Digital Circuits
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Lecture – 05
Number System (Contd.)

So far we were talking about positive numbers, fine? So, but the numbers may be negative also.

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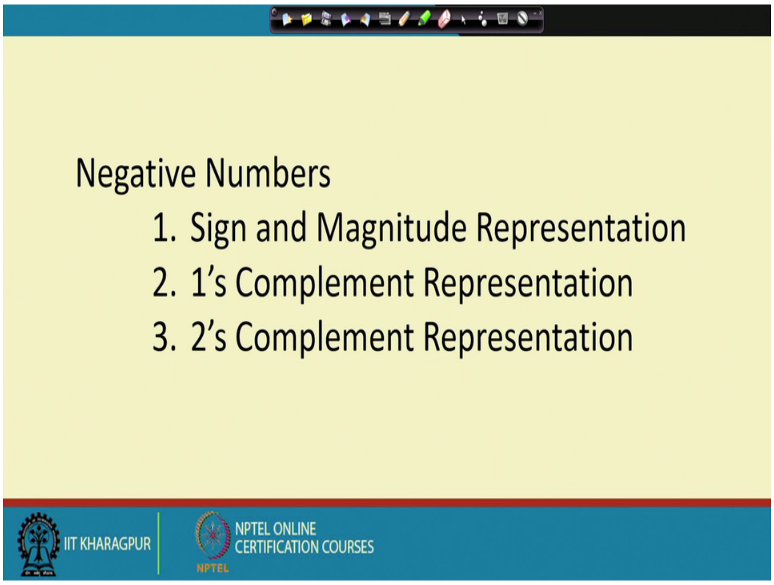

So, next we will be looking into how to represent negative numbers. So, there are 3 distinct ways in which we represent negative numbers in computer systems. One is called sign magnitude representation, one is called 1's complement representation, the other one is 2's complement representation.

So, we will be looking into these 3 types of representation in our next few classes.

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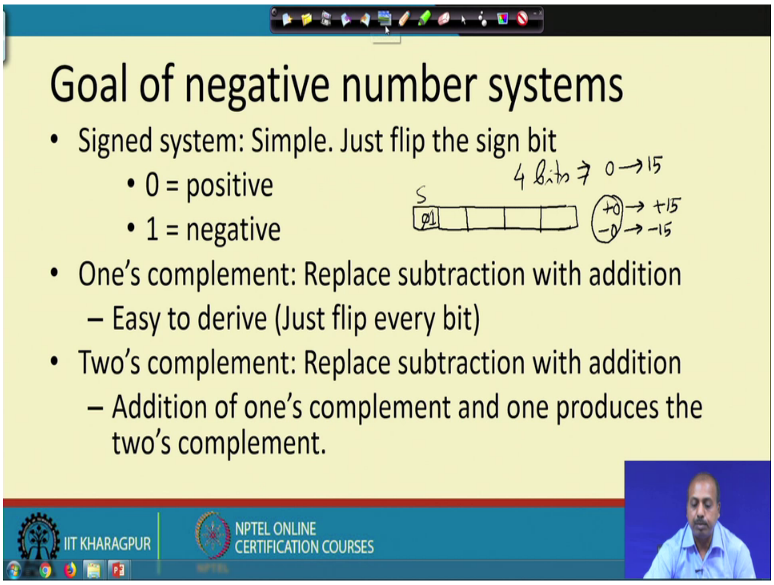

So, why do we need this? What is the goal of this negative number system? So, the sign system so, this is simple so, you just flip the sign bit so, 0 is positive and 1 is negative. So, if we have to if we. So, you can say that in your system. So, if I have got say 4 bits so, if I have got say 4 bits for representing numbers. So, in 4 bits I can represent the numbers 0 to 15, fine?

Now so, in sign system what will happen is that; so, we will have 4 plus 1 5 bits. So, so these 4 bits are there, plus there will be extra bit which is for the sign part, ok. Now this sign parts so, if it is 0; that means, I am representing positive numbers. So, naturally I can represents the numbers plus 0 to plus 15. On the other hand if this bit is one, then the number is a negative number. So, you can say as if it is minus 0 to minus 15, because this value can go up to 15.

So, of course, this is a bit confusing so, I have got a plus 0 and a minus 0, but that is their, but apart from that. So, this a representation is very simple. So, if we have to convert a number from positive to negative or negative to positive you just flip the sign bit of the number. So, that way the sign magnitude represent the sign resistance. So, this is very simple and it can be used for representing the negative numbers.

The 1's complement representation so, it is says that replace subtraction with the goal is to replace subtraction with addition. So, the this add this subtraction process so, we can do using addition. And here the idea is that we just flip every bit. So, if the number was originally say if I representing the number say plus 5 in a 4 bit representation if the number if the number is 0 1 0 1 for representing minus 5.

