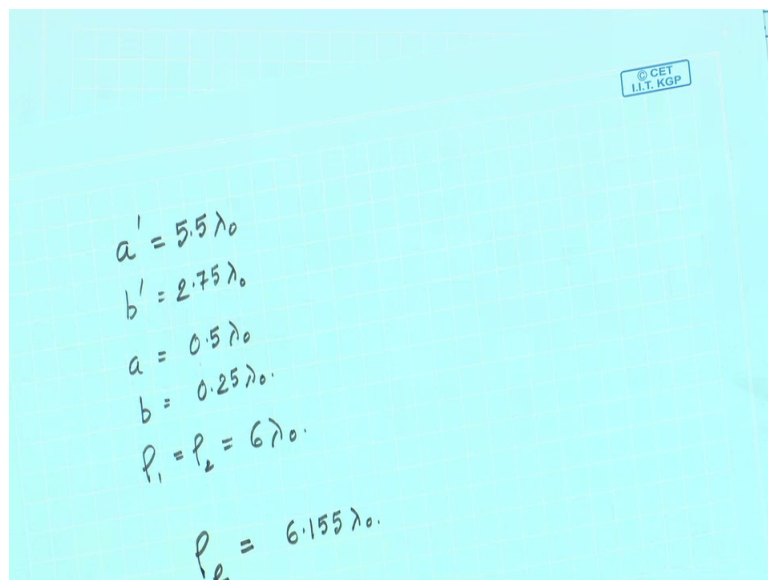


**Analysis and Design Principles of Microwave Antennas**  
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**Lecture – 25**  
**Horn Antenna (Contd.)**

Welcome to this lecture, we are continuing the design of Horn Antenna. So, before going into the actual problem, let us also find for the given horn since we have started that example what will be the gain of this horn.

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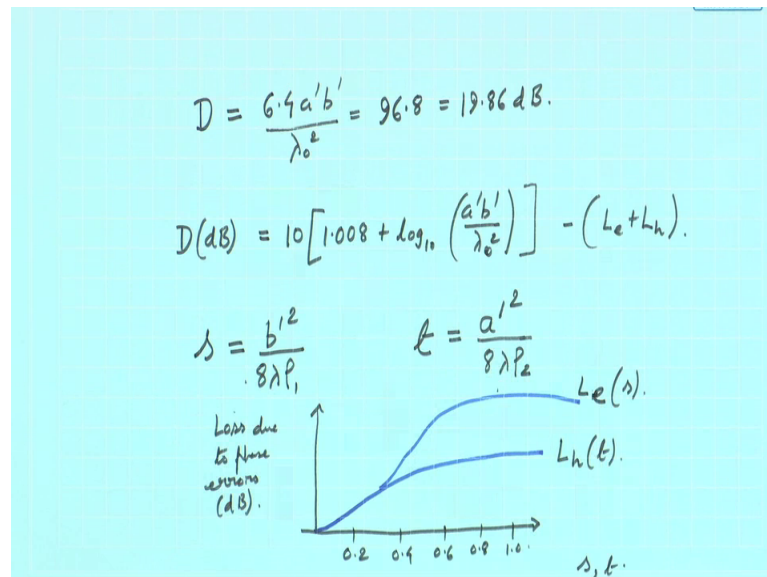
Handwritten equations on a grid background:

$$a' = 5.5 \lambda_0$$
$$b' = 2.75 \lambda_0$$
$$a = 0.5 \lambda_0$$
$$b = 0.25 \lambda_0$$
$$P_1 = P_2 = 6 \lambda_0$$
$$P_e = 6.155 \lambda_0$$

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You see I said that typically we use up to 10 lambda the flares. So, here it is 5.5 and it is oh sorry how much it is the highest you see it is 5.5 lambda 0, it is 2.7 5 lambda naught. So, there will be some gain. So, let us find out gain.

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So, our formula says D. Actually D and g are same for horn. So, D is 6.4 a dash b dashed by lambda naught square. So, you will if you put all these values, you will get something like 96.8 is the directivity. Through which is almost this 19.8 6 d B ok. Now, here actually this is one formula, another formula is there which also is used. So, that formula is somewhat empirical and this formula gives you good thing, but just in case someone wants to use it another formula is available that gives something like this.

