Analysis and Design Principles of Microwave Antennas

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Lecture - 01
Concept of Vector and Scalar Potential

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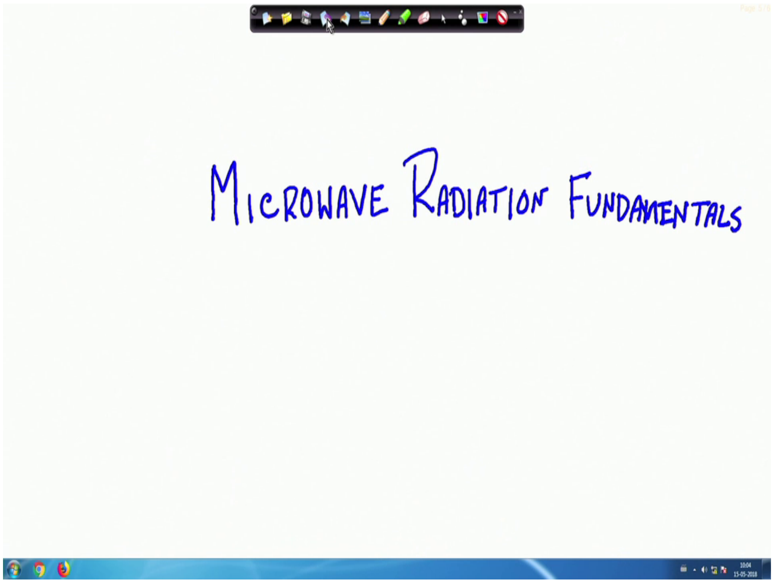

Welcome to this course on Microwave Antenna Design and Principles; now the first thing that we will see is microwave radiation fundamentals. So, you know that if we have a time varying current time varying electric current, then Maxwell's law predicted that there will be radiation. So, any structure that supports time varying electric current that will radiate, but that is not an antenna because that radiation may not be efficient; what do you mean by efficiency of radiation? That means, whatever power we are giving the whole thing should be or sizable amount of that should be radiated to the free space; that means, that there should be proper impedance matching; we know the free space impedance is 377 ohm.

So, if the impedance looking at the antenna port is not compatible to this 377 ohm, then the radiation of the power will not be radiated much there will be reflection of power. So, that is why one task of antenna is to efficiently couple the power to the free space, another job of antenna is to direct the radiation in the proper direction, because we do not want to radiate everywhere we want to radiate where we want it to be received by someone else.

So, these two properties the directive nature of radiation and impedance coupling this is antenna bring. So, we can design the structures, so that efficiently do this and then we call that is antenna. Now problem of antenna engineering is two fold first we know that what type of current density on the structure will produce what fields? We know that if there is a time varying current then both electric and magnetic fields they coexists, you cannot separate them.

So, these everywhere in space we want to find what is the E and H field, this is called the problem of antenna analysis. That means, given if we know the field if we know the current on the structure we should be able to predict what is the electric and magnetic field everywhere in this universe. But there is another problem that suppose I want that this should be the type of field, I want that at this direction there should be. So, much radiation compared to that in some other direction there should not be any radiation compared to that in some other direction there will be some suppose 30 percent of radiation of the maximum.

So; that means, if I specify that radiation I want this type of directive nature, then how do we design the antenna? That is called antenna synthesis problem. Now mainly in this course we will be first do the analysis there after we will touch briefly the synthesis problem. So, we first start whether Maxwell's equation, because Maxwell's equation you know that summarizes basically the behavior of electromagnetic fields when sources of excitation are present those are time varying sources.

So, and also we have Fourier analysis. So, we know that any electrical signal can be broken into various a super position of sinusoid. So, if we give a sinusoidal time variation; time varying current to a structure then let us first write the Maxwell's equation.