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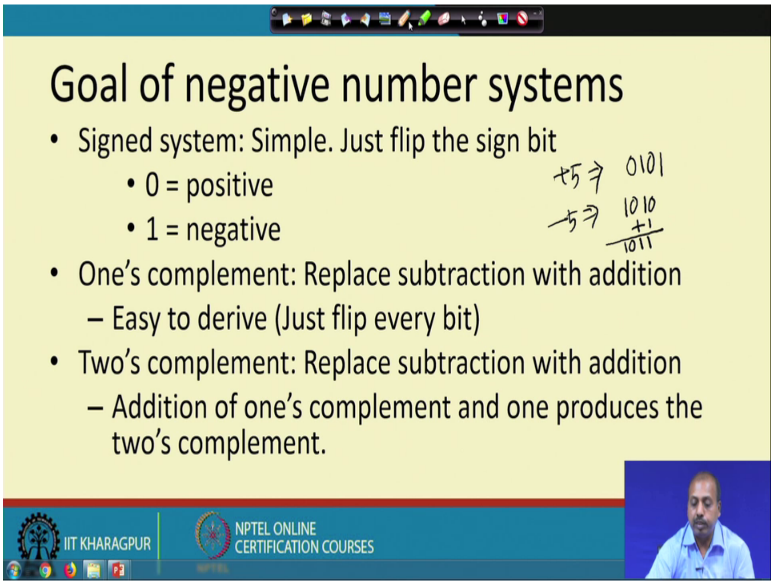

We just flip all the bit so, it is 1 0 1 0, ok. So, that is the it is the 1's complement number system.

In 2's complement number system so; again we will be replacing subtraction with addition. So, addition of 1's complement and one produces the 2's complement. So, this is the 1's complement representation of minus 5, we are looking for 2's complement representation, then with that you have to add a 1. So, if you were adding a 1 so, it is becoming 1 0 1 1. So, that is the 2's complement representation of minus 5.

So, this way this number systems they are the negative numbers can be represented, and we can use them for computer representation.

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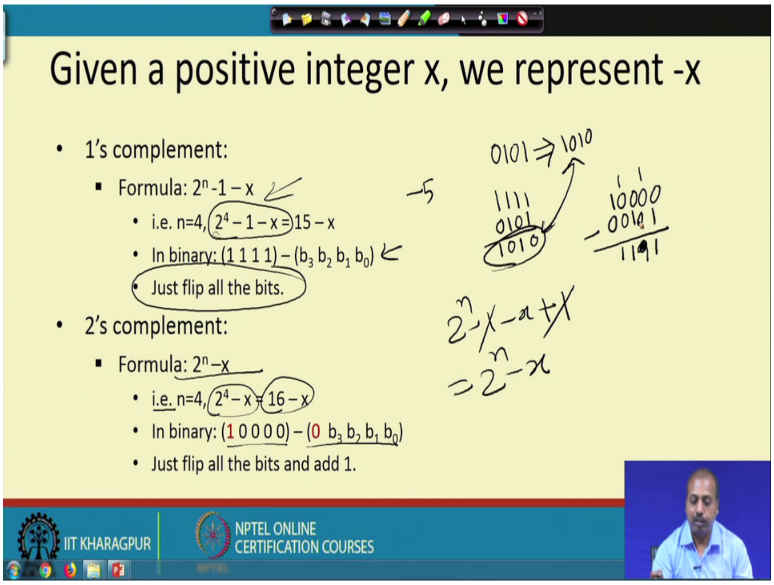

So, if we are for the 1's complement, the overall formula is like this, if the original number is x, and you want to represent minus of x now, then for 1's complement the formula is 2 power n minus 1 minus x. So, for example, if we have a got n equal to 4 then for the formula is 2 power 4 minus 1 minus x the 15 minus x.

So, in binary so, what is happening is the this 15 is 1 1 1 1, minus the x b 3 b suppose the x is b 3 b 2 b 1 b 0. So, essentially what will happen is that all bits will get flipped. So, you were subtracting so, this will be it is subtracting from one. So, the bits will be getting flipped ok.

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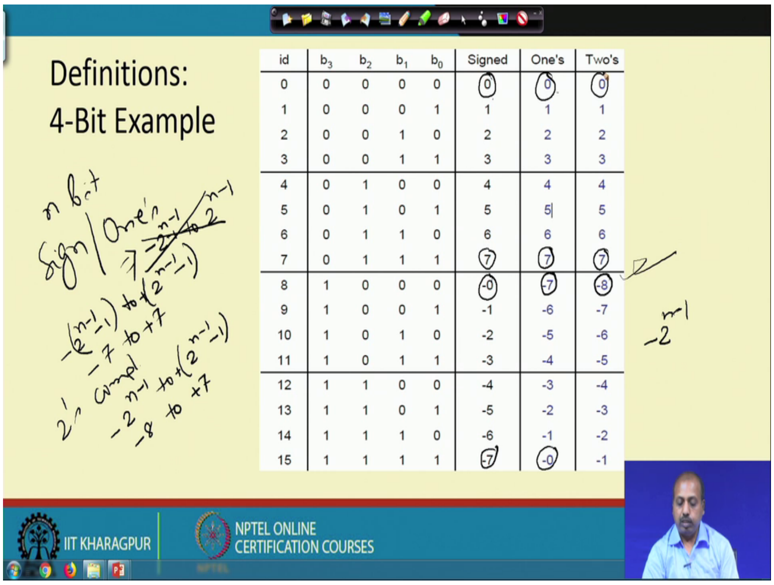

So, anything like say if I have got say, 5 sorry if I have got say 5 ah, then in binary number system 4 bit binary it is 0 1 0 1 now say ah. So, I have got the numbers so I want to represent minus 5. So, as per this formula it says 1 1 1 1, from there have to subtract 0 1 0 1. So, 1 minus 1 is 0, then this 1 minus 0 is 1 1 minus 1 is 0 1 minus 0 is 1. So, what you get is 1 0 1 0 which is the 1's complements.