So, here you see that this formula otherwise simple, but this L e and L h they are the um they represent specifically the losses in d B, due to phase errors in the e and h planes of the horns. Because, you do not know what is the value of a dash b dash. So, this is given and so, what they do? For these 2 planes, they also have some new variables s, which is b dash square by h lambda rho 1. And, another variable is t this is not time t this is a constant of these things a dashed square by 8 lambda rho 2 rho and rho 2 are the remaining.

So; that means, basically I can say that, this is the H plane and this is the a thing. And, what they no this is the E plane and this is the H plane horn screen. And, some gaps are available, where they give loss due to phase errors in d B versus this variables s or t both now generally there are 0.2 0.4 0.6 0.8 1.0. And the graphs are something like this, this is L. However, L e as a function of s this is L h as a function of t.

So, from this graphs from your because if a dashed and b dashed are known these things can be calculated s and t. So, from the graphs, you can find out the losses. So, if we do that for the same problem, then we can check whether our that value is near.

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I.I.T. KGP

$$s = 0.1575$$

$$t = 0.63$$

$$L_e = 0.2 \text{ dB}$$

$$L_h = 2.75 \text{ dB}$$

$$D = 18.93 \text{ dB}$$

$$D = \frac{\pi \lambda^2}{32 a b} D_E D_H$$

$\uparrow$       $\uparrow$   
 $D_E$     $D_H$

So, if you do that for the given things s will be something like 0.1575 and t is turn out to be 0.63 from the graphs, you will see that  $L_e$  is 0.2 d B and  $L_h$  is 2.7 5 d B. So, if you put that you get 18.9 3 d B. So, already you will see instead of this our that first formula that gave us 19.86 d B. So, that is not so, up the mark. So, what we will do? That is also another formula I would not go into that.

So, people use various designer like various formula just to mention I am writing the third formula is pi lambda square by 32 a b, then  $D_E$  into  $D_H$ . You see remember this is a b the feed guide;  $D_E$  is the directivity of the E plane horn  $D_H$  is the directivity of the H-plane horn. So, by that also people find out, but our that formula is effective. So, with that formula now you see problem is how I find out that, which direction to flare more whether I will put more flare on E side or more flare on H side etcetera; that means, what will be my a dash b dash.

Suppose, the problem is a gain is specified. That I want so, much gain from the pyramidal horn; the feed waveguide dimension is known, now how I do the flare? So, that will now attempt, that is the actual design problem.

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$$G = \frac{6.4}{\lambda_0^2} a'b'$$
$$= \frac{1}{2} \frac{4\pi}{\lambda_0^2} (a'b')$$

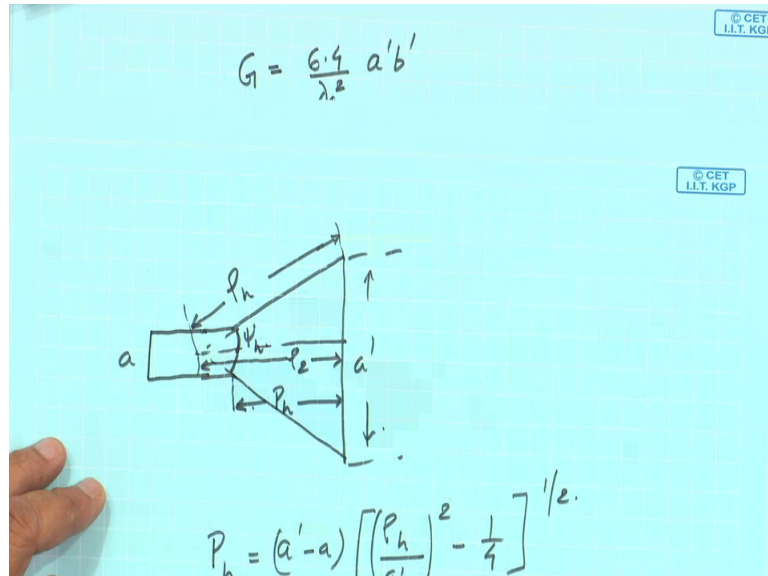
H plane horn

$$\frac{\pi a'}{\lambda_0} \tan \frac{\psi_h}{4} \leq \frac{3\pi}{4}$$
$$\tan \frac{\psi_h}{2} \approx 2 \tan \frac{\psi_h}{4} = \frac{3\lambda_0}{2a'} = \frac{a'/2}{\rho_h/2}$$

So, and here we will have to fix a formula. So, we take that same formula that we use, that is a very likable formula. Simply, this was our standard formula. Now, I will just manipulate this formula. I can write like this and then I will say that let us look at H plane horn, H plane horn. If  $\frac{\pi a'}{\lambda_0} \tan \psi_h$  is less than  $\frac{3\pi}{4}$ , it is maximum that (Refer Time: 08:16). Obviously, generally people go for this maximum thing. So, that gives  $\tan \frac{\psi_h}{2}$  is almost  $2 \tan \frac{\psi_h}{4}$  and this is equal to  $\frac{3\lambda_0}{2a'}$ , you see the geometry from that you can derive it.

