Analysis and Design Principles of Microwave Antennas
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Lecture - 37
Array Pattern Synthesis

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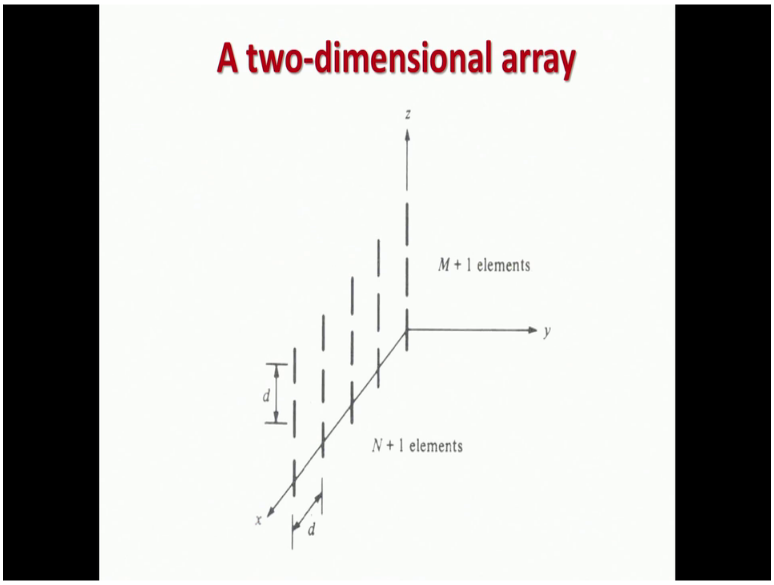

Welcome to this NPTEL lecture on antennas. Now, we will see today some advanced topics, the first we will see Array Pattern Synthesis. Actually, we have seen array antennas, but we have seen only one-dimensional antennas. Today, we will first see that two-dimensional antenna, and then see that there is a need for having a synthesis procedure in antenna array antenna design, so that will try to see though we would we would not go very deep in that. But just I will sketch that, whatever you learned if you want to explore that antenna synthesis problem, you can go with this background ok.

So, first let us consider a two-dimensional array, because we have not sorry we have not considered that. So, you see a two-dimensional array for the simplicity, I have taken that it is a one-dimensional array in x-axis and also in the z-axis, this one-dimensional array is repeated. So, I am considering that as before N plus 1 element along x-axis, now today this there are such M plus 1 number of such linear arrays. So, total elements are N plus 1 into M plus 1 ok, they are uniform spacing that is why uniform array, and also the excitations the same excitation.

So, we can that this array may be described as an array of M plus 1 linear arrays does by using the principle of pattern multiplication. The two-dimensional array factor will be the product of the array factor for M plus 1 linear array along the z-axis with the array factor for the N plus 1 elements array along the x.

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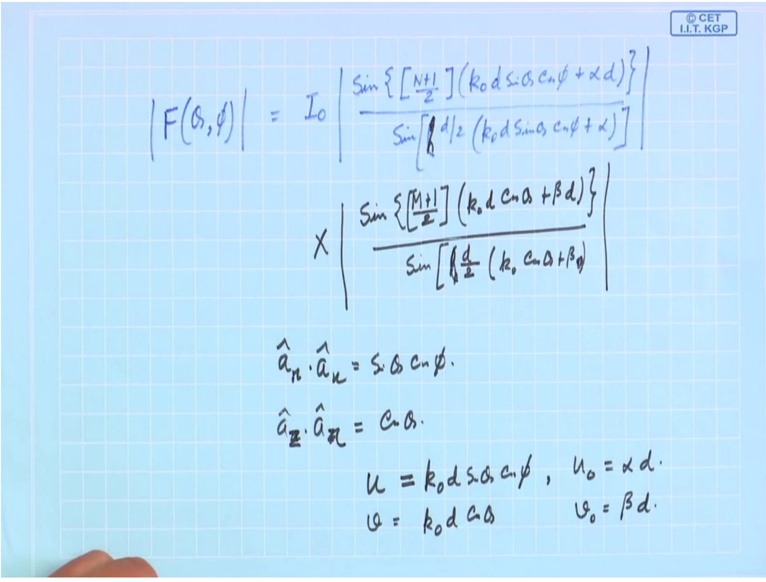

So, we can written the array factor since it is 2 dimensional, so we are writing explicitly the sin and l that we are expressing in terms of theta phi (Refer Time: 03:40), where you can note that we have used this that a r cross a x is sin theta cos phi, and a r cross a z.