So, if you take 1's complement of this. So, all the bits are flipped. So, you get 1 0 1 0. So, you see that these 2 are same this 2 are same. So, this is same as flipping all the bit. So, the this is the mathematics so, we have to take 2 power n minus 1 minus x, but ultimately, it boils down to a very simple state you just flip all the bits. Then in the 2's complement the formula is 2 power n minus x. So, in 1's complement we have already done this 1, 2 power n minus 1 minus x. So, that is why it says that if we with 1's complement if we add this 1 so, this is 2 power n minus 1 minus x. So, we if we that if we add one then these ones will cancel out. So, you will get 2 power n minus x, fine?

So, we have got this 2's complement representation. So, so n equal to 4, we have got 2 power 4 minus x so, that is 16 minus x. So, so, was a 16 is 1 0 0 0 0 minus this 1. So, in thus in the if we if we take the previous example that for the that minus 5 so, it is 1 0 0 0 0 minus 0 0 1 0 1 ok. So, so 0 minus 1 is 1 with 1 carry. So, so basically so, 0 minus 1 is 1, now 0 minus 0 is 0 0 minus 1. So, there will be one carry so, this 1 minus 0 will be this will be 1. And then this, then 0 minus 1 is one with a one carry then 1 borrow rather. So, 1 minus 0 is sorry this is not anyway so, this is I will I will come back to this later this is not very simple here anyway.

So, basically what we have to do is, we have to do a this is done by means of some addition it is not, it is not doing the subtraction in 2's complement form that is why it created some problem so, we will come back to this later. But this is basically flipping all the bits and taking and adding one to that. So, so in say the if these are number so, 0 so, in normal or representation is 0 0 0 sign representation.

So, this is 0 1's complement representation is also 0 2's complement is also 0. So, for up to this much there is no change. So, up to 7 since a 4 bit representation up to 7 all are same. But when you go to 8, then you have got this 1 0 0 0. So, since they have signed representation so, this will be using a sign bit. So, most significant bit is the sign bit so, this part is 0 so, it represents minus 0.

Similarly, 9 in the sign representation will be is minus 1, because these 3 bits represent 1. So, this sign bit is one so, this is minus 1, similarly this one is minus 2. So, this is minus 3 like that in 1's complement form so, this 1 0 0 0. So, that is your 15 minus x. So, 15 minus so, that is basically that 15 minus 8 that is 7 and that is a negative number so, it is minus 7. So, it is 15 minus 9 is 6 that is minus 6. So, like that in 2's complement number system it is 16 minus x. So, it is 16 minus 8 is 8.

So, this 1 0 0 0 is minus 8 1 0 0 1 is minus 7 like that. So, this way from the formula will directly get all these values. But the point that I would like to mention is the range of values that you can represent in the number system the so, in case of signed representation. So, you can represent the numbers minus 7 to plus 7 in a 4 bit if I give you only 4 bit, then you can represents the numbers minus 7 to plus 7. In 1's complement representation, your negative side you can go up to minus 7, and the positive side we can go up to plus 7; so, here also it is minus 7 to plus 7.

In 2's complement representation you can go from minus 8 to plus 7. So, this is something extra that you have got. And what was so, in so, I can say in general. So, if I have got say n bit. So, n bit representation, then in this signed and this signed and this 1's complement. So, they are giving you, you can represent numbers from minus 2 to the power n minus 1 to 2 to the power n minus 1. So, minus 2 to the power n minus 1, sorry, this is there should be a minus 1. So, minus 2 to the power n minus 1, that is for example, here it is to 2 to the power n minus 1 minus 1. So, minus 2 to the power n minus 1 minus 1 to plus 2 to the power n minus 1 minus 1. So, if we put n equal to 4 so, this is basically your minus 7 to plus 7.

In 2's complement notation in 2's complement. So, you have got the range, which is minus 2 to the power n minus 1 to 2 to the power n minus 1 minus 1, ok. So, for n equal to say n equal to 4 so, you have got here minus 8 to plus 7. So, as you see here so, you can go up to minus 8 on the negative side and plus 7. On the positive side on 1's complement and all so, I can go to minus 7 on the negative side at most. And plus 7 on the positive side at most similarly here also I can go up to minus 7 on the negative side and plus 7 on the positive side.

So, why this thing has happened you see that so, in 2's complement notation. So, I can represent one number extra. So, this minus 2 to the power n minus 1 so, this was not representable in 1's complement and signed representation, but it is representable in 2's complement form. This thing has happen because in case of this signed representation and 1's complement representation so; we are representing this plus 0 as well as minus 0. Similarly, in 1's complement also we are representing plus 0 as well as minus 0.

So, the value 0 it has got 2 different representations in signed representation and 1's complement representation. So, that is actually I should say utilizing one extra code ok; of the of the value so, but in case of 2's complement you have got only 1 0. So, there is nothing like minus 0 in 2's complement notation. So, we have got one extra patterned so, which can which is being utilize to represent say one extra number.

So, this way we can have this 2's complement number system to be doing better than this 1's complement, and signed magnitude there are of course, other advantage that we will see later, let us see.

