Analysis and Design Principles of Microwave Antennas
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Lecture – 17
Introduction to Antenna Array

Welcome, to NPTEL lecture on Array Antenna. Actually, we were seeing wire antenna; some of the wire antennas like dipoles, monopole folded dipole etcetera we have seen. The logical next step would be to see an antenna which by area radiates power that means an aperture type of antenna.

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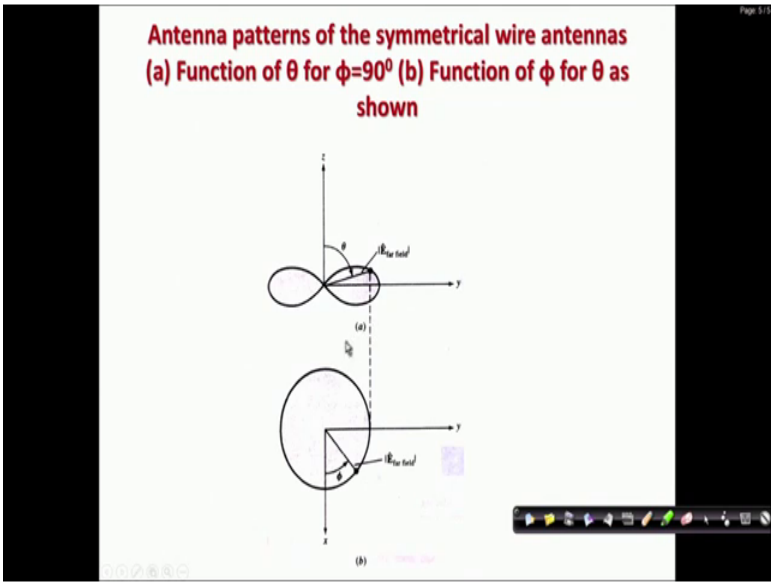

But, before that in these wire antenna discussion we want to point out one thing if we look at the wire antennas that we have seen till now the pattern wire antenna pattern looks like these. You will see that in the E field or in the theta play theta direction we have a directed beam, but in the phi direction or in the azimuthal plane, we have a non directional pattern. Now, that has some problem, problem or actually eh the wire antennas that we have seen all wire antennas are symmetrical now symmetrical from the field point of view and for that this is natural that in the azimuthal direction the there will be no directional nature in the radiation pattern.

Now, that is very good for broadcasting type of antennas because these wire antennas are they may be used for radio broadcast or TV broadcast sort of applications, but the moment point to point communication proliferated with the help of this cellular telephone etcetera another problem came that if I have a in the azimuthal direction; that means, I have an antenna now compared to the antenna axis in a perpendicular direct plane I have an uniform pattern then if there is an another source of radiation or another transmitter nearby, then there will be lot of interference coming.

So, unless and until I can shape these there will be problems because there will be long interference. Like the problem we are saying suppose Calcutta radio station is radiating and nearby there is Dhaka radio station. Now if Calcutta radio station does not put a null in the direction of Dhaka radio station, then Dhaka radio stations inference will be there. So, you can see to the next slide.

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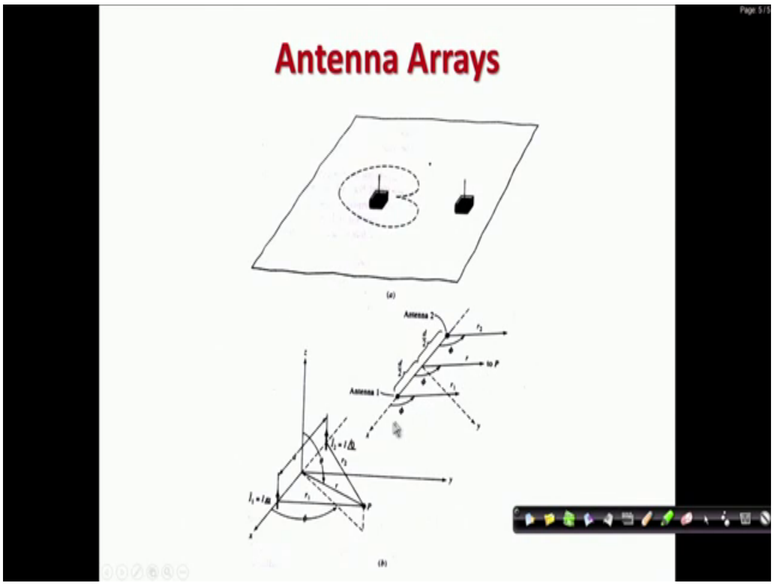

This is the problem that, suppose I have one of the this is one transmitter if there is another transmitter then it is required that in that direction I should have a null, but that is not possible with a wire antenna that we have seen till now because of their this azimuthal pattern being, that is why we have seen that the radiation pattern does not have a theta dependence radiation pattern is fully sorry radiation pattern does not have a phi dependence for the antennas that we have seen.