Now, as before, we generally go to the u plane. So, here since it is two-dimensional, we will introduce the two quantities u and v. So, u as before we will write as k 0 d sin theta cos phi, and u 0 is alpha d. And v you are introducing as k naught d cos theta v naught is beta d.

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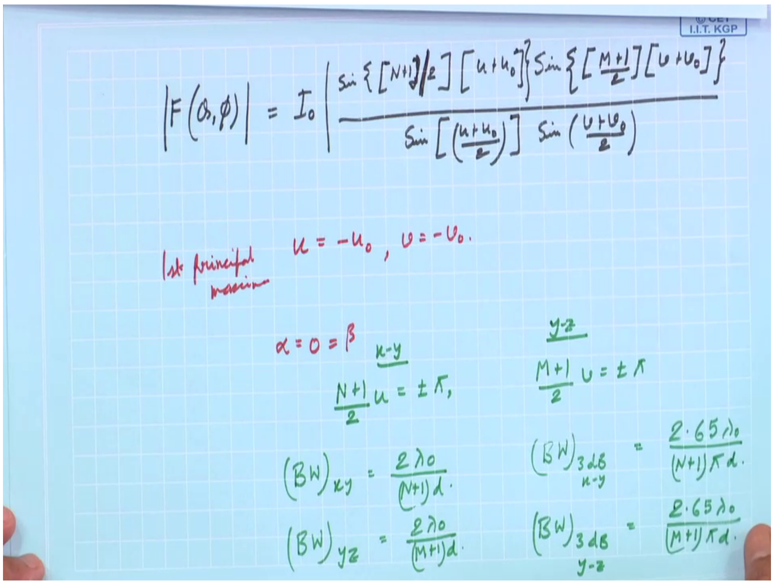

And so, we can rewrite this array factor two-dimensional array factor as I naught. So, these array factor as its first principle maxima, when u is equal to minus u 0. So, first principle maxima, when u is equal to minus u 0, v is equal to minus v 0. So, in that direction, it has the maximum radiation.

And if we take alpha is equal to 0 is equal to beta that means, the progressive phase shift, then this direction will be perpendicular to the plane of the array that is along plus minus y direction. For suitable values of alpha and beta, the beam can be directed in any direction. So, this is called the concept of phased array that if I want to move the beam that can be done by putting some values for alpha, beta. So, whatever in which direction you want to put based on that, alpha, beta can be put the values. So, since this is done electronically, it is also called electronic scanning.

In the in the case of a broad side array, the angular width of the lobe in the x-y and y-z planes is obtained by setting in plus 1 by 2 u is equal to plus minus pi, and so this is in the x-y plane. And in the y-z plane, M plus 1 by 2 v is equal to plus minus pi.

So, the same that we have done for linear array. So, it can be easily shown that bandwidth a beam width in the xy plane is given by 2 lambda naught by N plus 1 d. And beam width in the yz plane is so, in both the cases, you can see that as the case for broad side array, the beam width is this is the null to null beam width. This is inversely proportional to the array length in that axis.

In a similar way, the half power beam width can be shown to be that beam width 3 dB in the x-y plane that will be 2.65 lambda naught by N plus 1 pi d. And beam width 3 dB in the y-z plane, it will be 2.65 lambda naught M plus 1 pi into d.

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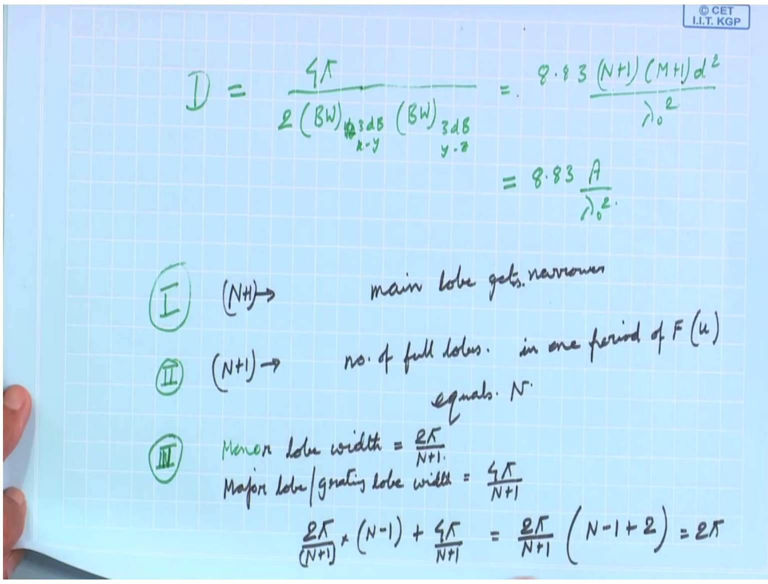

Now, from this, we can find out, since we have in two planes, we have the beam width, so directivity can be obtained. And please note that there are two beams. So, directivity will be 4 pi divided by 2 into bandwidth 3 dB that x-y into beam width 3 dB into y-z, so that if you do, it comes to 8.83 N plus 1 M plus 1 d square by lambda naught square.