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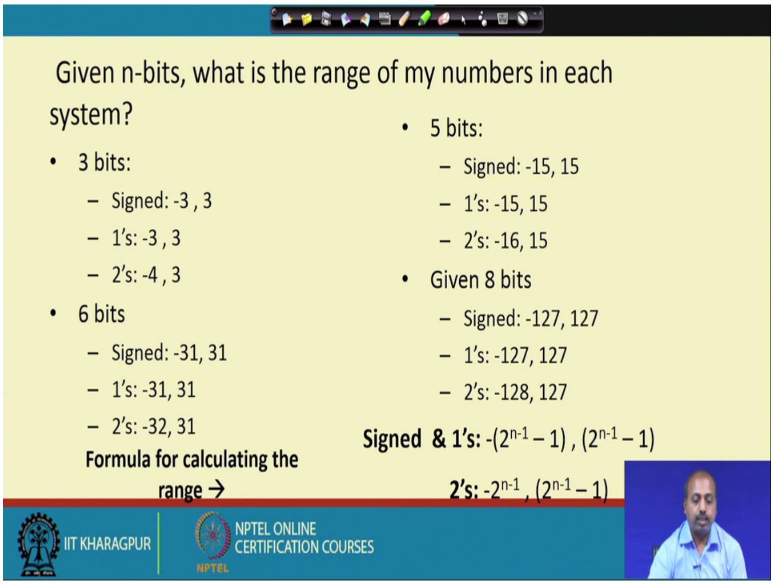

So, this is the thing that is given n bits what is the range of my numbers in each system, the 3 bits signed minus 3 to plus 3 1's complement minus 3 to plus 3 2's complement minus 4 to plus 3. So, if you have given 8 bits then signed is minus 127 to plus 127 1's complement minus 127 to plus 127. 2's complement is minus 128 to plus 127, and this is the rule that I was talking about formula for calculating the range signed and 1's complement is minus 2 to the power n minus 1 minus 1 to 2 to the power n minus 1 minus 1. And for 2's complement it is minus 2 to the power n minus 1 to 2 to the power n minus 1 minus 1 so, this way we can have ranges, ok.

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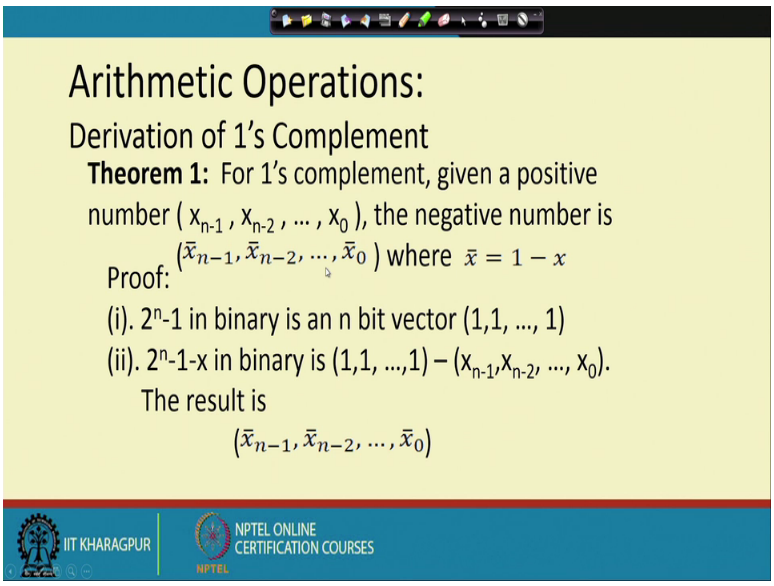

So, how to a derive 1's complement? For 1's complement given a positive number x one xn minus 1 xn minus 2 these are the individual bits of the number, xn minus 1 xn minus 2 and to x 0. The negative number is given by this xn minus 1 bar xn minus 2 bar up to x 0 bar where x bar is actually 1 minus x. So, so, xn minus 1 bar is 1 minus xn minus 1 like that. So, it is because of the fact that from the definition of 1's complement, it says that it is from the definition of 1's complement it says 2 to the power n, 2 to the power n minus 1 minus x.

So, this 2 to the power n so, so this 2 to the power n minus 1 is all 1 and so, 2 to the power n minus 1 minus x is 1 1 all 1 minus this so, naturally all the bits are getting complemented. So, for 1's complement so, we can get the number the 1's complement representation will be negative number in this fashion.

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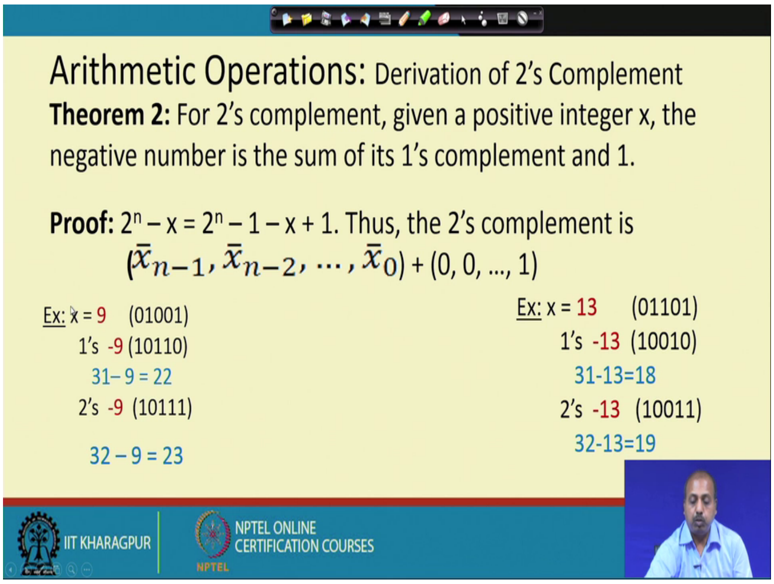

For 2's complement the operation is like this that you have to the negative number is the sum of it is 1's complement and 1. So, we can proof it like this that for 2's complement this is the formula 2 to the power n minus x. So, it can be written the 2 to the power n minus 1 minus x plus 1. So, now this part 2 to the power n 2 to the power n minus 1 minus x so, this is the 1's complement ok, with that we are adding this one. So, 2's complement is basically the 1's complement plus 1.