Now, only to rectify these you see if instead of a single antenna we use two antennas or more antennas more number of antennas then this problem can be rectified. That is why we will see that basically this communication antennas they are sealed in a box. If you look at the, this cellular telephonic base station antennas, you will see that they have a long box. Actually if you open the box actually that box is nothing, but it is a sealed thing, if you open the box there will be antennas they use something like patch antennas, but there are an array of patch antennas. Instead of patch antenna they could have used dipoles also. So, in some of the stations you will see number of dipoles together. So, that array antenna is to shape the pattern, so that the interference etcetera can be avoided.

Now, let us look at these first will what will do? We will now take what happens if two array antennas we place together; that means, let us say that the antennas are still in the z directed things suppose this is one of one dipole, this is another dipole or this is one monopole, this is another monopole etcetera and so, they are along the x-axis the antennas current is in the z directed and they are separated by a distance d and we are observing what is happening in the azimuthal plane; that means, in this in this P direction some arbitrary point P.

So, the center of this two antennas thing actually here now the antenna axis will be out of the array axis will be called the x-axis. Antenna is z directed, but antennas are radiant along x-axis. So, the x-axis is the array axis the center of that point that is the center of the coordinate also. So, the field point is P. So, center of this array axis that the radius vector of point P is r.

So, this antenna if I from this antenna is at a distance of r 1 from point p, this second antenna i 2 that is at a distance of r 2 from the field vector P and if we project this P in the xy plane then that projection has an angle phi; that means, this P is at an angle phi and also from here theta. So, this P is at a distance r theta phi, ok. So, now, we are interested that what happens is to this azimuthal or to the whole field pattern if we at the point P. So, we will first evaluate the field that point P this is the same view point. Obviously, we are interested that P is at the far field.

So, you see in this structure we have now just took a snapshot that at the axis antenna 2, though it is an wire antenna, we have meditate point just for the drawing sake antenna 1, antenna 2. Antenna 1 is at a distance d by 2 from the center, antenna 2 is at a distance d by 2 from the center. So, the inter antenna or inter element spacing is d this is point P.

Now, P is at far field. So, r 1, r 2 all are much larger then d. So, that is why they are all parallel rays and r 1, r, r 2 so, they are actually in the numeral in the amplitude part we can say that r, r 1, r 2 similar, but in the phase part we cannot do. So, we will have to evaluate what is r 2 and r 1 with respect to r. So, as we seen from these you can easily see that basically I can write r 1 is nothing, but r minus d by 2 cos phi and r 2 is r plus d by 2 cos phi, for the phase part so, that we will do. So, now, let us part finding what is a field.

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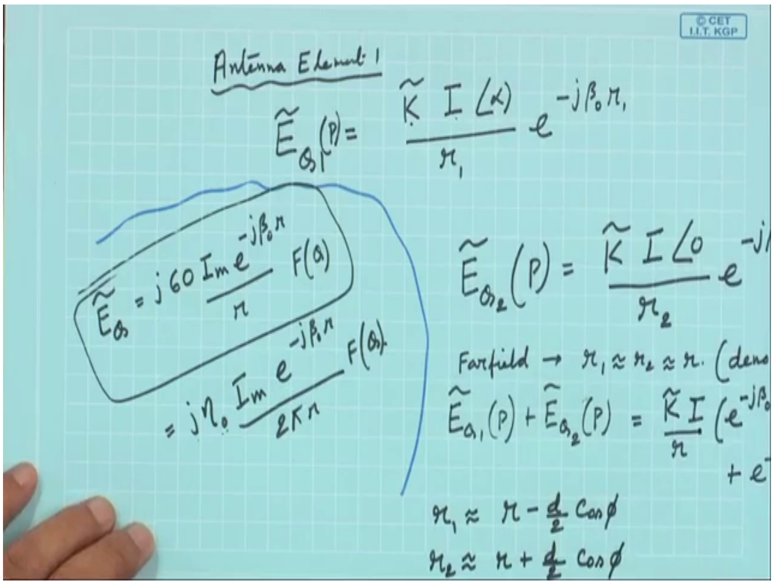

So, we know if it is a any wire antennas let us so, individual element suppose this antenna 1 or antenna element 1 antenna element 1. We have already seen that it is field far field can be far field is e theta directed. So, I can write that the far field far antenna 1 that can be written can I say if you look at that your actual expression that already we have found out these expressions earlier.

So, from there you can see that actually I am writing here the E theta for far field is something like j 60, I me to the power minus j beta naught r by r F theta. This was the expression that we evaluated I do not know whether we have seen it or at we can we have seen this one I think that this one we have seen. This becomes this if we put the value of this eta naught as 125, ok.