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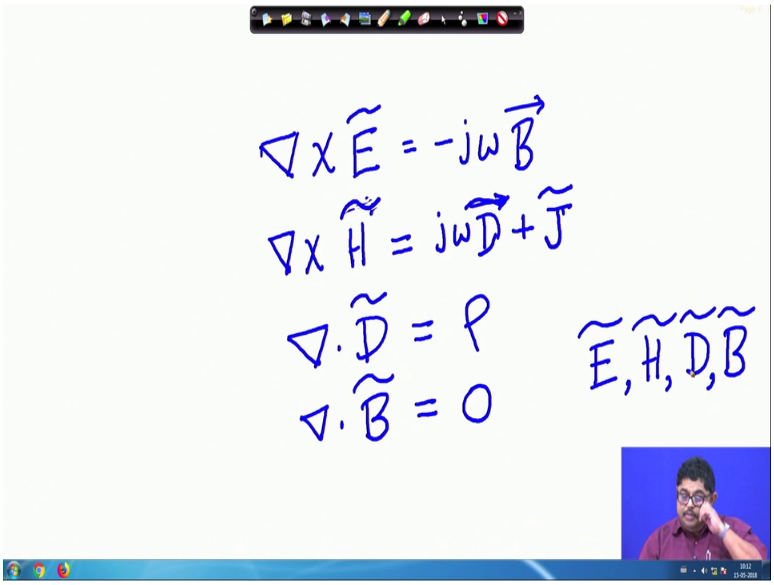

We know that Maxwell's equation are the curl of the electric field phasor, that I can write as this omega is the excitation that is coming in the sinusoidal frequency sinusoidal excitation frequency, then we know that curl of the magnetic field this is phasor. So, then we also know that the divergence of the displacement vector that is given by the volume charge density and we know that divergence of the V vector that is 0.

So, these are the four Maxwell's equation, but you know that this here we assume that we know the sources, sources here is the current density vector j and the charge density rho. So, if we know this we want to find what is E and H everywhere, but here you see there are various quantities we have the electric field, we have the magnetic field H, but we also have two more unknown quantities one is D, another we have B.

So, this is the electric flux density this is a magnetic flux density. So, there are four equations, but we are interested in EH. So, can we eliminate DB? Yes we can because we have constitutive relations.

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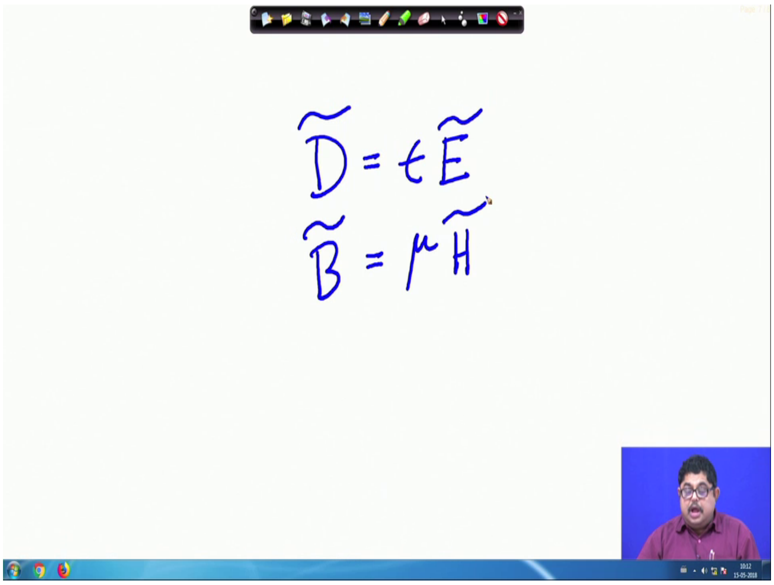

So, with the constitutive relation, we know that D is related to for simple materials D is related do the electric field and B, the magnetic flux density vector is related to the magnetic field by the constitutive relation.

So, now we want to go back and eliminate DB, so but here you see there is a problem that even if we eliminate B from here by using that constitutive relations. So, there will be H coming here, so the curl of E is specified in terms of basically the magnetic field H similarly, the curl of H is specified in terms of this D I can replace by E with the constitutive relations. So, curl of H is in terms of the electric field so they are coupled together.