So, these are some examples like x equal to 9. So, this is the or number 9 positive number 9. So, if we take 1's complement so, I will be getting this one. So, 31 minus 9 is

22 so, you can check that this number is really 22. So, it is one 2 4 8 16 so, 16 plus 4 20 plus 2 22. So, that is 31 minus 9 2's complement of minus 9 is 22 plus 1. So, that is 23 so, that is 32 minus 9 that is 23.

Similarly, here for the 13 also we can get it is 1's complement as 1 0 0 1 0, 2's complement as 1 0 0 1 0. So, this way you can do this you can prove this 1's complement and 2's complement rules for during the purpose.

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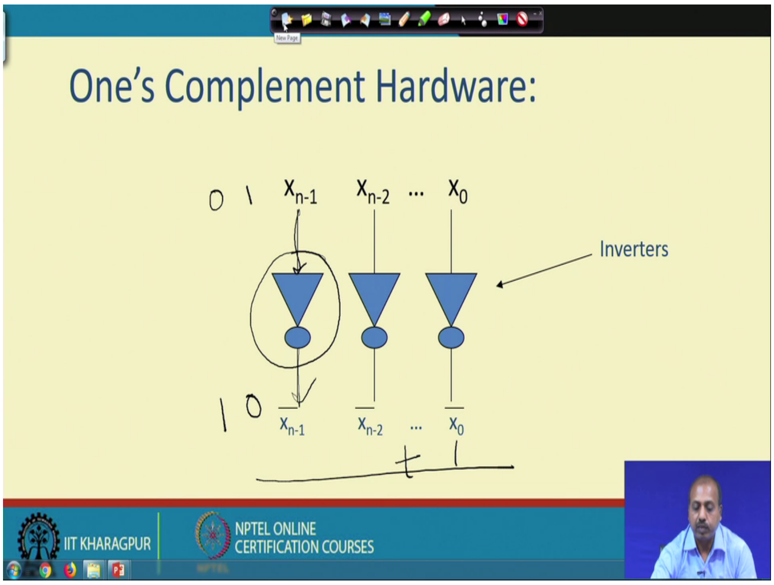

So, 1's complement implementation is very simple. So, we will we are not a familiar with this symbols, but we will see after a few classes, that they are actually the inverters. So, inverter means if you whatever give value you give if just complement that bit and produces the output. So, if you give a one here, you will get a 0 here, we will put a 0 here, you will get a one here.

So, that is what you need for 1's complement. So, for 1's complementing the hardware is very simple, you just do this thing. But for 2's complement it is not that simple because for 2's complement, after doing this you have to have some add up which will be adding a one with this. So, that way the 2's complement hardware is going be a bit complex compare to 1's complement. But anyway 2's complement is fine, 1's complement is fine.

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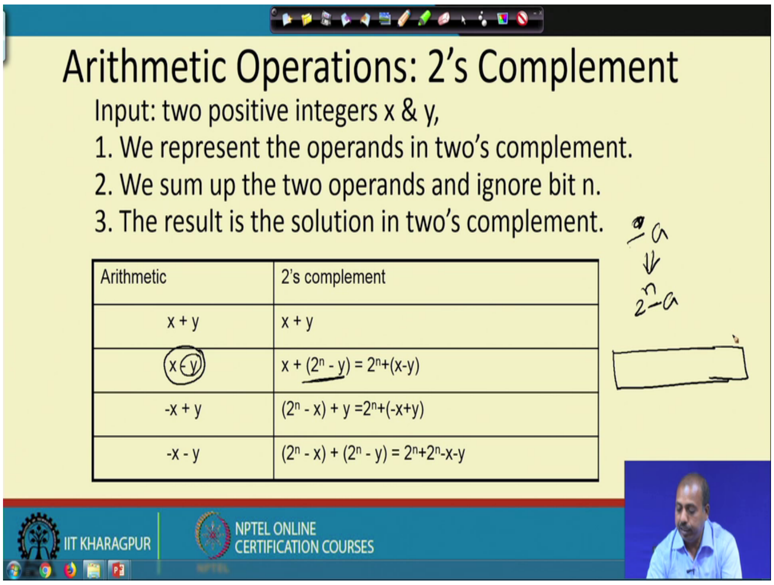

Now, whenever you are doing some arithmetic operation in 2's complement; suppose, we have 2 positive integers x and y has input so, we want to do some operations on x and y. So, for doing the operations first we represent the operands in the 2's complement form, and then we sum up the 2 operands and 2 operands and ignore the bit n. So, we it we do not need to consider whether it is they are addition a version of subtraction operation, like say x plus y. So, there is nothing to do so that is simply if the operation is x plus y. But this x minus y what we do is that we do not use a subtractor rather this minus y is represented in 2's complement form.

So, we have got the number y with us the positive y with us. So, we take the 2's complement of this y so, 2's complement of 2's y is 2 power n minus y. Now when is so, if you just simplify it further. So, it is 2 power n plus x minus y so, if this number turns out to be negative, then it will be subtracted from 2 to the power n directly.