So, based on these I think I can say this as for the dipole. So, based on these I think I can write the E theta 1 the field at point P E theta 1 at point P due to the antenna 1 that can be written as some constant I. So, this K actually everything here you see j eta naught, this 2 pi and F theta everything I have put here; that means, K is actually then it is a constant and it is a function of theta. So, also that depends. So, you have included here because I am not interested to say how what is that theta dependency of the resultant field. I am interested to see what is a phi dependence of that field.

So, K has absorbed all the coefficients and what is I? I is the current fed to this antenna. So, if we see here these this antenna 1 that has an fed with current I 1 and it is phase is with respect to some reference phase is alpha. So, this excitation of these is a thing actual thing in array antenna this complex excitation that is current excitation antenna that is in our hand.

So, here we are exciting I 1 with a current I and angle alpha whereas, I 2 is excited with the same current we could have given different current, but it has said to start with simple thing same current amplitude I, but angle is 0 with respect to I 2 I 1 current leads by an angle alpha. So, this current I alpha and the distance is r 1.

So, this is the far field the first antenna element the second antenna element can be written as I 0 r 2 E to the power minus j beta naught r 2. Actually this is true for any wire antenna instead of dipole also you could have taken. So, these thing only sometimes the terminal currents of the non resonance type of dipole they here these are the terminal currents or this maximum current which occurs at the field point in their case the maximum may not occur at the field point so, that by some constant relation they can be eh extrapolated ok.

Now, so, how this excitation is done you see that both are excited with the same current amplitude, but with the different phase; that means, this look at here this I 1 and I 2 we can feed with the same source, but between I 1 and I 2 in the path, there should be a phase shipping network. So, that these current is shifted by angle alpha that is the only trick. So, there should be a feeding network in the line feeding one of the things.

Now, other things are you see the far field we have seen the that is dependent varies as 1 by r, 1 the distance from of the observation point from the source and also it has a phase space phase E to the power minus j beta naught r 1, ok. So, now, what will be the total field at point P? So, it will be E theta 1 plus E theta 2 P and that we can say that again r 1 and r 2 say we are at the far field. So, far field means that I can say r 1 is same as r 2 same as let us say r for in the denominator or amplitude part not here.

So, I can write K then I, then r then here I can write E to the power minus j beta naught r 1 and also I will have E to the power j alpha because of the current excitation and here I have E to the power minus j beta naught r 2, ok. So, this thing now I will have to put these conditions that in the far field r 1 will be r minus d by 2 plus phi and r 2 is r plus d by 2 cos phi.

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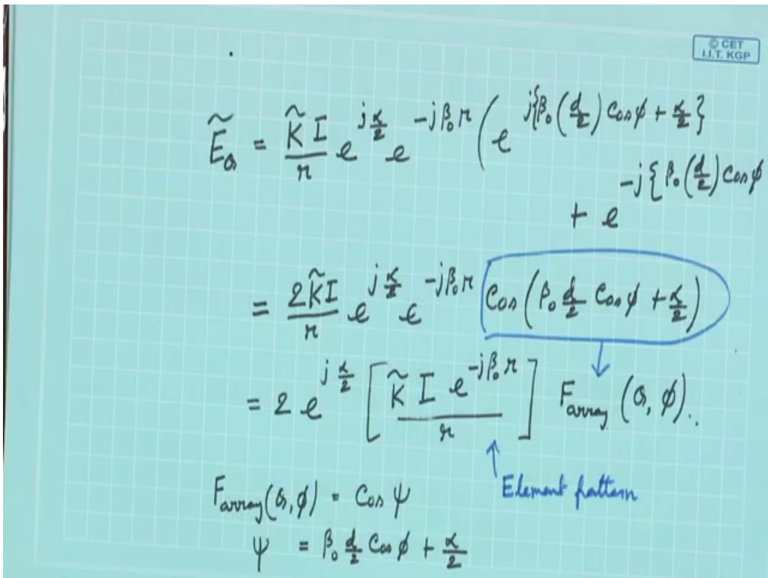

So, putting these I can write that E theta will be K I r e to the power j alpha by 2 e to the power minus j beta naught r e to the power j beta naught d by 2 cos phi plus alpha by 2 plus e to the power minus j beta naught d by 2 cos phi plus.

So, this can be. So, these from my convenience I can write it as 2 e to the power j alpha by 2 K I. So, you see that this one I have written that this whole part this part I am writing as an array factor previously you see in the dipole expression we have F theta actually this have to be small f. So, that this is from an element pattern. So, this is the theta variation here you see this thing is a function of only d and alpha. So, both are arrays parameter, that d is the separation between the two antenna elements and alpha is the progressive phase shift of the current of one of the antenna. So, apart from that all are constant terms. So, this is a called array factor capital F theta phi.