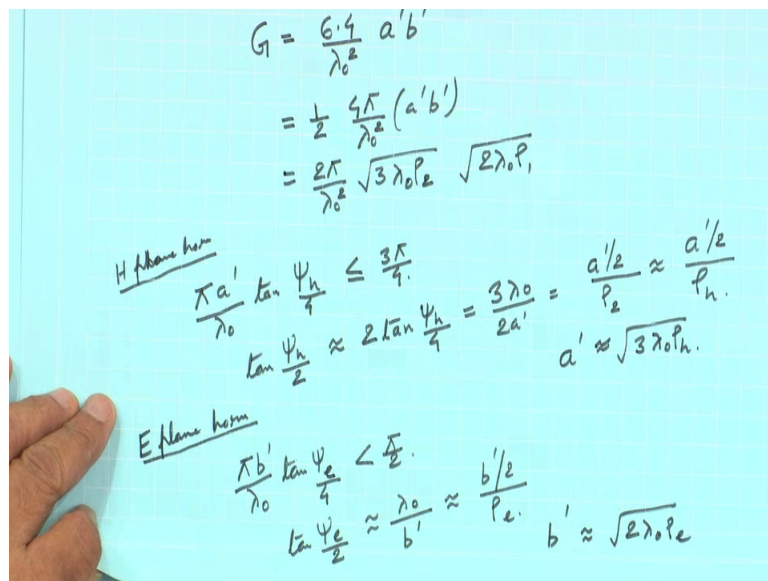
So, this is equal to  $\frac{a'}{2}$  divided by  $\rho_h$  and that we are approximating to  $\frac{a'}{2}$  by  $\rho_h$ . How, we are doing that? Because, if you see  $\rho_h$  and  $\rho_h$  means if I get those H-plane these are in.

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So, what I am saying? For long horns rho 2 and rho h this I am approximately saying same. For long horns you can get because this part is ok.

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Similarly, for E plane horns, now cha it is a mid-complicate this. So, tan h in so, from here I can say that tan be so, a dash will be approximately 3 lambda 0 rho h from here you see E plane horn let us come. So, pi b dashed by lambda naught tan psi e by 4 is less than pi by 2, that says that tan pi by 2 is lambda naught by d dashed and that is b dashed by 2 by rho e.

So, we can say that  $b$  dashed will be almost equal to  $2\lambda_0 \rho_e$ . This is our we are relating  $a$  dashed and  $b$  dashed to  $\rho_e$  and  $\rho_h$ . So, from this formula now, I will be able to write that this is equal to  $2\pi$  by  $\lambda_0$  square root  $3\rho_0$  naught by  $2$  to  $2\rho_0$  naught by  $1$  ok.

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$$G \approx \frac{2\pi}{\lambda_0^2} \sqrt{3\lambda_0\rho_h} \sqrt{2\lambda_0\rho_e}$$

$$\rho_e = \rho_h$$

$$(b' - b) \sqrt{\left(\frac{\rho_e}{b}\right)^2 - \frac{1}{4}} = (a' - a) \sqrt{\left(\frac{\rho_h}{a}\right)^2 - \frac{1}{4}}$$

$$\left(\sqrt{2x} - \frac{b}{\lambda}\right)^2 (2x - 1) = \left(\frac{G}{2\pi} \sqrt{\frac{3}{2\pi}} \frac{1}{\sqrt{\lambda}} - \frac{a}{\lambda}\right)^2 \left(\frac{G^2}{6\pi^3} \frac{1}{x} - 1\right)$$

So, I am writing  $G$  is  $2\pi$  by  $\lambda_0$  square,  $3\lambda_0 \rho_h$   $2\lambda_0 \rho_e$  this is gain is completely related to  $\rho_e$  and  $\rho_h$ . Now, I enforce on this or another condition for this I will have to satisfy that, if I choose this  $\rho_e$  and  $\rho_h$ , the horn should be physically realizable. That means, I will have to now I put it here, this I will use later. Now, I will put that  $P_e$  should be equal to  $P_h$ ; that means, where I left that time  $b$  dash minus  $b$  root over  $\rho_e$  by  $b$  dash square minus  $1$  by  $4$  is equal to  $a$  dash minus  $a$  root over  $\rho_h$  by  $a$  dash square minus  $1$  by  $4$  ok.