So, you will directly get the 2 power n representation. So, basically so, you remember that if a number is negative, if a number is negative, then suppose I have a number a which is a negative number. So, it is represented as 2 to the power n minus a in the 2's complement notation.

Now, you see that I am doing this x minus y. So, what I am doing? I am taking 2's complement of this y. So, accordingly I am getting this value 2 2 power n minus y. Now after doing this operation if the number is positive then there is no problem, but if the number is negative ok. So, if the number is positive then this 2 power n, so, this 2 power n is basically so, if you look into this individual bits ok.

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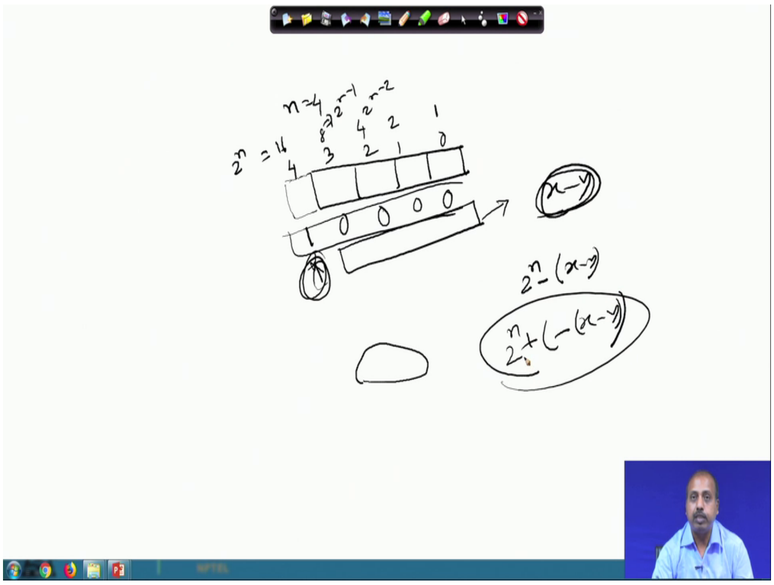

Let me take it fresh page and so, if I have got say n equal to 4 fine. So, if n equal to 4 then I have got the bits 0 1 2 3. And this bit this bit is the 4th bit. So, that is basically for if you look into the corresponding weights. So, this is one 2 4 8 16. So, this is basically this is equal to 2 to the power n, this is equal to 2 to the power n minus 1 this is 2 power n minus 2 like that.

Now, the operation that I was doing their the operation that I was doing there is this one. So, 2 to the power n plus x minus y. So, this 2 to the power n so, where is it going to affect? So, it is 2 to the power n is represented by this one, fine? Plus x minus y so, if x minus y is a positive quantity if x minus y is a positive quantity. So, it will be restricted to this 4 bits only so, it is not going to affect this bit. So, whatever the bit is coming, if I simply ignore these bit, then I will get the sum I will get the result x minus y.

If this my this result is negative if x minus y is negative then of course, that is no problem, because then these whole number is going to be 2 power the this the as per the representation. So, this is 2 power n minus of x minus y so, I am getting the result directly. So, if the number x minus y is negative, then I am. So, by doing this addition I am what I am essentially getting is 2 power n plus minus of x minus y so, I am getting the same result.

So, by doing this subtraction, instead of doing this subtraction so, if I do addition then also I am getting this subtraction done ok. So, then this minus x plus y if I have to do it, I do not need to do subtraction, I just take 2's complement of this x which is 2 power n minus x, and then with that I add y.

So, as a result I will get 2 power n plus minus x plus y; the same logic if this minus x plus y is a positive quantity then this n th bit can simply be neglected. And if this result is negative then this will be this is anywhere the 2's complement representation of that negative number. And minus x minus y so, this is I am taking the 2's complement of them, and then doing the addition. So, again I am getting the same thing. So, raised 2 power n plus 2 power n minus x minus y so, that is the 2's complement representation.

So, whatever we are doing whenever we are doing this subtraction operation. So, I can just take the 2's complement representation and do the addition operation.

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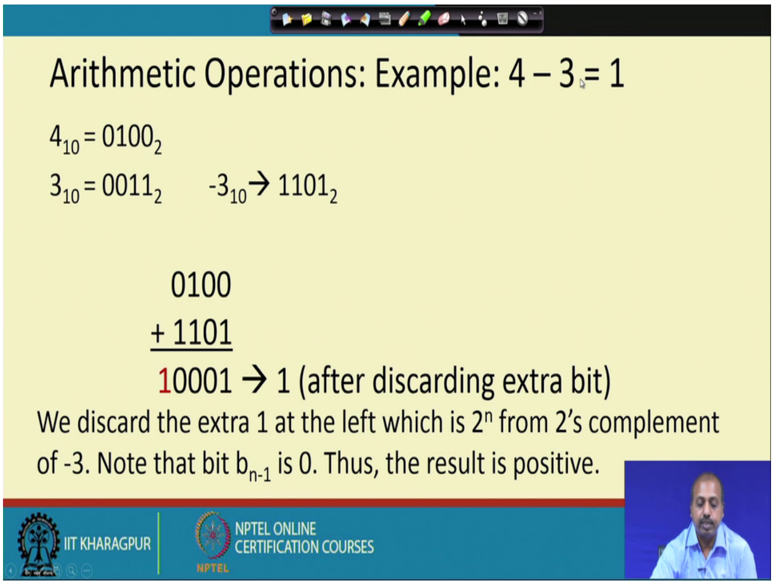

So, for example, this one is a 4 minus 3 is equal to 1 are we getting it by the formula that we are doing. So, 4 in 2's complement is it is a positive 4. So, this is 0 1 0 0 3 is 0 0 1 0, now I need minus 3. So, 2's complement representation of that is 1 1 0 1.