Unless and until I cannot solve them because, I want that the left side should be unknowns, right side should be known that is not here present because both E and H they are unknown to me, but they are present both in the left hand side and right hand side. So; obviously, there are J rho also I will make use of them, but I need to now do something.

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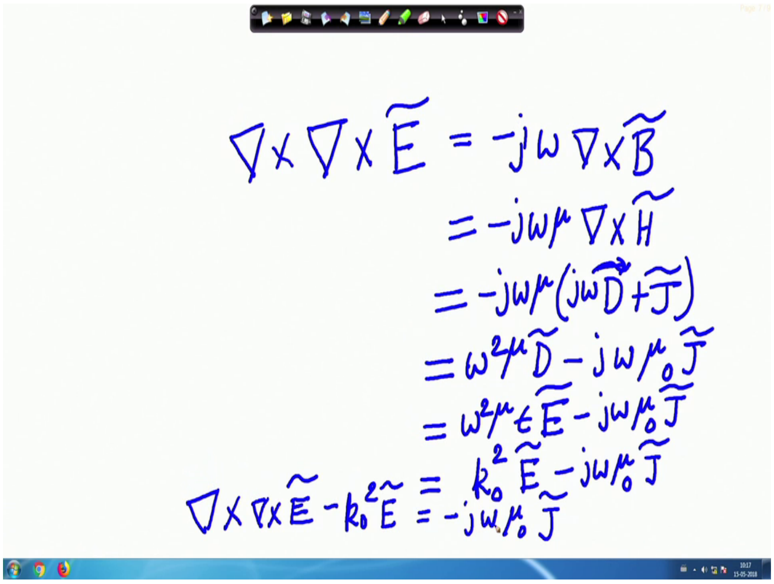

So, that we do by actually you know that if we take the call of the first equation then we get so that will be del and here we can put the constitutive relation ok, then del cross H was there in the second Maxwell's equation. So, I can put that minus j omega mu then in place of that I will put j omega sorry. And then again by putting the constitutive relation; I can write these has omega square, mu D, again I am using constitutive relation. And now this omega square into mu into epsilon this is a constant quantity for a given source of excitation and given medium.

So, that we generally denote by a constant called k naught, so k naught is web number naught is for free space I am writing. So, now you will see that this equation here simply if I take this k naught square E here. So, I can write del cross del cross E minus k naught E is minus is j omega mu 1 j. So, this is amenable for solution because you see left hand side is unknown e and right hand side is the unknown source quantity the current density that the structure is supporting others are all constant I know; so, I can theoretically solve this.

So antonym analysis is from this and once I know E I can put that into Maxwell's equation and find H. So, that would have made the whole antenna engineering is can be solved by this, but actually if I want to go and solve me because this is a vector relation. So, if I want to do it will be problematic because the j and E they are not so well related mathematically; I will find some problems in many of the cases.

So, in antenna engineering we uses alternative route we use the concept of potentials to simplify the solution. Here I recall that if you remember in electrostatics also the Coulomb's law that gives us that charge density and electric field. So, that can be related, but many times we do not want to we are at problem to find that. So, there we since electrostatic fields where conservative we define the concept of potential which is a scalar quantity.

And which many times simplified because looking at the charge distribution there we could have found potential and fund potential which would have always find that the gradient of that potential that is the electric field. Similarly, here also we bring the concept of potential; concept of potential is a fictitious concept, but that reaches the solution because instead of solving this equation we will get solving through the use of potential. So, that will be the topic of this first lecture mainly that what is the concept of potential?