So, this is varying as the phi in general it can vary also with theta that is why we are writing theta phi what is this you see this is nothing but, can you recognize these? This was nothing, but our element pattern. This is the pattern of the array by the and one element you see it is far field. So, that means, now total field you see this is the constant part and if I take the magnitude of these then can I write that this is the element pattern? Obviously, this is a field pattern because this is E theta and this is the array pattern.

So, by arrays geometry am getting this pattern from the element am getting this pattern. So, what is E theta? E theta or the total this array antennas pattern is nothing, but multiplication of that element pattern with the array pattern. This is the general result this is called principles of pattern multiplication. So, the resultant field is the product of the pattern of the individual identical elements and the array factor.

So, this is principal pattern multiplication for two elements we have we are seeing these just later I may be in this lecture or in the next lecture will prove it generally, that for any type of array this is the general thing that element pattern and array patterns gets multiplied. Now, what was our interest our interest on the magnitude of each theta as a function of phi because we started motivated to have a array of two elements from there. So, let us see that.

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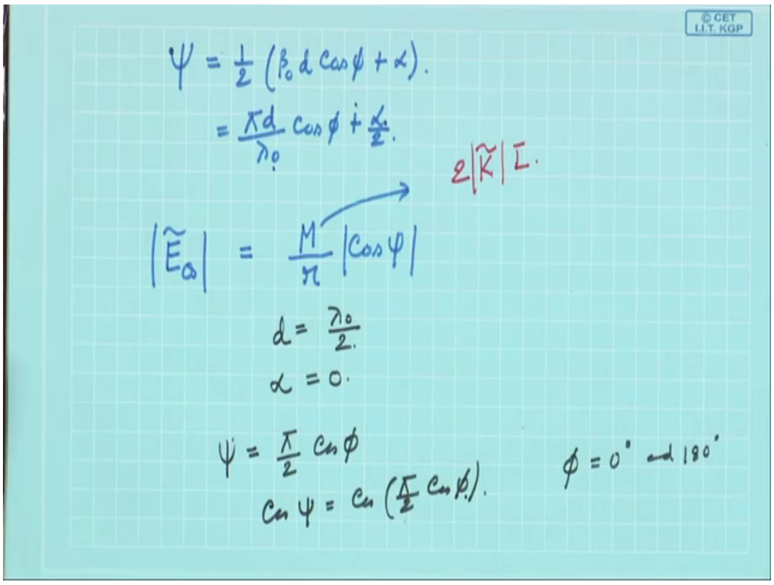

So, let me again rewrite the argument of the array factor; that means, this angle psi that was that was one very psi here psi ha. So, I can write half beta naught d cos phi plus alpha. So, by putting beta naught value, then pi d by lambda naught cos phi plus alpha by 2. So, you see what is the psi? This variable psi is nothing, but it is a function of the electrical separation distance of two antennas. d by lambda naught is the electrical separation distance and also it is a function of alpha, which I already explained as the progressive phase shift.

So, now, I can easily write what is the magnitude of the total field E theta that is can be written something like m by r cos phi. What is m? This m is 2 K I phi, some constant. Already seen this these are dependent on one is the excitation another is what type of element I am using the.

So, what will be the shape of the power pattern? Obviously, the shape of the because radiation pattern is basically power pattern so, in the far field we know the edge phi the magnetic field that will be some constant in to these; that means, eta naught into these. So, that means, square of this pattern will be the power pattern. So, this one you see this cos depending on cos phi. So, various values you will see these pattern will have various maxima and minimas in this pattern.

So, the pattern will be symmetrical about a line joining the antennas because in the x-axis the we have taken the two antenna elements symmetrical. So, that means, what is the adjust what is the adjustment we have the adjustment we have is that I can play with the inter element spacing. So, two elements what is the spacing that if I adjust I am changing basically this psi and from that this cos psi value is changing. So, the pattern will change, also I can adjust this value of progressive phase shift alpha and by that I can change the pattern.

Suppose, the antennas spacing I take as for an example, that let me take d is equal to lambda naught by 2 half wavelength long, half wavelength distance separation and let us take that alpha is 0, no phase difference both the things are fed in phase. So, obviously, physically we can say that if there is no phase difference then at point P there will be the maximum of the thing because r 1 and r 2 are symmetrical if we again look at the this (Refer Time: 26:36).

So, if I have no phase difference that means, alpha is 0. So, it will pick at this point because this pattern also picks at a point here this one picks at a point here. So, here there will be pick let us see that for these case what is becomes the value of phi? Phi becomes you see pi by 2 cos phi. So, now, the pattern will be the array factor becomes cos psi means cos pi by 2 cos phi.

So, for a fixed distance r the this cos phi or this pattern will have null at point when pi by 2 cos phi becomes pi by 2. So, when that can become so that means, for values phi is equal to 0 degree and 180 degree there are nulls.