Now, if I am adding now I am adding this, ok. So, this is the addition and this is the result ok. So, after doing the addition, so, if we discard this since the result is. So, a since this so, we discard this extra one at the left. So, this extra one at the left is discarded. So, which is 2 to the power n from 2's complement of minus 3. So, and this b n minus 1 bit is 0 so, the result is positive. So, I will get a positive number 0 0 0 1 so, this bit can always be neglected.

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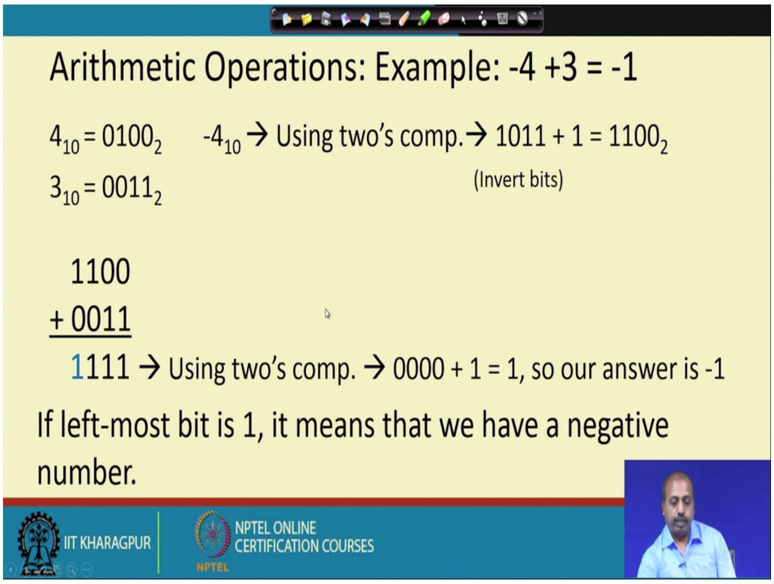

So, we will see with another example like say minus 4 plus 3 that is equal to 1. So, how is it happening? So, 4 is 0 1 0 0 so, minus 4 in 2's complement representation is; so, this is 1's complement plus 1 so, 1 1 0 0 3 is 0 0 1 0. So, 1 1 0 0 plus 0 0 1 0 so, this produces this, ok.

Now, there is no carry produce here to be neglected so, that carry is 0. Now you see that since this bit is 1 so, this means that this is a negative number and if we want to get the original the decimal numbers. So, what you do is you take the 2's complement of this. So, this is 0 0 0 0 plus 1 that is 1 so, our answer is minus 1. So, if the left most bit is 1, so that means, it is a negative number, and if you convert it if you convert this into it is 2's complement. So, you will get the corresponding positive number. So, here I am converting this 1 1 1 1 into 2's complement. So, I am getting one so, which is the corresponding positive number.

So, in this way we can convert so, I never done any subtraction operation; though, it was necessary that I will be doing a subtraction operation of 3 minus 4, but that was not necessary.

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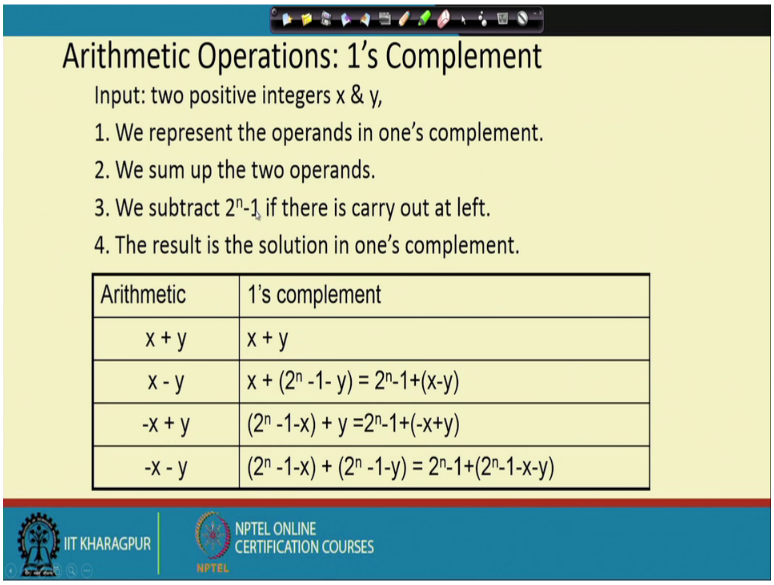

What about to 1's complement operation? So, 1's complement operation is a bit complex. So, here we have got if we have got 2 input integers x positive integers x and y. So, first of all we represent them in 1's complement, we sum the 2 operand, and we subtract 2 to the power n minus 1 if there is a carry out left. So, in case of 2's complement we did not have to do anything. So, if we represent a 2's complement, then sum up. So, it was over, but in 1's complement it is not over at that point.

So, if there is a carry, then you have to subtract 2 to the power n minus 1, and the result will be in 1's complement formula. So, this x plus y is x plus y x minus y is x plus. So, this is 1's complements, and if then you are getting this thing and after that if there is a carry, then you have to subtract 2 to the power n minus 1.

