

**Analysis and Design Principles of Microwave Antennas**  
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**Lecture - 03**  
**General Properties of Radiated Fields from an Antenna**

Welcome to this third lecture. In previous class we have seen that if we have a current element  $I dl$  directed along let us say a  $z$  axis then how to find the vector potential. The vector potential  $A$  will also be  $z$  directed. However, I will tell you that always it is not so easy to find the, what is the current density vector direction and what is this distribution. For some simple cases particularly were antennas with canonical structures that means, well known geometrical shapes we can find  $j$  or by measurement experiment we can find.

But if we cannot find then we have other methods mode approximate free transmission equation by which we can find. But nowadays with modern computers there are numerical techniques, one of that is method of moments there also we can find out for any complicated structure what is the current density vector. So, from that we can now, find out.

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$\odot P(r, \theta, \phi)$

Hertzian Dipole

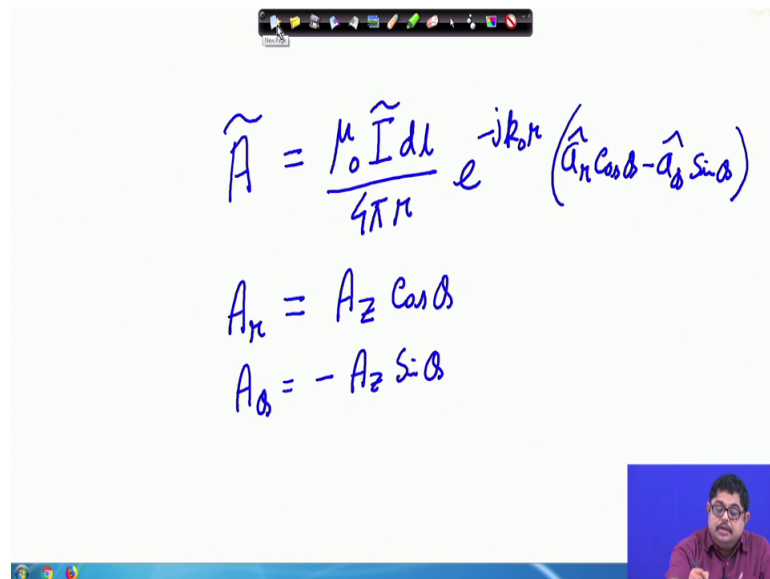
$$\tilde{A} = \frac{\mu_0}{4\pi} \tilde{I} dl \frac{e^{-jk_0 r}}{r} \hat{a}_z$$

$$\hat{a}_z = \hat{a}_r \cos \theta - \hat{a}_\theta \sin \theta$$

However, we will now, find out from these vector potential the electric and magnetic fields at a distance point  $P$ .  $P$  is our observation point it is customary to put these in terms

of the distance then elevation angle and azimuth angle or theta phi. So, spherical coordinate. So, these vector potential that we have derived in Cartesian coordinate because of its use because in the source side we have used Cartesian coordinates, but. Now, this z needs to be converted to spherical coordinate so that we can represent the fields from it electric field and magnetic fields. So, it is well known that this z is nothing, but in terms of the spherical co-ordinate unit vectors a r cos theta minus a theta sin theta. So, if we put this then A, the phasor A becomes.

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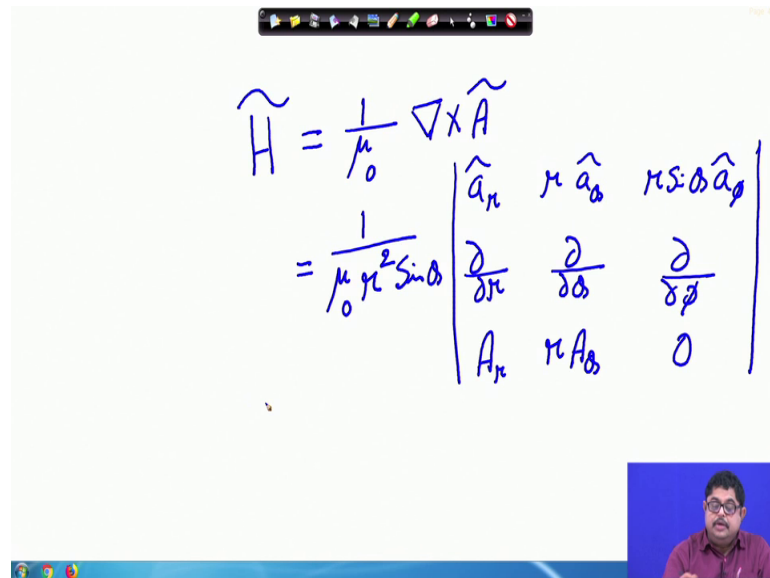
$$\tilde{A} = \frac{\mu_0 \tilde{I} dl}{4\pi r} e^{-jk_0 r} (\hat{a}_r \cos \theta - \hat{a}_\theta \sin \theta)$$