This condition I am putting in the  $G$  formula. This  $\rho_e$  or  $\rho_h$  and in this if I put I get an equation like this, because here you that is up to you find out either for  $\rho_e$  or  $\rho_h$  you  $\rho_h$  you express in terms of  $G$ , and  $\rho_e$  also you express in terms of  $G$ . Here, you put in place of  $\rho_e$  this in place of  $\rho_h$  is this. So, that will give you a formula where you will get putting that into this you get something like this. A complicated formula, I will introduce a new variables  $x$  minus  $b$  by  $\lambda_0$  whole square  $2\pi$  minus  $1$  is equal to  $G$  by  $2\pi$  root over  $3$  by  $2\pi$   $1$  by root over  $\lambda_0$  minus  $a$  by  $\lambda_0$  whole square into  $G$  square by  $6\pi^3$   $1$  by  $x$  minus  $1$ .

Student: (Refer Time: 14:37).

So, I have indicated that this, but ultimately after a bit manipulation, because this is required, because when you do I did actually. So, just before this lecture I did. So, unless and until you be there you would not be able to go to the next one.

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Hom Design Eq.

$$\left(\sqrt{2x} - \frac{b}{\lambda}\right)^2 (2x-1) = \left(\frac{G}{2\sqrt{2}\pi\sqrt{x}} - \frac{a}{\lambda}\right)^2 \left(\frac{G^2}{18\pi^2 x} - 1\right)$$
$$x = \frac{\rho_e}{\lambda}$$
$$\rho_h = \frac{G^2}{24\pi^2} \left(\frac{1}{x}\right)$$

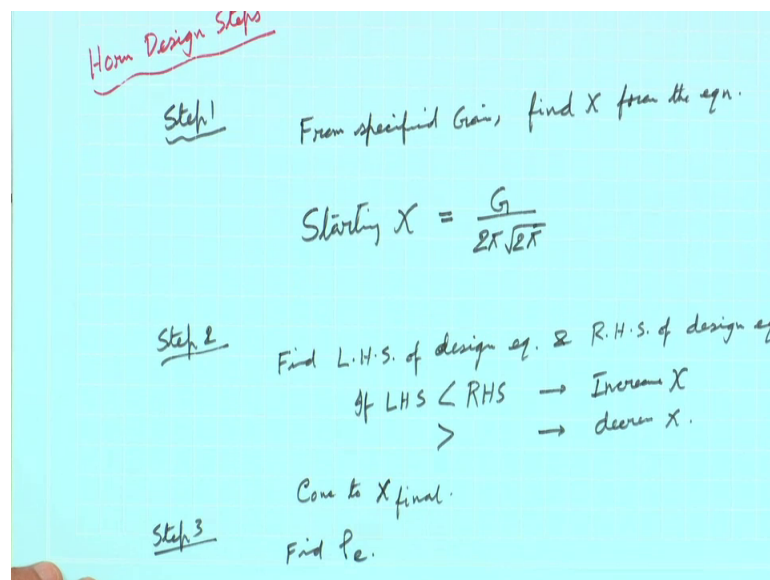
So, final formula is from here after a bit more manipulation  $2x - b/\lambda$  by  $\lambda$  whole square into  $2\psi - 1$  is equal to  $G/\sqrt{2}\sqrt{2\pi} \cdot 1/x - a/\lambda$  whole square into  $G^2/18\pi^2 \cdot 1/x - 1$  same formula only some more constant. So, now, what is  $\psi$ ?  $\psi$  is important it is  $\rho_e/\lambda$  and. So, what is what I should have said  $\theta x$  is this and so,  $\rho_h/\lambda$  becomes  $G^2/24\pi^2 \cdot 1/\psi$ .