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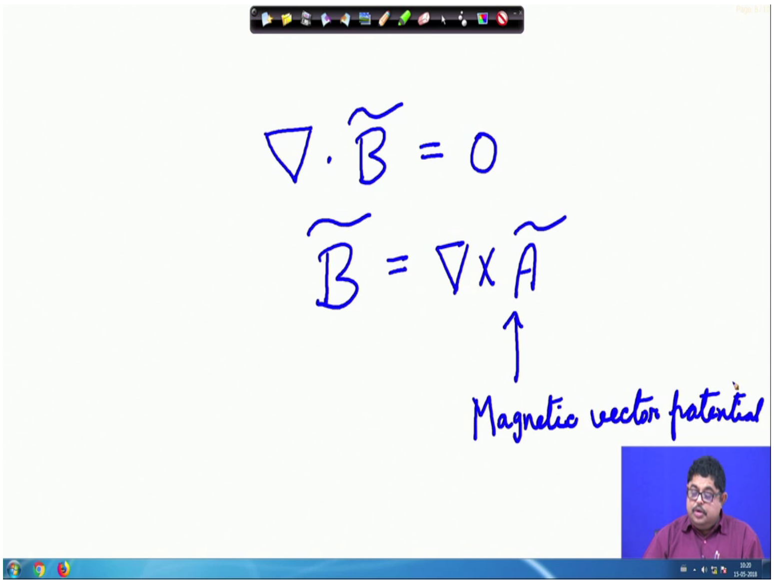

So, again we look at the fourth Maxwell's equation that divergence of the magnetic flux density is 0. So, this was Maxwell's fourth equation from there from our knowledge of vector analysis; we can say that B then should be expressed curl of a vector quantity, because we know that divergence of curl of any vector is 0. So, we can say that B should be curl of a vector and these vector we call this is a vector because curl can be taken only with a vector field. So, this is called magnetic vector potential ok, so if we put this into Maxwell's first equation which in earlier cases I solved.

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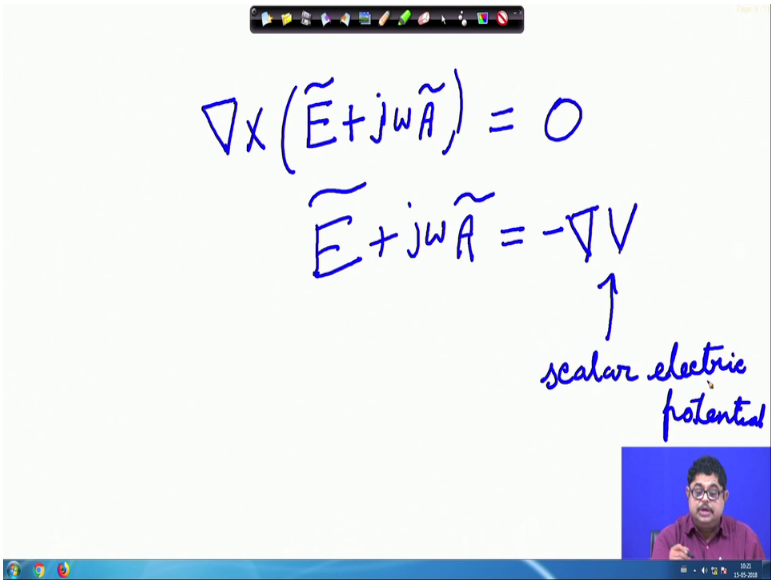

Again I will see that that if I put it into Maxwell's first equation the del cross E there, if I put and change the size you see that; I will get these equation and these equation say that curl of something is 0. So, again from our vector analysis we can then say that this quantity; this should be then gradient of a scalar field V. So, this V is a scalar this is another potential function so this one is called scalar electric potential.

This is scalar that is why it is scalar; it is coming from the definition of electric field that is why electric potential. And you see the previous one was magnetic vector potential the magnetic was coming from this because this definition was coming from magnetic field definition, that is why magnetic vector potential. Now, with that definition both these vector and scalar potential; if we put to Maxwell second equation because I have already used first equation and fourth equation; I am now putting it to the magnet second equation.