$$A_r = A_z \cos \theta$$

$$A_\theta = -A_z \sin \theta$$

Now, this from here we can say that our radial component A r of the vector potential that is nothing, but A z cos theta and A theta is minus A z sin theta.

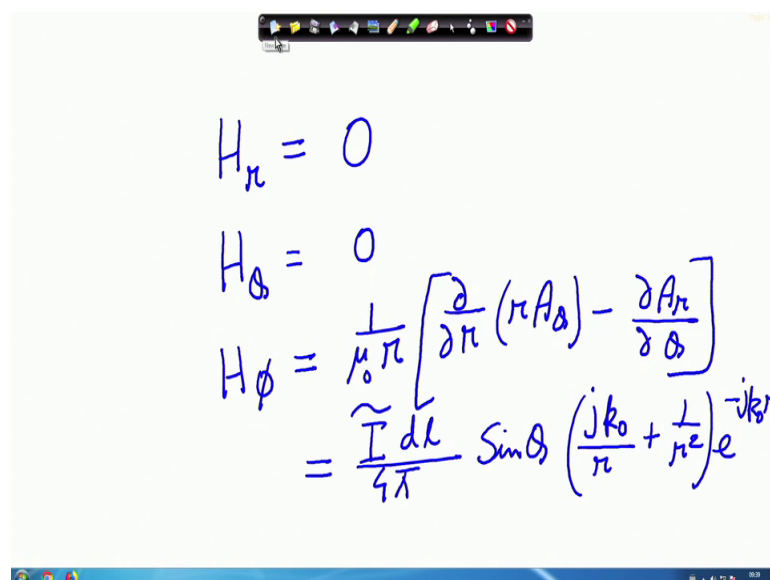
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$$\begin{aligned}\hat{H} &= \frac{1}{\mu_0} \nabla \times \hat{A} \\ &= \frac{1}{\mu_0 \pi^2 \sin \theta} \begin{vmatrix} \hat{a}_r & \pi \hat{a}_\theta & \pi \sin \theta \hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & \pi A_\theta & 0 \end{vmatrix}\end{aligned}$$

Now, once we know vector potential the first step is to find the magnetic field H. So, H phasor is  $\frac{1}{\mu_0} \nabla \times \mathbf{A}$ . So, just we will have to perform this del operation. So, del operation we know it is in spherical co-ordinate. And we have these A r component we have A theta component, we do not have any phi component. So, from this it is easy to find H r, H theta and H phi components.

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$$\begin{aligned}H_r &= 0 \\ H_\theta &= 0 \\ H_\phi &= \frac{1}{\mu_0 \pi} \left[ \frac{\partial}{\partial r} (\pi A_\theta) - \frac{\partial A_r}{\partial \theta} \right] \\ &= \frac{\tilde{I} dl}{4\pi} \sin \theta \left( \frac{jk_0}{r} + \frac{1}{r^2} \right) e^{-jk_0 r}\end{aligned}$$

So, you do it I am. Now, writing the results  $H_r$  component will be 0,  $H_\theta$  component will also be 0, but  $H_\phi$  that will be given by  $\frac{1}{\mu_0} \nabla \times \mathbf{A}$ . So, this if we evaluate we

can find out this value. Now, we will modify it because here  $k$  naught by  $r$  terms. So, we do not want that we want  $1$  by  $r$ ,  $1$  by  $r$  square like that terms, so we just manipulate. And just write this very important magnetic field, the phi component.

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$$\tilde{H}_\phi = \frac{\tilde{I} dl}{4\pi} k_0^2 \sin\theta \left( j \frac{1}{k_0 r} + \frac{1}{k_0^2 r^2} \right)$$

$$\tilde{E} = \frac{1}{j\omega\epsilon_0} \nabla \times \tilde{H} e^{-jk_0 r}$$

So, this is our  $H_\phi$  that means, the magnetic field only have an azimuthal component it does not have any radial or elevation components. So, from this it is easy actually there are two roots, one is from the vector potential itself you can find, but also if I put it into the Maxwell's equation because now, I know  $H$ . So, there is a curl of  $H$  the right side will be so that equation which amperes law Maxwell's second equation.