So, you can say that, the you see  $\rho_e$  is proportional to  $x$  and  $\rho_h$  is inversely proportional to  $x$ , but the best thing is this is actually my design formula. Because, here you see that I have only 2 variables  $G$  and  $\psi$  other things are known,  $a$   $b$  are known to me that those are feed waveguide thing. So, the specification will give me  $G$  I will find  $\psi$  from  $\psi$  I will find  $\rho_e$  as well as  $\rho_h$  and from  $\rho_e$  and  $\rho_h$  I will go back to a dash and  $b$  dash. That already guaranties me that physically constructed. So, my that confusion that how to choose whether a dash more or  $b$  dash more. Actually, you see they are counter balanced here.

That for a given  $\psi$   $\rho_e$  is proportional and  $\rho_h$  is inversely proportional. So, that is difficult to do initially, but you can do it. Now, the point is this equation does not have a solution. So, iteratively we will have to solve it. What, you will have to do this has 2 eqs on 2 sides, you will have to always check suppose from a given specific gain you find  $\psi$  put it here. You find  $\psi$  means, you start with the  $\psi$  you put it here, you find out which side is less. Accordingly, you either increase  $\psi$  or decrease  $\psi$ . So, go on doing that finally, when you are more or less equated this that is your final answer. So, I will write the steps. So, this equation is this is the I will say horn design equation very important equation.

Student: (Refer Time: 18:32).

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And, now I will write horn design steps, horn design steps. Step 1 I will say that from specified gain find  $x$  from the equation, but you cannot do because this equation you cannot solve. So, what you do starting  $x$  and  $\psi$  starting  $\psi$ , that you take as  $G$  by  $2\pi$  then put in this equation.

Student: (Refer Time: 19:43).

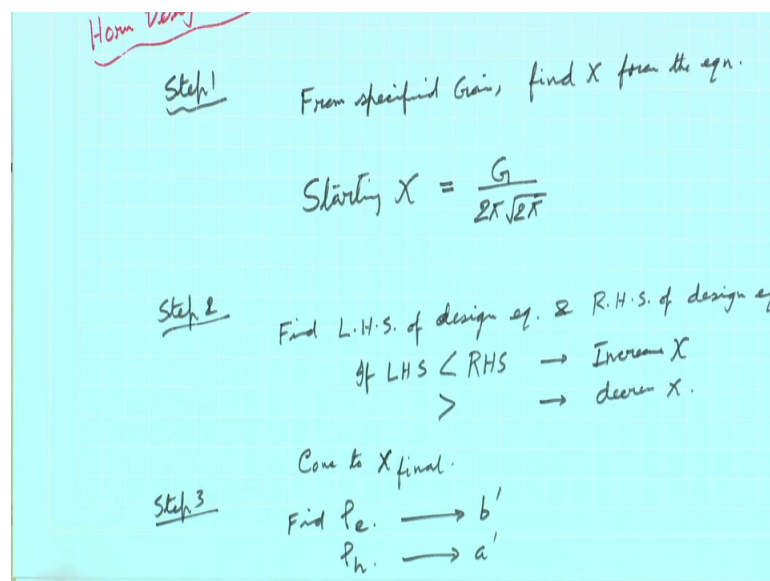
Check LHS RHS step 2 I will say find left hand side of the equation of design equation and R.H.S of design equation; if LHS is less than RHS, you can see lhs is less. So, you



see here  $x$  is coming in the numerator; here  $x$  is coming in the denominator. So, the obvious choice is if lhs is less than RHS increase psi.

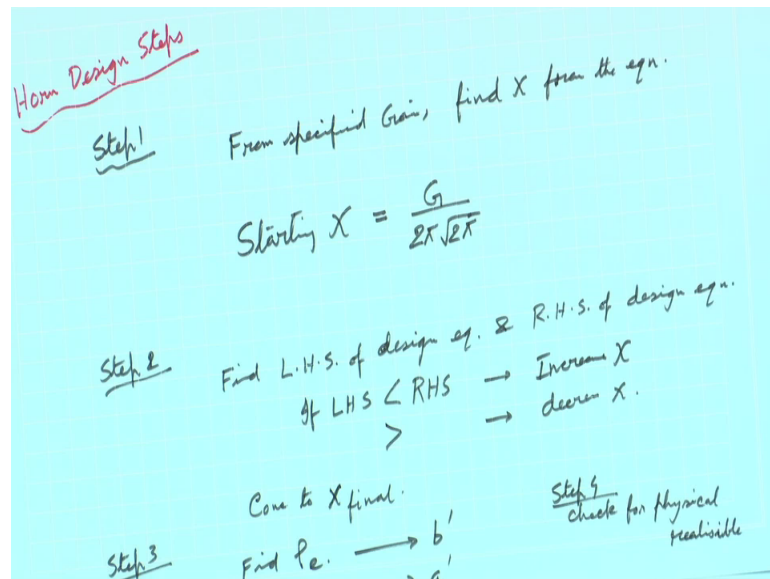
And, if greater then decrease psi. So, iteratively come to  $x$  final or psi final ok. The moment you do that then you will find next step is you will get here find rho e rho h, because they are directly from the defining things from these equations you get rho e and rho h. And, once you know rho e you can find out b dashed once you know rho h you can find a dash.