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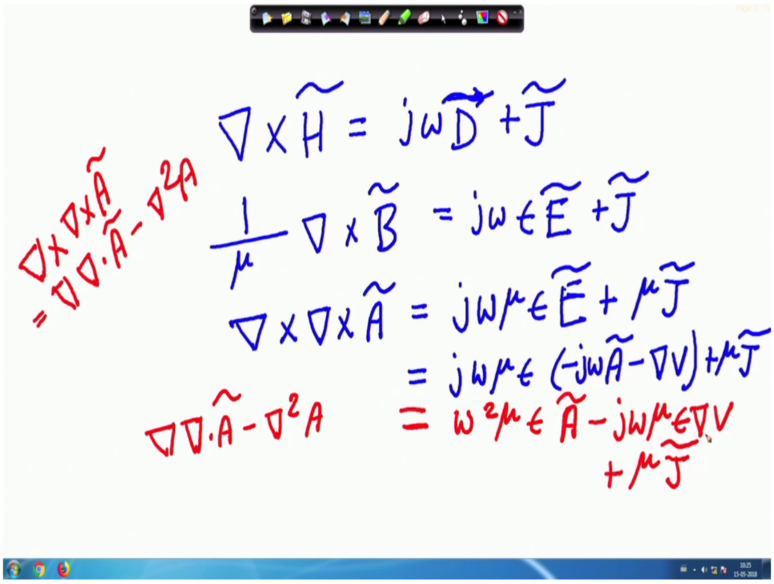

Which is Ampere's law or Maxwell's modified law, so there if I put I am again writing that equation; so, that you understand that now constitutive relation I am making use and here again we have already seen the vector potentials. So, here I will put the vector potential; magnetic vector potential in place of B. So, I will get j omega mu epsilon E here I will put the vector scalar potential V.

Now, we can use the vector identity del cross del cross A is del this is Laplacian, this is gradient of divergence. So, if we use this then what we get in this side I will write that del del dot A minus del square A is equal to this quantity. So, here if I simplify; that means, we open the bracket, I get omega square mu epsilon A minus j omega mu epsilon del V plus mu J ok.

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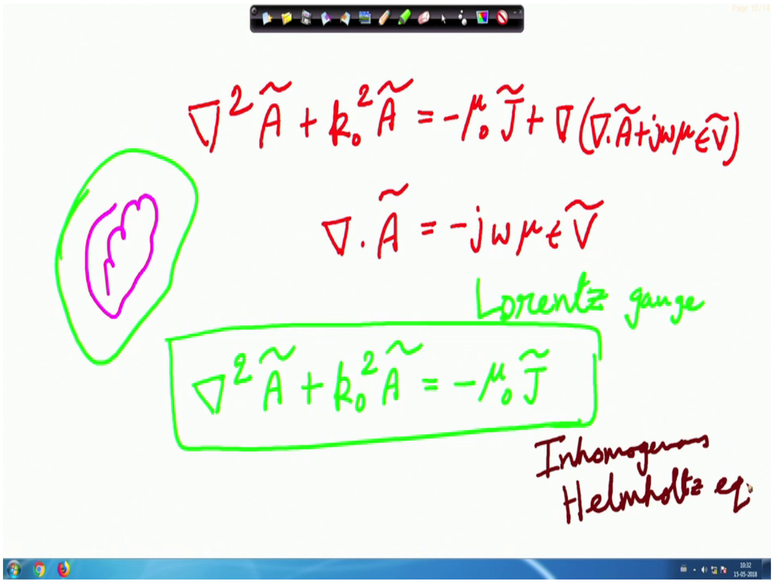

So, here I can now put this that if I just change sides I can write it as. So, you see that accept these portion; this portion I will have to do something otherwise I have got a familiar 1. And this one now there is a theorem called Helmholtz theorem which state there any vector or any yes vector field that gets uniquely specify, if we specify its divergence and curl. Actually that is why Maxwell's equation contains divergence of E field and curl of E field divergence of magnetic field and divergence curl of magnetic field four equations are there.

So, that electric and magnetic fields gets specified; similarly here we have already put for the vector potential A we have defined its curl, but we have not defined its divergence there is a chance we can put these divergence to choose divergence in such a way; so, that this terms becomes 0, so that is why we can choose the divergence of A to be minus j omega mu epsilon V.

If we do that this condition this condition is called Lorentz gauge condition. So, Lorentz gauge condition is this with this we come to our, so these equation was well solved instead of solving the electric field equation directly, we will solve this equation and this equation has a beauty you see that Laplacian of a vector that is a scalar. So, the what is the direction of A? Direction of A direction of A is simply the direction of J.