If we put that then and also noting that this is a field point actually this is the antenna  $I dl$  and this is the field point  $P$ . So, here there are no current, current is here. So, in this point there are no current. So,  $j$  at this point is  $0$ . So, now, we can write the electric field phasor from the Maxwell's equation has  $1$  by  $j$  omega epsilon naught del cross  $H$ . So, if we do that again then we can find out the components.

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$$\tilde{E}_r = 2 \frac{\tilde{I} dl}{4\pi} \eta_0 k_0^2 \cos\theta \left( \frac{1}{k_0^2 r^2} - \frac{j}{k_0^3 r^3} \right)$$

$$\tilde{E}_\theta = \frac{\tilde{I} dl}{4\pi} \eta_0 k_0^2 \sin\theta \left( \frac{j}{k_0 r} + \frac{1}{k_0^2 r^2} - \frac{j}{k_0^3 r^3} \right)$$

$$\tilde{E}_\phi = 0 \quad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

There we will see that it has E r component the E r component is given by 2 there is an E theta component all these are phasor quantities, and it does not have any phi components. The magnetic field has phi component, the electric field does not have any phi component here the new term is this eta naught eta naught is the intrinsic impedance of free space and given by mu naught by epsilon naught.

So, now, here if we look at the E r E theta also if you look at the previous expression of H phi, so they can be you see the variation with distance instead of writing it 1 by r we have seen it k naught by k naught into r k naught r k naught r whole square. Then here also you see k naught r k naught r whole square k naught r whole cube we have a specific purpose for this that I will explain later. But what is this k naught r? Let us see the k naught r.

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The image shows a whiteboard with handwritten mathematical derivations. The first equation is  $k_0 = \omega \sqrt{\mu_0 \epsilon_0}$ , which is simplified to  $\frac{\omega}{c}$ . The second equation is  $k_0 r = \frac{\omega r}{c} = \frac{2\pi f r}{c} = 2\pi \left( \frac{r}{\lambda_0} \right)$ . A small video inset in the bottom right corner shows a man with glasses and a red shirt speaking.

So, we have what was our  $k$  naught value  $k$  naught was  $\omega$  if you remember  $\omega$  then root over  $\mu$  naught  $\epsilon$  naught, so that we can write as  $\omega$  into  $\mu$  naught  $\epsilon$  naught, so that I can write as  $\omega$  by  $c$ , where  $c$  is the free space velocity of light. So, what is  $k$  naught  $r$ ?  $k$  naught  $r$  is  $\omega r$  by  $c$ .

Now,  $\omega$  is angular frequency, so I can write it in terms of the frequency  $2\pi f$  by  $c$ . Again  $r$  by  $c$  is  $\lambda$  naught, so it is  $2\pi r$  by  $\lambda$  naught, where  $\lambda$  naught is the wavelength corresponding to the operating frequency. So, you see this is considered as that instead of the distance  $r$  is the radial distance of the observation point on the antenna.

We are writing it as a electrical distance  $r$  by  $\lambda$  naught because  $\lambda$  naught is the electrical quantity, actually the source is oscillating with these of frequency  $f$  naught the corresponding wavelength is  $\lambda$  naught, so where is also propagating. So,  $r$  by  $\lambda$  naught  $\lambda$  naught is the wavelength. So,  $r$  by  $\lambda$  naught is the electrical distance. So, we can say that electric and magnetic fields that is coming out from the current element those are functions of the electrical distance of the antenna, from the antenna.

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$\xrightarrow{\text{dc}} k_0 = \frac{\omega}{c} = 0$   
 $\tilde{H}_\phi = \frac{\tilde{I} dl}{4\pi r^2} \sin\theta$   
ac Mag field  
 Two terms  $\rightarrow \propto \frac{1}{r}$   $\rightarrow$  farfield  
 $\propto \frac{1}{r^2}$   $\rightarrow$  dominates near antenna  
 $\rightarrow$  induction field.

