update both the x coordinate and the y coordinate so that the point can move appropriately in space. Now once you are able to have both of these update rules solved and get down a complete confirmative of them comes down the fun over here.

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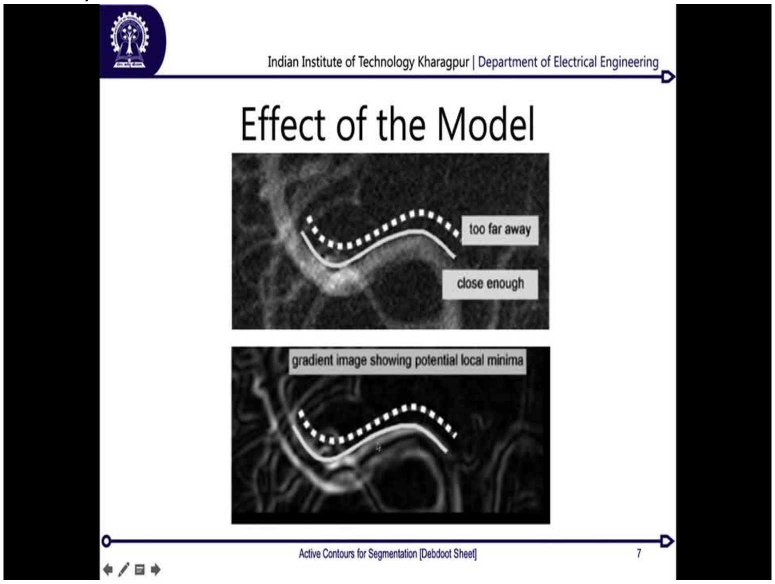

Now say that this is an example where we are looking at a angiogram in a blood in the heart so it is a cardiac angiography image which you are seeing over here and this is one of the blood vessels on which the contour has to snap down actually, okay. Now, if this is the contour which you initially start with or say this is the contour you initially start with, in either of the conditions it would actually come and snap onto this contour which is close enough over here.

So you can start with a point which is very far off, you can start even with a very wired shape and that will come down and the reason why it comes down is because you have this gradients which are the external energy over there. One is the internal energy which just restricts the points from flying away so that it looks like a very confirmative contour over here and it does not fly away.

The other point which you need to look over here is basically that it again restricts itself to the actual shape of the object on which you are trying to look over here. So this external energy or the derivative over the image basically gives you the idea about the shape of the contour on which you are supposed to lie and until you come down on to this one you will never be at a minimization condition in anyway.

So this is the beauty of active contours of how they can snap down the actual model where they are supposed to go. Now based on this so this is not just where it ends because you can look into one major problem over here is that say that you have a very short objects short span of contour on which you want to fit down then and you are very close to the actual closing criteria then you can snap down very easily.

But say that you have something like a lesion, okay and you have drawn a closed contour which is a small circle which is enclosing that whole lesion and now this circle is supposed to snap down onto that object but if you just use this kind of a energy function as we have used only for an active contours with the snakes model it will never be going down and snapping on to this object confirmatively. So most likely it will just be dangling somewhere in between and there are also conditions that if the contour is much further apart from the actual point where it is supposed to snap down, then it can start wobbling as well.

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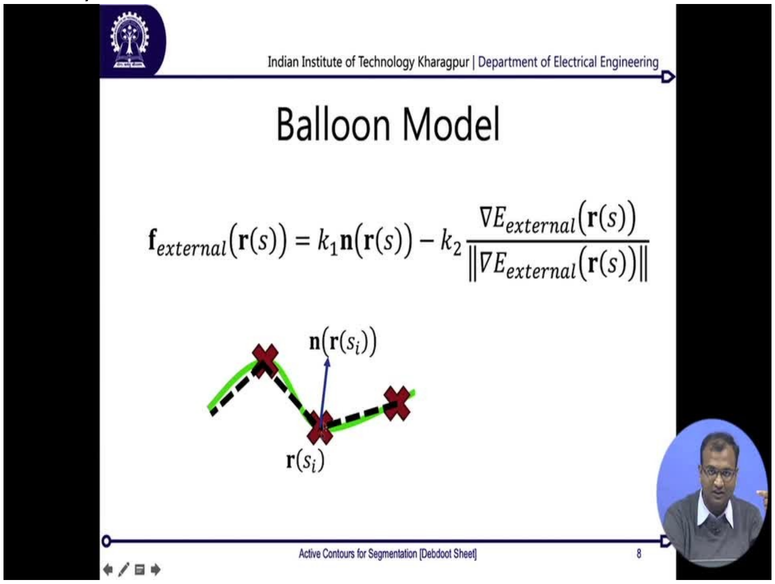

Now in order to get rid of that, we have a new kind of a model which introduces another force which is a drift force and this is called as Balloon Model. Now what it does is assume that you have some sort of a balloon which can conceit itself to a volume of 0. So there is a balloon which you can inflate and then it can constit itself. So now this inflation will be dependent on what you are going to fit. Now imagine that you have a balloon and you are blowing it out so generally it will be a convex spherical shape on which it blows out, right?

So there can be different kind balloons which blow down as cylindrical shapes as well or in different kind of artistic shapes. Now, if you are not blowing this balloon or trying to fit this balloon inside a rock. So the rock is inside the balloon, then this balloon is basically going to stick to the surface of the rock and follow that contour and this whole thing happens because the balloon has a natural tendency to constrict itself.