So, the if from some measurement or somehow if we can for by physical understanding if you can find what is the direction of J, we are finding immediately that the vector potential A that will also have that same direction. That was not possible in the earliest case, if we have not taken the help of vector potential; then if you look at the electric field we cannot say that j is this direction so electric field is also been this direction.

Now that is the beauty that is why we have a difficult problem of finding n H that we have put a less difficult problem that let us find J. Now how that is easier? Because in any case here we will have to now find J finding J is; that means, what is the current on the structure we will have to find. Now that why we are considering that this is easier? Because, as I said that physical reasoning sometimes helps on, measurement helps also another thing is this structure.

Suppose I have a radiator, now this radiator suppose on this there are the electric electric current is like this, now and fields are there everywhere in the universe. So, instead of finding the fields; it is easier to find this is on a localized thing. Because this current is localized because the current the; this conduction current that will be only on this structure it will not be in the free space.

A free space cannot support current, so since it is localized; it is localized over the structure that us radiating that is why it will be an easier problem and in fact, it turns out that this is an easier problem, so we will solve these equation. Once we know A, we can go to the defining equation of A; by that we will find the magnetic field and from that magnetic field; we know we will put to the Maxwell's equation and find out what is the electric field; so this will be our journey.

Now, this equation you know this comes several times in all branches of engineering. This is wave equation and we know its name this name the first person who did it the Helmholtz. So, this is an Helmholtz equation only thing please remember that this is an inhomogeneous Helmholtz equation, because right hand side is not 0; it is the source. So, the source quantity will have to find or we will have to have the particular solution for this.

But the general expression of this we know and we will now write it that this is inhomogeneous Helmholtz equation ok. So, this is one step forward that inhomogeneous Helmholtz equation; now it can be argued that we had actually if we look at Maxwell's equation, you say in Maxwell's equation there are two source quantities, one is this J and another is this rho. So, why we have taken J; we could have taken rho also yes that is true that we can start from we can consider rho as the source also.

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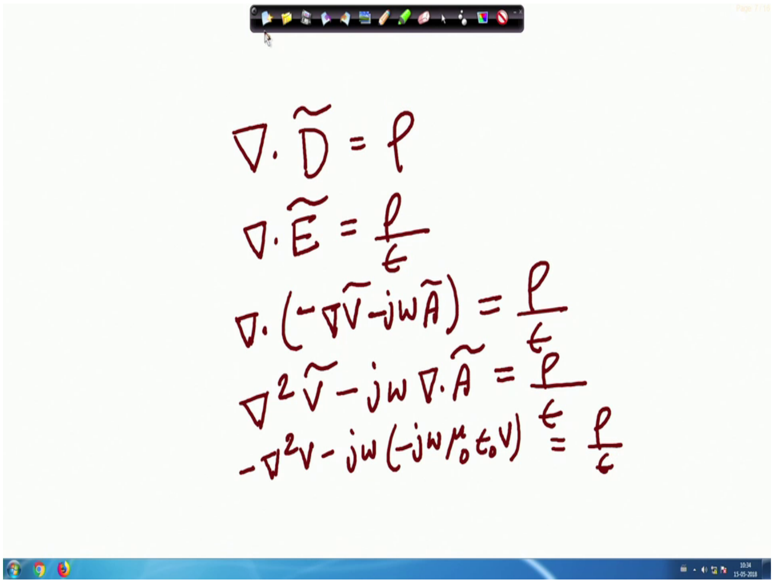

And that we know this is the Maxwell's third equation or Gauss's law and from here, I am hurriedly doing putting the scalar potential definition. I can write this as please remember in electrostatics this is simply minus del V. In electrodynamics it is this better potential also should be added here, because that was our definition if you remember. And then these becomes, all these are present quantities sometimes in hurry; this is I am putting the divergence that Lorentz gauge condition here.