Now, let us consider the field components more closely. So, first consider the magnetic field you look at your magnetic field expression and what happens if I send an dc current for dc current  $k_0$  we know it is  $\omega/c$ . Now, for dc  $\omega$  is 0. So,  $k_0$  becomes 0 so it is the dc condition,  $k_0$  becomes 0. So, what happens to  $H_\phi$ ? If we look at its expression  $H_\phi$  is become  $I dl$  by  $4\pi r^2 \sin\theta$ .

So, if you send dc current the magnetic field is this if you remember that this was nothing, but the magnetic field when we have considered magnetostatics if we send the current of  $I$ , then for an infinitesimal length  $l$  on that the magnetic field will be azimuthal and is this came from Biot-Savart (Refer Time: 14:10) if you remember. So, it boils down to Biot-Savart (Refer Time: 14:14) for dc, but what time we are actually antennas radiation is a time varying current. So, we do not have dc, we have our ac current. So, if we look at the magnetic field expression let me go to the magnetic field expression, you see that these are all constant terms this is a phase term, but magnitude wise you see I have a term varying with electrical distance  $r$  and square of the electrical distance  $r$ .

So, there are two terms one is varying inversely with electrical distance another is varying inversely with the square of the distance. So, we can write that. So, we have two terms in the magnetic field, magnetic field two terms, one is varying with one by  $r$  another is varying with one by  $r^2$ .

Now, close to the antenna these  $r$  value is very small or  $k$  naught  $r$  that is very small. So, this one dominates over this one,  $1$  by  $r$  term. So, dominates near antenna you can write that. Now, this component is also called induction field. Why I will not be able to explain. Now, later I will explain why this is called induction field and this one this  $1$  by  $r$  term if I go further away from the antenna at a sufficiently large distance this term will start dominating over this one square term. So, this term is called far field of the antenna. Now, what is far field etcetera we will discuss again but this is their nomenclature. So, this is induction field, this is far field.

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$$\tilde{H}_{farfield} = jk_0 \frac{\tilde{I} dl \sin\theta}{4\pi r} e^{-jk_0 r} \hat{a}_\phi$$

So, I can write this far field component for later use. So,  $H$  far field I can write as  $j k$  naught  $I dl \sin \theta$ , these shows it is a way it is going out the azimuthal direction and already we have seen the induction field. Now, what is the distance where this induction field component and this far field component they become equal that means, let us look at the a field expression.

So, basically I will have to equate this the magnitude will become equal that means, I will have to make  $1$  by  $k$  naught  $r$  is equal to  $1$  by  $k$  naught  $r$  square at the distance of  $r$  it will be equal. So, that let me equate that that  $k$  naught  $r$ , I can say a border  $r_b$  is equal to  $k$  naught square  $r_b$  square.



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$$k_0 r_b = k_0^2 r_b^2$$
$$r_b = \frac{\lambda_0}{2\pi} \approx \frac{\lambda_0}{6}$$

Electric field  
→ only  $E_r$  and  $E_\theta$

So, value of that  $r_b$  is if you do this it will come to be  $\lambda_0 / 2\pi$ . So, roughly I can say that  $\lambda_0 / 6$ . So, at a distance from the current element at a distance of  $\lambda_0 / 6$ , the induction field and the far field they are being equal. So, this is one of the conclusion we can make from the magnetic fields.

So, magnetic field is only having azimuthal component magnetic field has two terms, one is induction field another is far field. Now, the boundary where induction far field start dominating the induction field is roughly at a distance of  $\lambda_0 / 6$ . Now, let us come to the let us come to look at electric fields more closely. So, electric field has only two components mainly  $E_r$  and  $E_\theta$  it does not have any  $\phi$  component.

Now, if we look at this  $E_r$  component this  $E_r$  component. You see  $E_r$  component does not have any  $1/r$  term. It has  $1/r^2$  term  $1/r$  term.  $E_\theta$  component has  $1/r$  term it also has  $1/r^2$  term  $1/r^3$  term that means, I can say that the  $E_r$  component has an induction term again induction field is one is called. So,  $E_\theta$  component also as an induction field component  $E_r$  does not have a far field  $E_\theta$  component has a far field component. But both of them has one more term that is varying as  $1/r^3$ . So, let us see those fields more closely.

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$$\tilde{E}_{\text{farfield}} = \frac{j n_0 k_0 \tilde{I} dl \sin \theta}{4 \pi r} e^{-jk_0 r} \hat{a}_\theta$$

Both  $\tilde{E}_{\text{farfield}}$  &  $\tilde{H}_{\text{farfield}}$  → same time phase.  
 → orthogonal.