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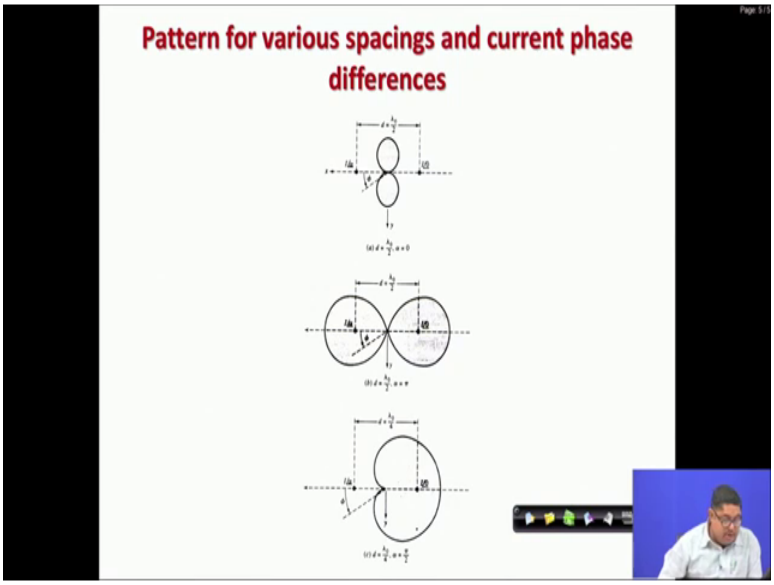

So, you can see that this is the pattern this first one you see we have taken d is equal to lambda naught by 2 and alpha is equal to 0. So, you see that this is the array axis, so, this is 0 degree; that means, here there is a null this is 0 degree, this is 180 degree because this angle 180 degree. So, here also we have a null in the pattern.

Now, where is the where is the maxima?

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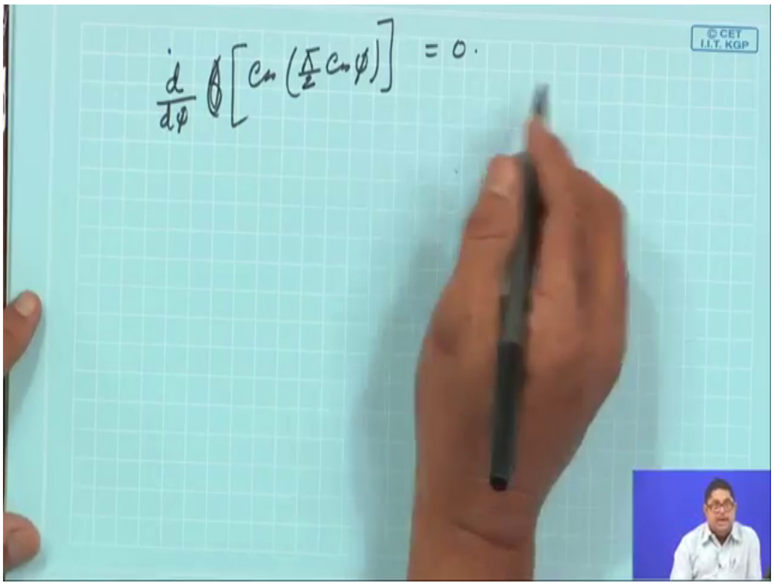

So, maxima you know that we can find out the maxima by differentiating this pattern with that cos, sorry, d by d phi cos pi by 2 cos phi you can do. Now, this can be minima maxima that you can check whether it is minima.

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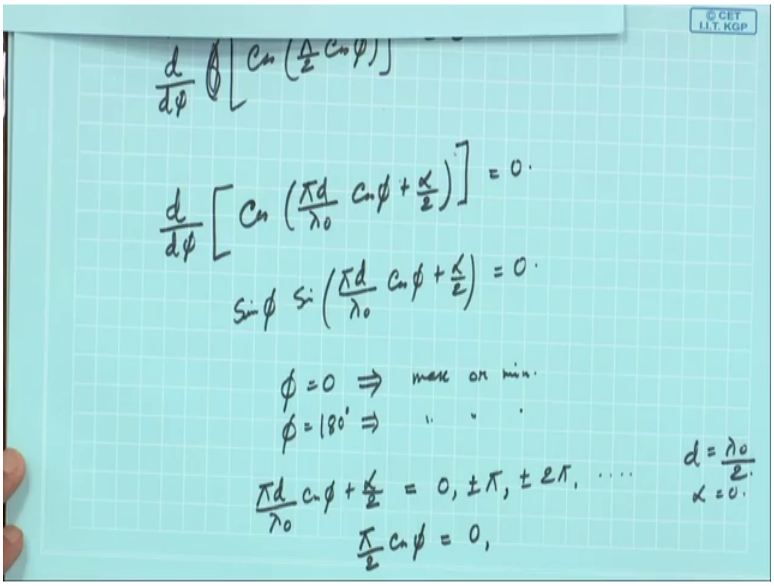

So, or in general if we do that that because this is a special case and come here that in general let us do d d phi cos pi d by lambda naught cos phi plus alpha by 2 is equal to 0. So, this if you do it becomes sin phi sin pi d by lambda naught cos phi plus alpha by 2 is equal to 0. So, the one of the solution is sin phi 0. So, that means, the phi is equal to 0 gives you either a maxima or minima.