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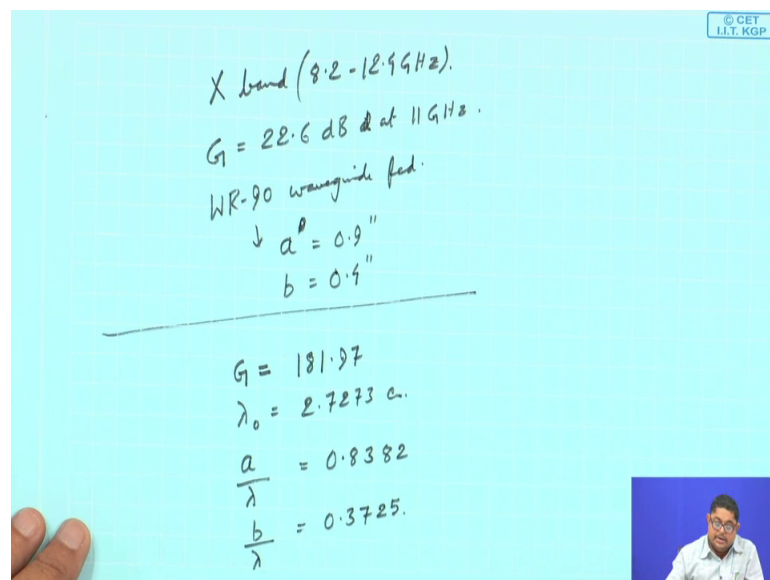
So, let us do it for that or let us take a problem to underscore how we do it? And finally, also you can do one thing I will say that after this I should add another step.

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That step 4 is check for physical realizability, because whether that is achieved that you can you should check. Though, we have taken it in the that guarantees inbuilt, but you should check, because when you have found out the final one, there may be slight error. So, that should not be such that your mechanical tolerance would not be able to handle that. So, for that this final check also should be there.

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So, the problem is design an optimum gain x band, x band means 8.2 to 12.4 gigahertz horn. So, that it is gain is 22.6 d B at 11 gigahertz. The horn is fed by a WR 90 waveguide. And this WR 90 waveguide has the inner dimension of the waveguide is a is 0.9 inch and b the narrower dimension is 0.4 inch. So, from here we can get what is the

absolute, but gain is given in so, we are starting the design. What is gain 22.6 d B, that if I put to absolute thing it is 181.97 or 182. And, what is my lambda 0 it is 2.7273 centimeter and this a by lambda is 0.8 3 8 2 and b by lambda is 0.3 7 2 5. Now, the first step is we can we have to start the iterations.

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$$X_{\text{start}} = \frac{181.97}{2\pi\sqrt{2}\pi} = 11.5539.$$

$$X_{\text{final}} = 11.1157$$

$$\rho_e = 11.1157\lambda = 30.316\text{ cm} = 11.935''$$

$$\rho_h = 12.0094\lambda = 32.753\text{ cm} = 12.895''$$

$$a' = \sqrt{3\lambda_0\rho_e} \approx \sqrt{3\lambda_0\rho_h} = 16.37\text{ cm} \quad (6.445'')$$

$$b' = \sqrt{2\lambda_0\rho_e} \approx \sqrt{2\lambda_0\rho_h} = 4.715\lambda_0 = 12.859\text{ cm}$$

$$\rho_e = 10.005\lambda = 27.286\text{ cm} = 10.743'' \quad 5.063''$$

So, zeta start that will be 181.97 by 2 pi into root 2 pi. So, that will give me 11.5539, then you go on doing the iteration you will see that initially the lhs is greater than RHS. So, we will have to decrease now that decrease you finally, do when I did I got x final is something like 11.1157. So, started 11.55 got it 11.11. So, that gives me a rho e of rho e simply from here you can find rho e will be psi into 8 lambda. So, it will be 11 point see rho e psi into lambda 11.1157 lambda. So, that is 30.316 centimeter or 11.935 inch rho h if you calculate 12.0094 lambda.