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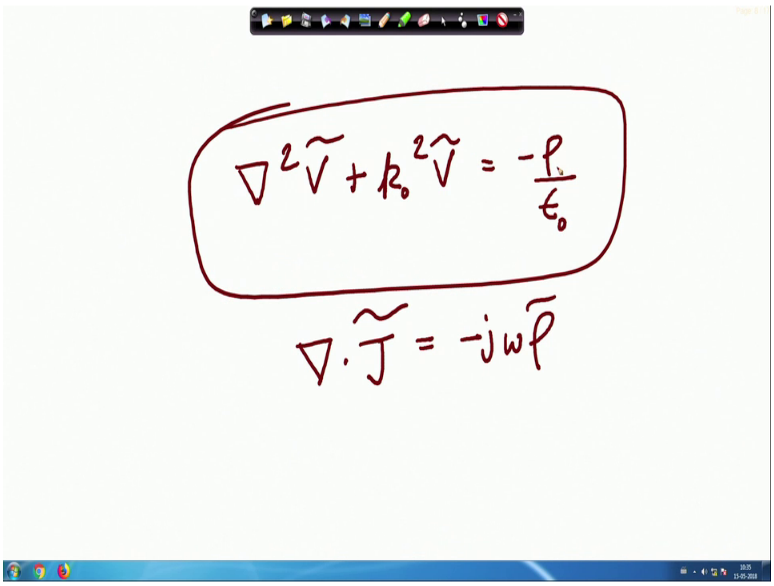

And by that it becomes del square V plus k naught square V; so you can solve this is also a equation. And so, from here also you can find then you can argue that this is a simpler equation because this is a V is a scalar quantity. So, I can find it, but the problem is finding rho and remember that for radiation we require a current. So, finding charged or by physically reasoning charge is not so easy; comparatively finding current is much more easy also these two equations are not independent.

The equation that involved magnetic vector potential A del square A plus k naught square A is equal to minus mu J and these both are wave equations in the form of inhomogeneous Helmholtz equation, but they are related who related them? Because we know that J and rho in an time varying speed they are related by the continuity equation which is del dot J is equal to minus rho del del rho del t that is j omega rho.

So, there is nothing new here, but finding rho over characterizing rho for a radiating structure is not so easy comparatively characterizing the J is easier that is why we do it.

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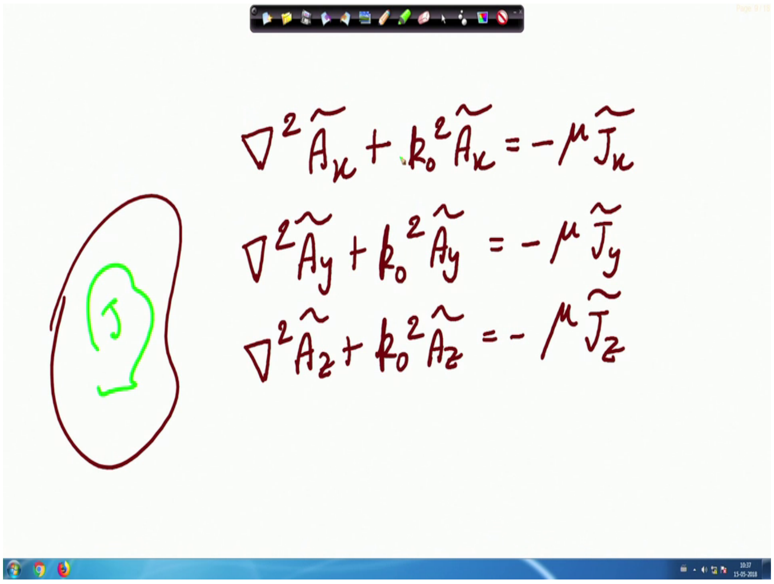

Also, I say here that actually the inhomogeneous Helmholtz equation in magnetic vector potential that boils down to three scalar equations. So, these three scalar equations so if I can somehow break the on the structure that time I am shown that suppose this is our current J. So, if I can break it into in rectangular coordinates z j y j z then I will have to solve these scalar equations. So, in any case we will have to solve this and these solution will give us into that what is the how to find E and H for the antenna case is that will be taking up in the next by taking a very conceptual current distribution.

Thank you.