So if it has a natural tendency (to) any contour which has a natural tendency to constrict itself is defiantly going to follow down the normal at any point so the normal vector which is pointing towards any point and that is where we get down this new energy function and what this does is there is an external force which is supposed to be applied over there and this external force is a weighted function of the normal force and the external energy over there.

So while the external energy is trying to drift apart and pull the balloon pull the contour to come outside, this force in the normal direction is forcing it in the opposite direction you can look at this sign changes on the vectors over here. So this what basically signifies what is going to happen over here. Now basically what you try to do is if you look at this particular contour over here, then the normal will be something which is forcing itself inward, right?

So you have one force which is on this side, another force of tension which is on this side so your normal force is you just pulling it inside and the external energy is something which is pushing it on this side. Now it will come down to and converges in that kind of a criteria and these are very useful in medical image segmentations when you are trying to look into a very closed object and a closed contour in totality.

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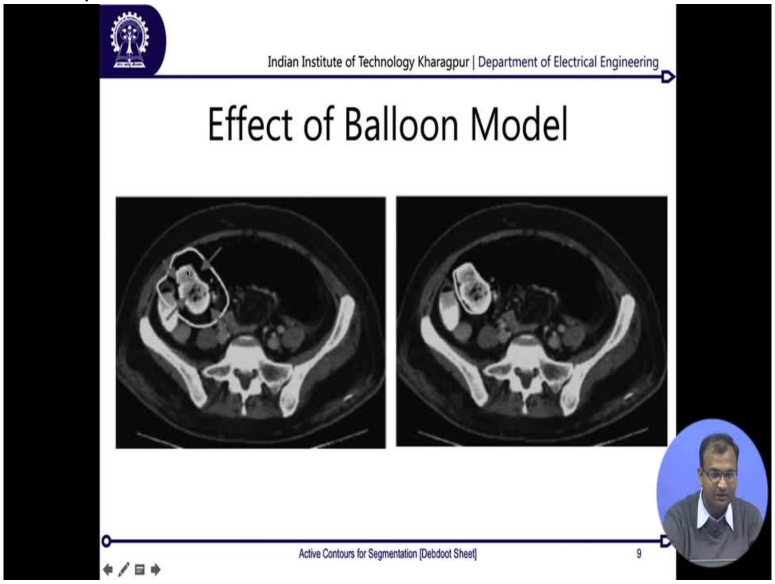

So for one of the examples you can look over here so initially so this is about a city if the upper abdomen and you have a lesion over here somewhere around your lunges and now the objective was that you draw an initial estimate of the contour which is over here and you are supposed to snap it down to this actual lesion and this can be used for basically finding out what is a lesion area or lesion volume over there and other characteristics along the edge or the perimeter of the lesion.

Now if you are not using a balloon model in general case you will have a contour which will just be spherically located over here or there are even chances that it can actually blow out and enclose this part over here which is not an actual lesion which is supposed to be encapsulated because if you look over here this is supposed to drift down and come down over here as well. So that is the wrong thing, whereas when you are using a balloon since it is always trying to retain a very convex shape in itself it is forcing itself inward the tension is pulling itself inwards and the force of intensity is pushing it outward.

In that case you would always find out a conformity to become convex over here and that is where this converges to a effect condition.

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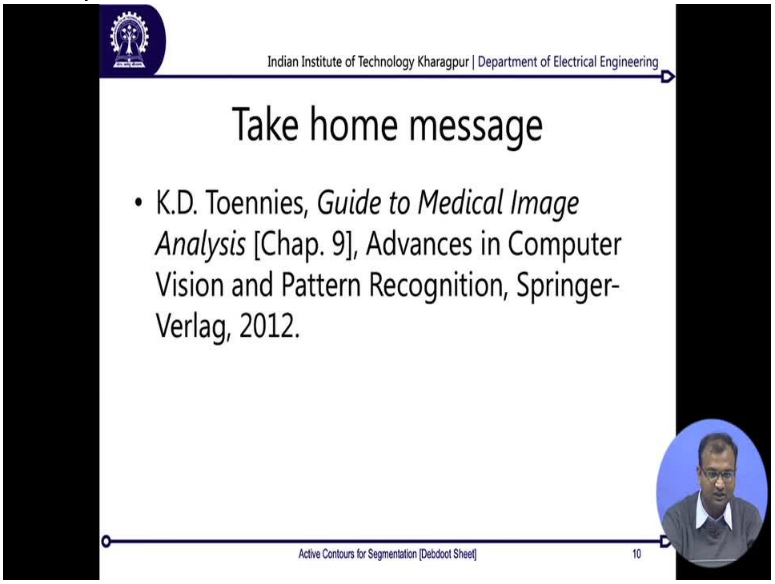

So with a balloon model your active contours are in a much better position to actually converge down on a particular point. So with that we basically come down to an end of this short lecture. You can read about more details in this particular book by Toennies on Guide to image medical image analysis on chapter 9 which is on active contours and active surface models.

So although we are concentrating only on active contours which exists on 2d but obviously you can extend this to the 3 dimensional space. In case you are on 3d space you will have something like a surface which is the boundary of objects over there so you can extend all of these criteria to your 3d if you have a 3d object and if you have a 3d datasets on which you try to do in that case since everything is extended onto 3d this contour now boils down to become something called as an active surface and that is where it converges.

So all start with a snakes, it goes down to balloon models and then you have some more advanced topics like level sets, you can have other techniques based on level sets as well but snakes as such and balloon together are some of the most robust and most widely used techniques for segmentations as far as active contours and surfaces are concerned. So with that we come to an end and thank you.