Now, we can say that N plus 1 into M plus 1 into d square that is the area of the whole array, so 8.83 area by lambda naught square. The same thing that physical area as a electrical or in terms of wavelength that means, the electrical physical area into the some constant term, so that is directivity is proportional to the area measured in wavelength square. So, this is a general property of area antenna, so in this case also that came.

Now, the question is that this is two-dimensional array that that time we have not covered, but in general, two two-dimensional arrays are common, if we want to have a face directives better to have a two-dimensional array. Now, we will again go back to the linear array, and we will see some interesting results.

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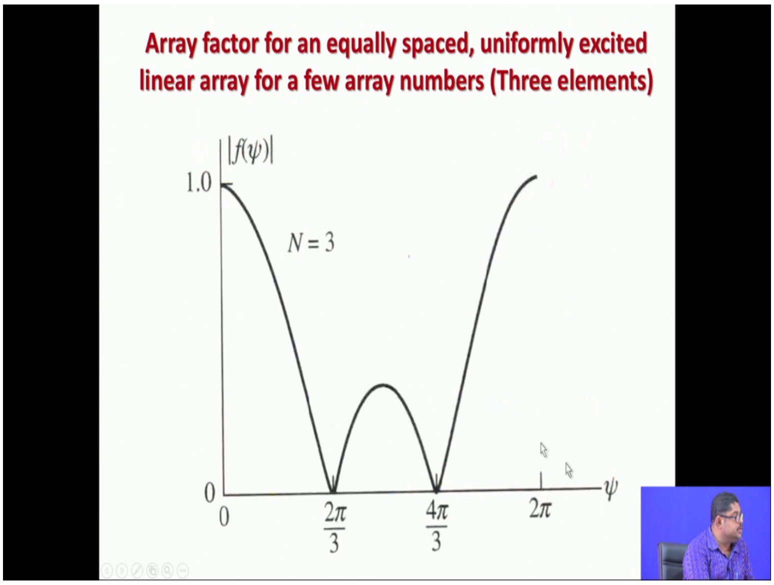

So, let us see the array factor for an equally spaced, uniformly excited linear array for we will see few array numbers. So, here let us say that we have three element array. This is the array factor turns out, here it should not be psi, actually it is u f u, u versus u please correct it, it is wrongly written. Because, we are our thing is u or you can say function of u plus u 0, and this is u plus u 0, so that axis. So, this is for a three element array.

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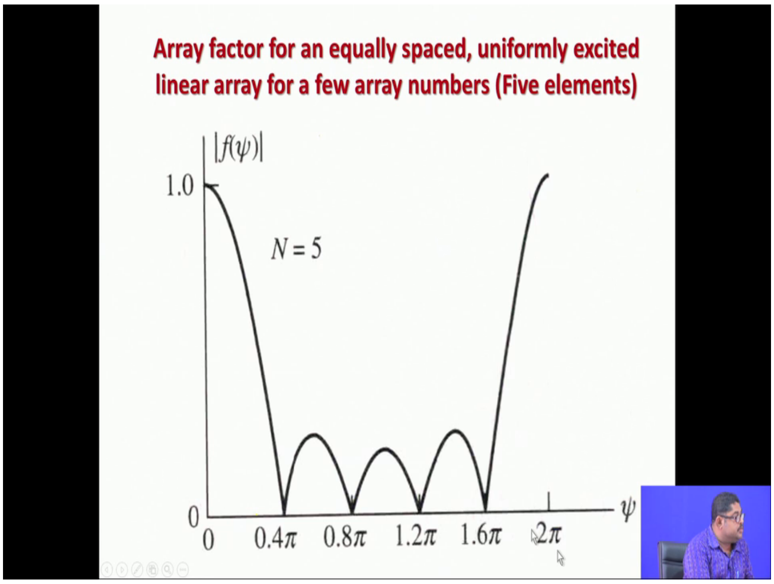

Then if I do for a five element array, N is equal to again it is not N, N plus 1 is equal to 5. So, you see this is the array factor. So, compared to the previous one, you can see that we are having the let me show the previous one, compared to that the N is equal to 5, the beam width is reducing.