So, that is 32.753 centimeter 12.895 inch from here you can calculate a dash, what is a dash 3 lambda naught rho 2 already I showed for long horns, 3 lambda naught rho h. So, rho h I have found here. So, it becomes 16.37 centimeter or 6.4 4 5 inch. And b dashed is root over 2 lambda naught rho 1, that is 2 lambda naught rho e that is 4.715 lambda naught, that is 12.859 centimeter or 5.063 inch. And, if I check P e is equal to 10.005 lambda or 27.286 centimeter, 10.743 inch and same as P h.

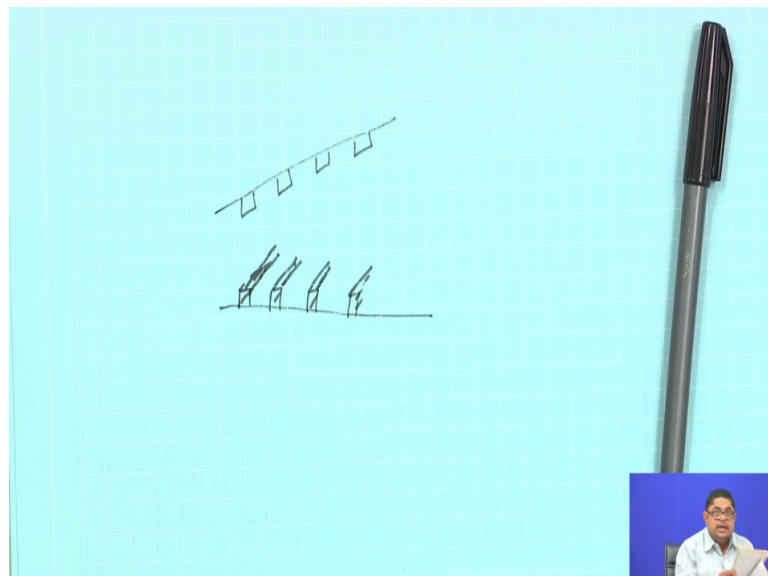
So, that will be a from the open ended waveguide mound there will be flaring of 10 inch, 10 inch is quite long for an x band ok. So, this completes the design of a pyramidal horn.

Now, actually you see that the phase field a actually the since horn aperture has a large dimension both the narrow dimension and a they have been flared. So, there will be sizable amount of cos polarization generated, you know this that in a dominal waveguide we keep the because of the narrowness of the things we keep the cross pole lobe. But the moment that gets flare there will be cross poles, cross polarization components start getting generated. So, there are sizable cross poles in the any pyramidal horn.

So, that creates in certain cases some problem and also in the there are sizable since it is a large aperture, there are reflection and deflection etcetera. So, horns that is why they do not have very high efficiency though the I am not saying that loss is there more, but due to this cross pole there are losses, which are the polarization losses. So, that is why efficiency of horns are typically 50 to 60 percent.

So, to improve this efficiency to 75 to 80 percent in particularly in radiometer applications distance star a thing people use better horns with 75 80 percent thing. Those are called corrugated horns. Actually, you what happens this? Cross poles they get generated. So, if that cross poles can be suppressed, then the purity polarization purity improves.

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So, what happens in this is the horn. So, inside the horns actually when a this is a side view corrugation means, you know that corrugated sheet etcetera. So, some protrusion some metallic protrusion actually these are metallic protrusions like this these are

metallic structures. So, these structures actually makes those cross polarization things absorbs. So, these protrusions it is dimension etcetera by manipulating that, you can cut those things those are called the corrugated horns.

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$$\vec{E}_{far} = \hat{u} E(\theta, \phi) e^{j\psi(\theta, \phi)} \frac{e^{-jk_0 r}}{r}$$

$\uparrow$  amplitude f.       $\uparrow$  phase f.