Similarly, phi is equal to 180 degree also max or minima and this other condition gives you that pi d by lambda naught cos phi plus alpha by 2. This will also give maxima or minima at 0 plus minus pi plus minus 2 pi. So, you can put the value of our thing and suppose, if we put d is equal to in that particular case d is equal to lambda naught by 2 and alpha is equal to 0. So, you can put pi by 2 cos phi is equal to 0 gives you that. So, phi is equal to pi by 2 and if you check this will be a maxima which happens here, you see pi by 2 is this maximum is at pi by 2. So, this is pi by 2 this is eh you can say minus pi by 2 or plus pi by 2. So, there are so, this.

Now, you see that if you go on these instead, suppose I take lambda naught by 2 and alpha is equal to pi; that means, progressive phase shift pi. You see instead of the pattern picking in the perpendicular direction to the array it picks along the array axis. Actually this will prove later, this type of array the first type is called box side array where the maximum takes place perpendicular to the array axis. This is an example of an end fire array where the maximum takes place at the along the axis, ok, just by changing the progressive phase shift instead of no phase shift we have given a 180 degree phase shift between the two element and that gives you end fire.

Similarly, you can take progressive phase shift pi by 2 distance pi by 4 here you see the pattern is here. So, there is a null. Suppose, if there is a null there is an interference from this direction you can put a null here other direction almost circular type of thing. So, this also helps to do an actually you will see that by using these and principles of pattern multiplication you can spill the beans. So, when we say that nowadays me mu antennas etcetera or an antenna has antenna array has 4 beams, 5 beams all these are become do from here, ok.

So, we would not pursue more because that term multiplication is general, but actually if we derive by the general principles of array antenna we can get much better views.

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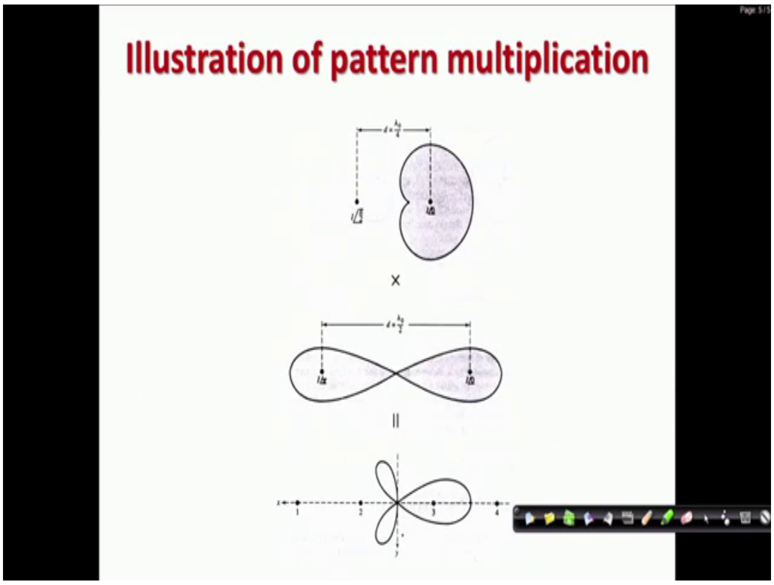

So, let me see, this is illustration of pattern multiplication you see from that if I have four such beams, suppose this is lambda naught by 4 and these are the two antennas if their separation is lambda naught by 2 this is an end fire type of it. So, now, if we take 1 2 3 4, four antenna elements and so, this one and this one they are separated by lambda by 4, this one and this one lambda by 4, this is lambda by 4.

So, you see there are two groups here; one is this 1, 2 and 3, 4 if I do like this. So, 1, 2 will give me a pattern like these, but this pattern will come here. Similarly, 3, 4 will come this pattern here and then that group it was inter elements spacing of these lambda naught by 2. So, there will be a due to the, that array that sub group array you can have these. So, this pattern will be multiplied by these and another one here. So, that will result in this type of pattern. There will be three such beams, ok. So, this, but the this is eh you can take the help of pattern multiplication, but more important this we can generally derive also if we have these (Refer Time: 34:56) thing.