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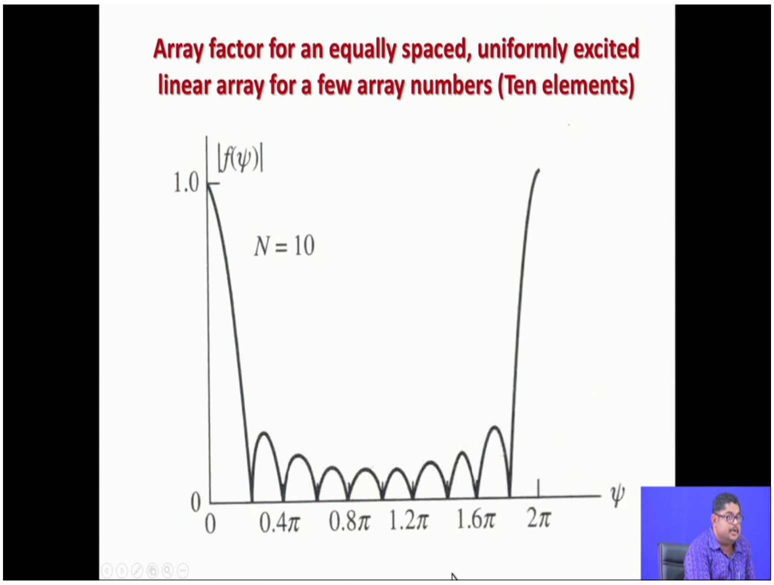

If I go to N is equal to 10, beam width is reducing, but also you see that number of side lobes are also increasing and this. Now, from these three graphs, I am again showing these, we can draw several conclusions. The first one is that as N increases N on N plus 1 in our case, we are considering N plus 1 always odd number, because we are placing a central element at the coordinate of array thing. So, as N plus 1 increases, the main lobe gets narrower. So, this is one of the observation for an uniform linear array, it is always true.

Now, again if N plus 1 increases, the number of side lobes also increases. But, how many, can we say that? Yes, we can say that number of full lobes full lobes means, number of full lobes full lobes means one main lobe plus all side lobes. In number of full lobes in one period of F u one period of F u that equals n, where N plus 1 is our element number that means, if I have three element array, I should have two lobes. You see three element, so this is one full lobe; this is one, so two lobes. Go to five, so we should have four 1, 2, 3, 4; ten, 1, 2, 3, 4, 5 1, 2, 3, 4, 5, 6, 7, 8, 9. So, this is a general thing that number of full lobes will be n, if we have N plus 1 element array.

Now, the 4th property is that or 1 2 sorry this is 3rd property is that the minor lobes if you calculate, it will be minor lobe width minor lobe width is 2 pi by N plus 1. And the major lobe or the grating lobe both major lobe or main lobe and grating lobe major lobe as well as grating lobe width is double of that is 4 pi by N plus 1.

So, you see that total how many things, so I have 2 pi into N plus 1 into can I say that N minus 1 side lobes in a period, because I say total N full lobes. So, N minus 1 side lobes (Refer Time: 17:50) plus 4 pi by N plus 1. What is this? You can take 2 pi by N plus common that gives you N minus 1 plus 2. So, it is N plus 1, so it is 2 pi. And we always say that F u is a repeats periodic function with 2 pi is the period. So, this is also one testing thing.

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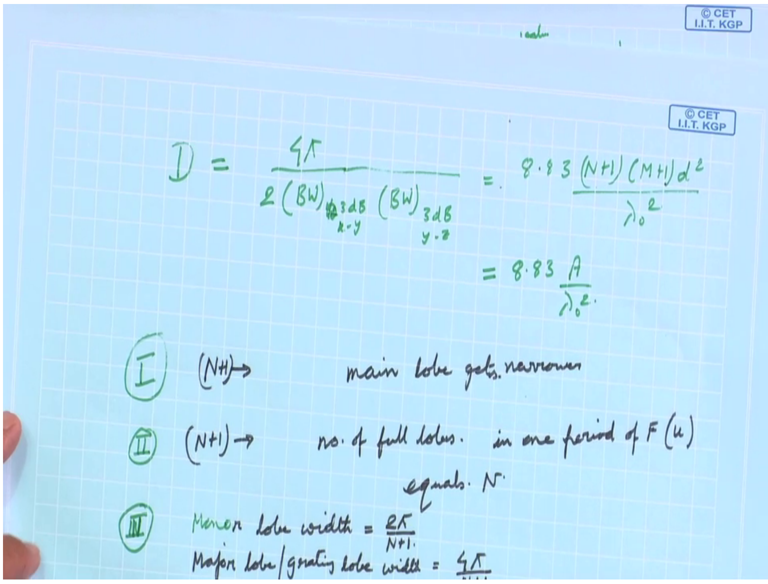