$\psi(\theta, \phi) = \text{const.}$   
 Phase centre

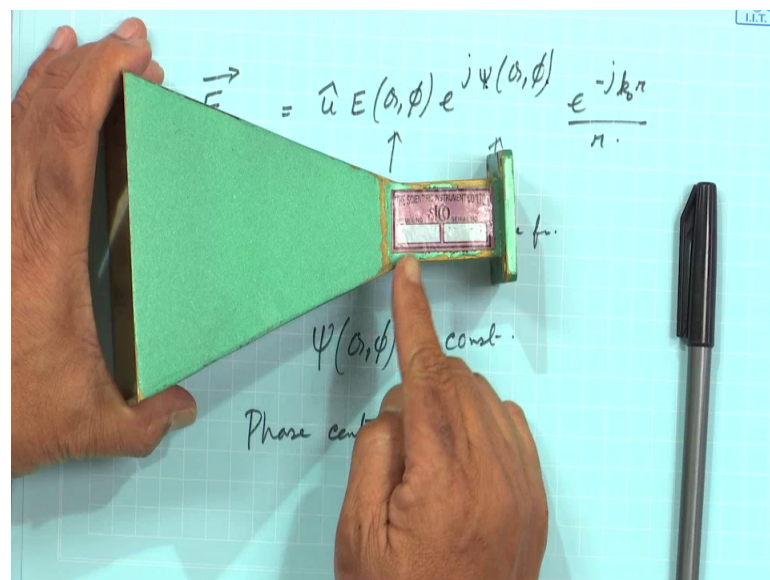
And, also I want to say that there is a concept called phase center. Actually, each of the far field radiated by antenna can be written in a general form that E far field far is. Now, what is this? You see this is an amplitude function so; that means, far field has an variation with theta phi that is amplitude function and this is the phase function. Now, in chalking or navigation, landing system etcetera it is required to assign to the antenna a reference point, that actually from a point from which point antennas radiation is coming. Now, actually in this type of aperture antennas there is no such point, because the point is possible only for a spherical waves, this aperture antennas generally do not produce spherical waves, they produce they line sources in cylindrical wave, but for reference purposes a hypothetical point is assumed.

So, what is that point? That reference point should have a characteristic that, for a given frequency this psi theta phi; that means, this phase function, this should be independent of theta and phi. That means, whatever theta phi I look at that point, this psi theta phi so; that means, psi theta phi should appear to be a constant from that point, for a given frequency this is independent of theta phi this point is called the so, the point at which this is valid that is called the phase center of the antenna. So, when reference to the phase

center the fields radiated by the antenna are considered to be spherical waves with equi phase surfaces.

For practical antennas such as array reflector and others a field fingle a single phase center valid for all theta and phi does not exist. However, in many systems a point can be found for which this constant for most of theta and phi, especially the main lobe theta and phi range. For reflectors feed the antennas space center is put at the focus, that we will see later because a otherwise how you determine focus? So, that is why you need to know the phase center; that means the point from where you can think that antennas radiation is coming.

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Usually the phase center of the horn is not located at it is mouth; mouth means where from it starts this is look at this is the mouth this is the aperture it is not at the mouth not at the aperture, but actually between the imaginary apex and aperture.

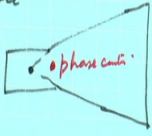
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$$\vec{E}_{\text{far}} = \hat{u} E(\theta, \phi) e^{j\psi(\theta, \phi)} \frac{e^{-jk_0 r}}{r}$$

↑ amplitude fr.  
↑ phase fr.

$\psi(\theta, \phi) = \text{const.}$

Phase centre:



Large  $\psi \rightarrow$  closer to apex

So, this is the aperture, this is the imaginary apex, the phase center is somewhere here. The exact location of the phase center of the horn depends on its flare angle. For large flare angle phase center is closer to apex as flaring decreases the phase center moves towards the aperture. So, where is the phase center of the open ended waveguide, which is nothing, but flaring is 0. So, that is why it is at the aperture with that I hope that you will be able to design a horn yourself already we have seen how to design the dipoles etcetera particularly folded dipoles etcetera.

So, you have a fairly good amount of knowledge from this another antenna aperture antenna, that is very useful that is dish antenna which is used in, now a day all the even TV receptions. But, invariably used in satellite communications etcetera where we want not 60-70 percent efficiency something like 90 percent efficiency.

So, that we will see in the next class what is those things, we would not see in details because that detailed thing is quite involved, but we will see that what are the salient features of those antennas, there are various efficiency concept how to do that. And then what is the gain of that system given the geometry that we will see. And then we will see actually that if you do not have a, this type of known antennas, but you have designed an antenna or you are given an antenna, how to analyze that if time permits we will also take up that issue.

Thank you.