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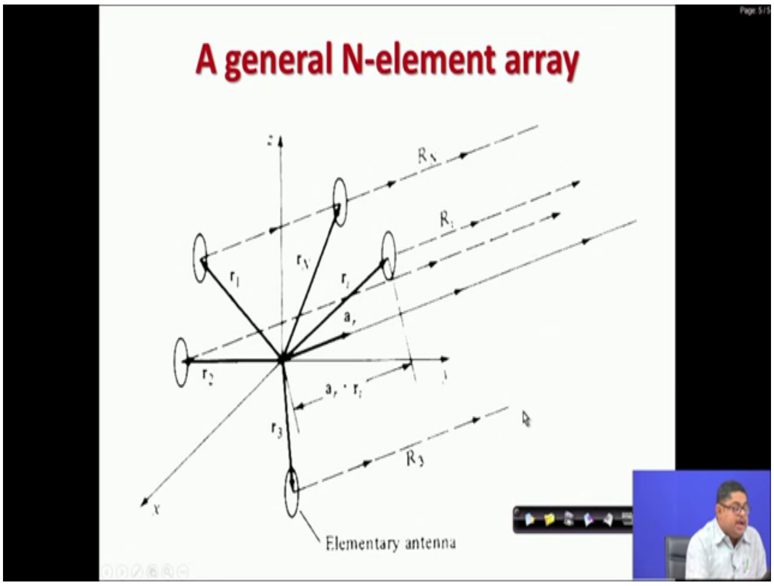

So, next we will start with N-element array. So, suppose I have these N number of radiators, all identical radiators, but at different distances from the coordinate system are available and I am looking at the far field. So, here there is let us say that this i-th radiator is along these, sorry, here i-th radiator is along these direction and so, the field point is here at P. So, this a r vector is the unit vector along the field point.

So, any i-th radiator that will have a difference from these a r vector; that means, this is a path, but suppose I am having these i-th radiator so, i-th radiator will have a difference of a r dot r i because that will be the a r is this, r i is this vector. So, this part; that means, cos of these a r r i cos of this angle that will be the path difference between the two, ok. So, with that suppose if I have r 3 then it will be a r dot r 3 a r dot r 1 etcetera. So, I can also tell you because the figure is not so good this is taken from Collins book.

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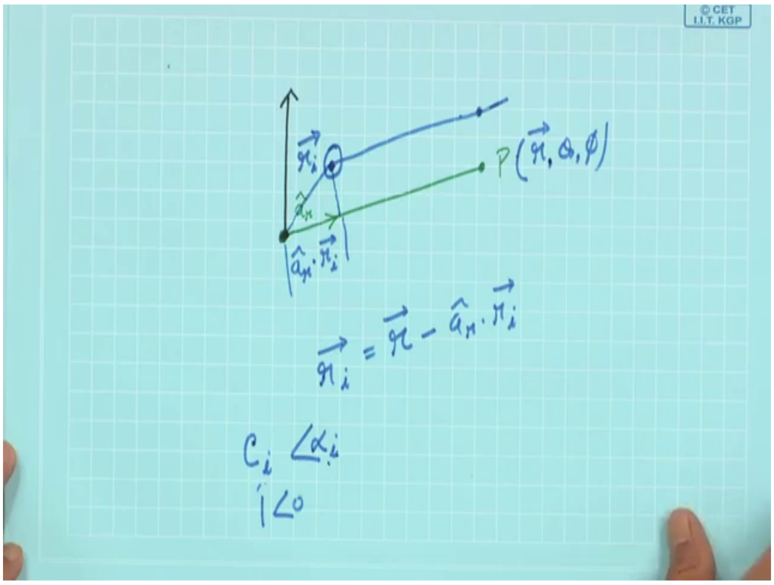

So, I can say this the same figure what I mean is this is the coordinate origin let me say this is the point P. So, along point P I have an a r vector unit vector from these direction and let us say that I have a radiator here. Here is an current element let me call it r i. So, this will have a path length like this will have a. So, this path at this point how much is the difference between this path length and this path length? Can I say that this is nothing, but this difference and that difference is what is the value of this difference? This is nothing, but a r dot r i that is what I was saying.

So, that means, when will write what is path for r i will write it is r minus a r dot r i this is the r i is equal to r minus a r dot r i this much I was trying to prove actually here the figure did not come nicely that is why, but otherwise if I look at here. So, all these are elementary antennas I have N such elementary antennas to start with I am distributed them haphazardly at each I know their coordinates etcetera. So, I can find at the far field the total field will be for position of all of them.

And, also there is you know that apart from this geometry there is another important thing that every antenna element is getting excited. So, let us call that the excitation actually this excitation is a complex excitation I will call it c i. So, or I will call it amplitude c i and phase alpha i. So, that means, each excitation of current for each radiator, am not exactly writing current because these concept is general this is true for wire antennas or aperture antennas etcetera. In case of aperture antenna we will see we do not have conduction current there we will have some other type of thing, may be those of you know it we can we generally like to call it magnetic current etcetera.