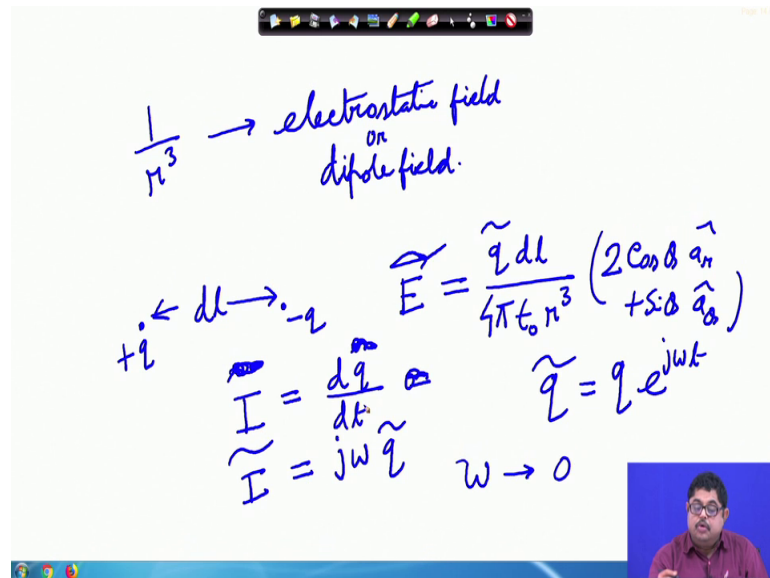
$$\frac{|\tilde{E}_{\text{farfield}}|}{|\tilde{H}_{\text{farfield}}|} = \eta_0$$

So, before going further I can say that E far field in this case I can write for later use is  $j \eta_0 k_0 \tilde{I} dl \sin \theta e^{-jk_0 r} \hat{a}_\theta$ . So, this is one thing and we can see that E far field has a time phase of  $j$ . Let us see the magnetic far field if I have written. I have not written. So, this also has a time phase of  $j$ . So, can I say that both. So, E far field is this also I can say that both E far field and H far field they are at same time phase because both are  $j$ . So, there in same time phase.

But E far field is  $\hat{a}_\theta$  directed H far field is  $\hat{a}_\phi$  directed so that means, they are orthogonal to each other. This two far fields are orthogonal to each other and what is the ratio of this far fields E far field its magnitude by H far field its magnitude, that ratio you will see is  $\eta_0$  because e far field is  $\hat{a}_\theta$  component magnitude you take H far field is  $\hat{a}_\phi$  component magnitude if you do that that is equal to intrinsic impedance of free space.

Now, as I said earlier is that  $1/r^2$  term is present in both  $E_r$  and  $E_\theta$ ,  $E_\theta$  component, so induction fields are present.  $E_r$  and  $E_\theta$  also contain  $1/r^3$  term now, which obviously, will dominate at very close distances, much closer than the domination of induction fields.

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They are called  $1/r^3$  terms they are called electrostatic field or dipole field. Now, why this name I will now explain that why this nomenclature electrostatic etcetera. Now, here I will recall one thing that if I have a dipole with point charges plus  $q$  and minus  $q$  separated by a distance  $dl$ , then the electric field you can easily find out that electric field that will be given by this in the present form say. This I think you have done in your electrostatic classes that the dipole static dipole, that the field is like this.

Now, what is this relevant to our case? Our case is obviously, not this electrostatic dipole case but we have in our field expressions we have an  $I dl$ . So, what is  $I$  this pressure  $I$ , I can write it as  $dq/dt$  and so sorry  $dq$ . So, pressure  $I$  is  $dq/dt$  now, I can in the phasor form plus sorry I will not say this  $I$  is equal to  $dq/dt$ . In the phasor form I can then write that  $I$  pressure will be  $j\omega q$  pressure. And what is  $q$  phasor?  $q$  phasor is if the charge is oscillating that means, charge is going. So, we can give  $q e^{j\omega t}$  to the power  $j\omega q$  is a time variation charged at a place that the time variation you know  $q e^{j\omega t}$  then I can write this.