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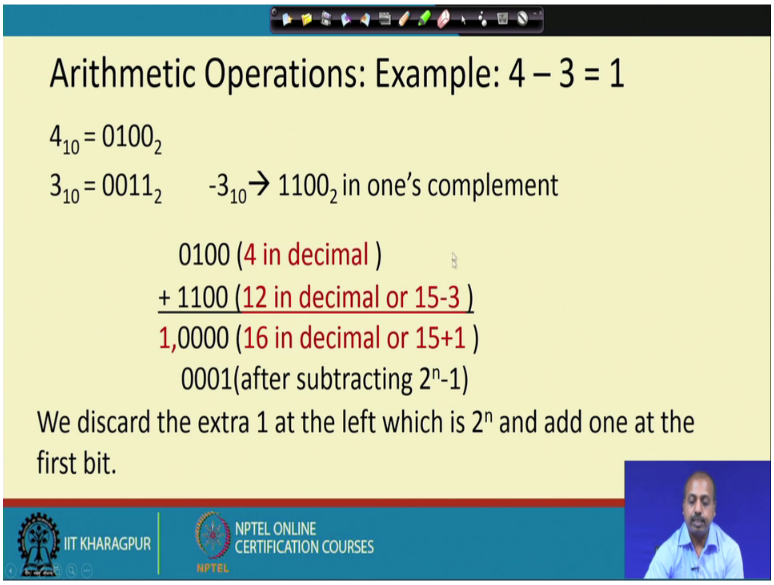

Let us take an example suppose 4 minus 3 ok. So, 4 is 0 1 0 0 3 is 0 0 1 0. So, minus 3 in 1's complement is 1 1 0 0. Now we are adding this so, after doing this addition you see there is a carry. So, carry one is generated so, it says that to get the result you have to subtract 2 to the power n minus 1. So, 2 to the power n minus 1 is. So, you have to subtract 15 from here so, after subtracting 15 you so, this value is 16. So, 2 to the power n minus 1 is 15 so, from 16 I have to subtract 15 I will get a one.

So, we discard the extra one at the left which is 2 to the power n and add one at the first step. So, this is the process of subtraction. So, you discard this so, that way your loosing this 16. So, your subtracting 16 and then your adding one. So, essentially your subtracting 15 so, essentially we are subtracting 2 to the power n minus 1 that is 15 in this case.

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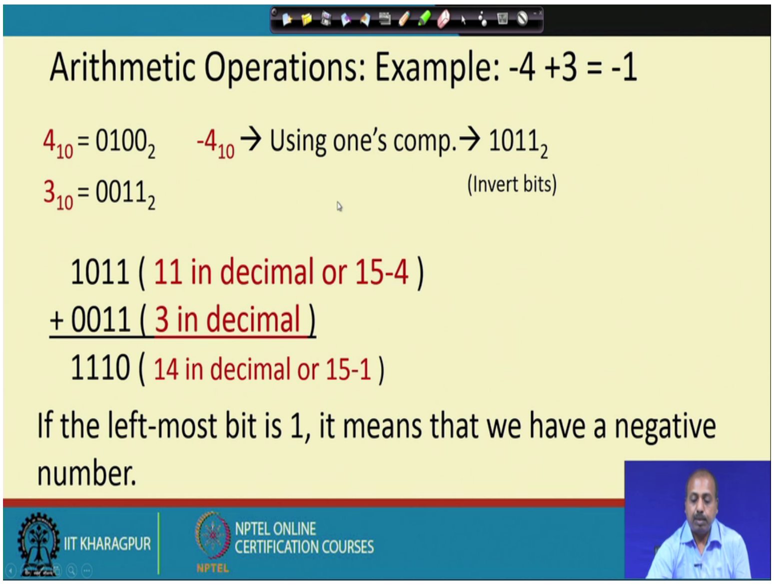

So, another example so, minus 4 plus 3 is minus 1 how is it coming. So, 4 is 0 1 0 0 so, minus 4 1's complement is 1 0 1 0, 3 is 0 0 1 0. So, you are doing this addition so, after doing this addition it comes 1 1 0 0. Now so, there is no carry generated. So, I do not have to do anything so, know the 2 power n minus 1 subtraction that is not necessary. So, it is the so, these itself is the number that I have got the, but it is in the 1's complement form. So, to get the actual number, what you have to do is that you have to convert it to 1's complement that is 0 0 0 1. So, that is one, but since this bit is one so, it is a negative number so; the value that you get is minus 1.

So, if the left most bit is one so, you were getting a minus 1. So, the rule is so, the rule is this one that after doing the summation if there is a carry, then you subtract 2 power n minus 1. Then either case the result will be available in 1's complement notation. So, this way in the in the in this example there was a carry generated. So, we had to subtract 15, and for subtracting 15 the clever way that we did is we ignore this ones. So, as a result I have subtracted 16, and then I have added I have turned on this I have added one at the least significant position.

So, that way I have made plus 1's so, minus 16 plus 1 so, actually 15 is subtracted. So, in this case since this carry was generated so, I have to subtract 15, but in this case we have got there is no carry generated. So, no subtraction is necessary so, this result is fine, but only thing is that to get the corresponding value what it to understanding what is the corresponding value. So, we have to convert it take it as 1's complement you take the 1's complement of this so that is one and since this bit is one. So, this is a negative number so, it is minus 1 so, minus 4 plus 3 is minus 1.