So, that is that is why I am writing these excitation amplitude is c i and the phase of that with respect to any reference is alpha i and the central element or the central point that is there we are placing a eh reference antenna and that reference antenna radiates with c i 1 and alpha i 0. So, with respect to that one that excitation these are c i alpha i is the excitation. The radius I have already said the radius vector of the observation point is r and obviously, theta phi is there. So, it is radius vector is r and it is unit vector I have shown here.

So, now, let us write the reference antennas field.

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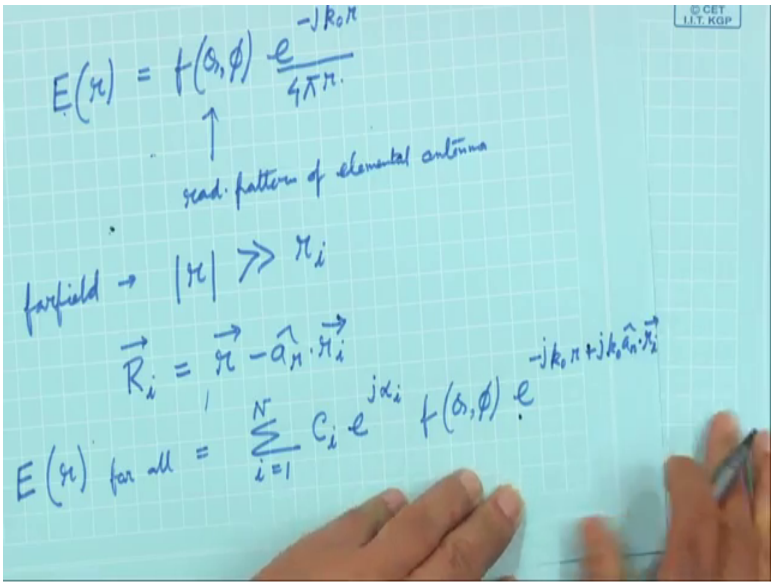

So, reference antenna at the center of the system. So, that is radiating a far field E r that I can write as f theta phi E to the power minus j K naught r by 4 pi r. So, what is a theta radiation pattern of elemental antennas? Elemental antennas means they maybe dipole, they maybe rectangular wave guide they may be horn, whatever, but elemental means element of this array and we assume that all are having same type of thing that is called this here the in an array generally the elements they are same though very sophisticated array is there may be difference, but here will not treat that, all are having the radiation patterns phi theta.

And, in the far field we know that r will be much much greater than r i magnitude. The rays form all the radius they are essentially parallel and I have already written that R i is the distance that I can write as R i r minus a r dot r i. So, the far field produced by the i-th antenna will suffer a propagation phase delay by an amount k naught in to this smaller than that of the reference antenna.

So, now, I can write the resultant field. So, this was for a single element. So, E r for all will be i is equal to 1 to l c i e to the power j alpha i f theta phi e to the power minus j k naught r plus j k naught a r dot r i.

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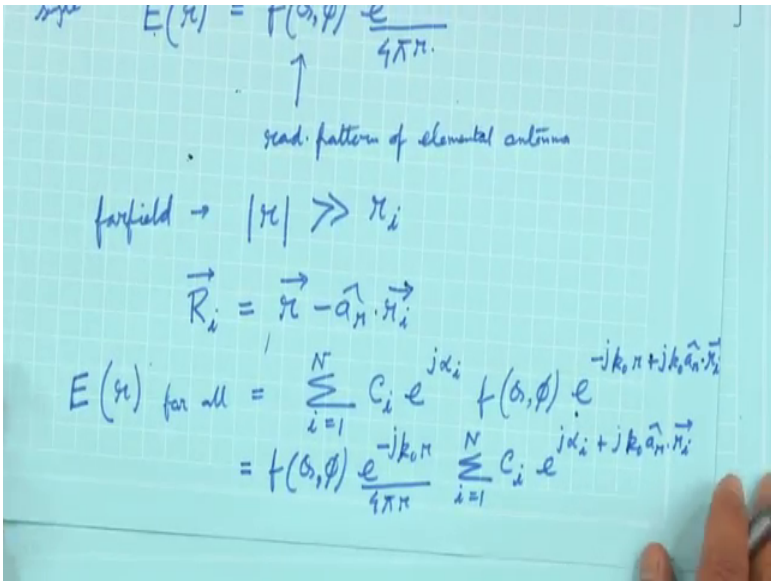

I can take out f theta of phi e to the power minus j k naught r and i dependent answer inside c i e to the power j alpha i plus j k naught a r dot r i, ok.

So, with this, this lecture is concluded now. In the later lecture we will see that, what is the application of this part.

Thank you.