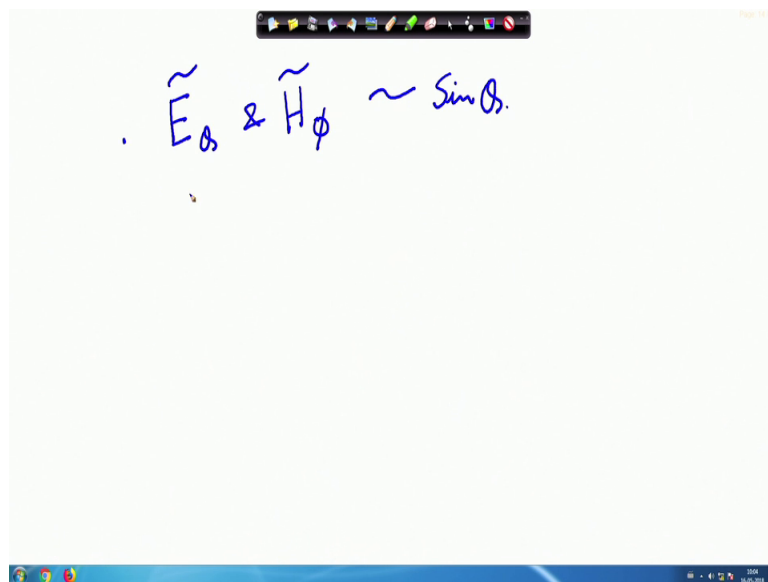
So, this if we put here that this  $q$  instead of that if I go for this phasor if I introduce this here, and if I let  $\omega$  goes to 0 if I force  $\omega$  goes to 0 then we get this field in our expression. That means, you do this go to our electrostatic field electric field. So, here in this electrostatic field you instead of  $I$  you write the  $j\omega q$  and that  $\omega$  you puts

to 0 you will see will get these two fields. So, that will come to the same as this E r component and e theta component they will be same as the dipoles thing.

So, that is why this name came that this terms they look like the dipole term. But the question is where is dipole coming here in our case you consider that we have a current element. So, I have a current uniform current I here time varying, but uniform current. Now, after going here the current is oscillating, but what the charges will do here. So, there will be a accumulation of charge that charge is changing the polarity because with the change of the current polarity they will change.

But there is an accumulation of charge here there is an opposite accumulation of charge here. At this instant if this is plus this is negative next instant this is next cycle this is minus q this is plus q this is plus etcetera. But there is an accumulation of charge. So, there is a dipole from here. So, in our current element also due to this finite discontinuity here there is a dipole form and that contributes here, but its effect does not go much it comes near. So, this is the origin of this dipole (Refer Time: 28:32) and but let us go ahead and see more things that.

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The image shows a whiteboard with a toolbar at the top and a Windows taskbar at the bottom. The whiteboard contains the handwritten equation:  $\tilde{E}_\theta \text{ \& \ } \tilde{H}_\phi \sim \sin\theta$ .

If you look at the expressions both E theta and H phi that means, our far field terms they adheres their variation is sin theta. So, we can say that the radiated fields that E theta and H phi far fields we are saying, they are not spherically symmetric. You see sin theta means that has a theta variation, so it is not spherically symmetric, whereas if you recall

that our magnetic vector potential was spherically symmetric. Now, this is important, actually that is why we did not solve  $E$ ,  $\theta$ ,  $H$ ,  $\phi$  directly.

We took the help of  $A$ , because with that spherically symmetric nature the solution becomes easier only because it becomes a function of  $r$  it was not a function of  $\theta$ ,  $\phi$ , but  $E$  and  $H$  they are functions of  $\theta$ .  $E$  is a function of  $\theta$ ,  $H$  is a function of  $\phi$ . So, if you want to get apart from their dependence on  $r$ . So, that was the difficulty in solving  $E$  directly that is why we took the help of the vector potential.

Actually this is always true that radiated fields cannot have spherical symmetry because and that shows that you cannot have an antenna which is radiating equally in all azimuth and elevation directions which later will be named as an omni antenna. So, an omni antenna is never possible always the radiated fields they will have some physical distribution.

Now, we will come to the power distribution that, what is the proof that this antenna is getting power because if it is radiating means power is taken from the current distribution antenna that means, current element to some space, to all the places the power is flowing. So, for that we will have to consider the pointing vector of this because now, we have field expressions so we can find the pointing vector, from the pointing vector we can easily find what is the power and will see that it radiates or not. That will be seen in the next class, ok.