And the 4th property is; what is the side lobe level? We have already defined side lobe level in the basic parameters that is the maximum value of largest side lobe largest value side lobe by the maximum value of main lobe.

So, as N increases, this SLL decreases. So, if you calculate, then for N plus 1 is equal to 5, we get SLL is minus 12 dB. N plus 1 is equal to 20, SLL is minus 13 dB. If you have larger than this, SLL will approach the value that we have already derived 13.3 dB or roughly we call it minus 13.5 dB. So, side lobe level cannot be put below minus 13.5 dB by an uniform array.

Also there is another property that you can easily prove that F u is symmetric about pi. This thing you can see that if this is pi, you see this is symmetric, this point is I think pi.

Then this point is pi, you see it is symmetric. This point is pi, you see it is symmetric, also it is periodic with 2 pi 12 dB.

Now, the question is actually these properties are interesting, but this is actually a drawback, because you see that if I have an uniform array, I cannot put side lobes at my specified values. Supposing some application if I require it to be 20 dB, I cannot do. And in radars, in various applications, we require it to be much lower. And also we want that the you see the gain also, the directivity is also fixed for linear arrays, it is inversely proportional to the given array length, but it cannot be changed, so that actually puts a bottleneck that ok, we can get it.

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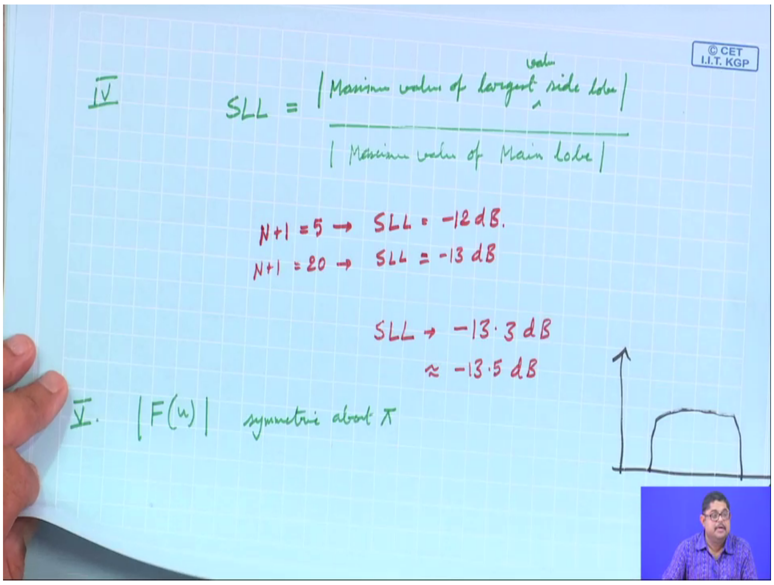

So, we will have to have more elements to decrease the value, but then the problem is that size may not be available so much array length may not be available for an antenna. And also if I want to increase the number of elements, then the element spacing because already there is an restriction on element spacing that I cannot go beyond lambda naught by 2. So, if I want to put more number of elements, I will cross that, and I will invite grating lobe, which is again not desirable.

So, actually you see there is nothing to do from our side much. As a designer, we do not have much to do, if we choose an uniform array, so that is the motivation for going to that can we synthesize any given pattern, suppose someone wants some exotic patterns, suppose I want F u by u. In many cases, we require F u by u to be like this. This is called sector beam that in a particular sector, I want uniform outside that I do not want. But, that type of thing, is it possible or someone might say that I want that I do not want any side lobe or I want side lobe to be very small, I do not care about what is the directivity.

So, these things are not possible with an uniform may, always there will be side lobes, always there will be that throughout the whole space, the beam is extending, may not be the main beam, but the other beams they are always radiating. So, from that place, if there is a strong interference that it will pick up, a strong noise that it will pick up, etcetera. So, this is the motivation for going to array pattern synthesis.

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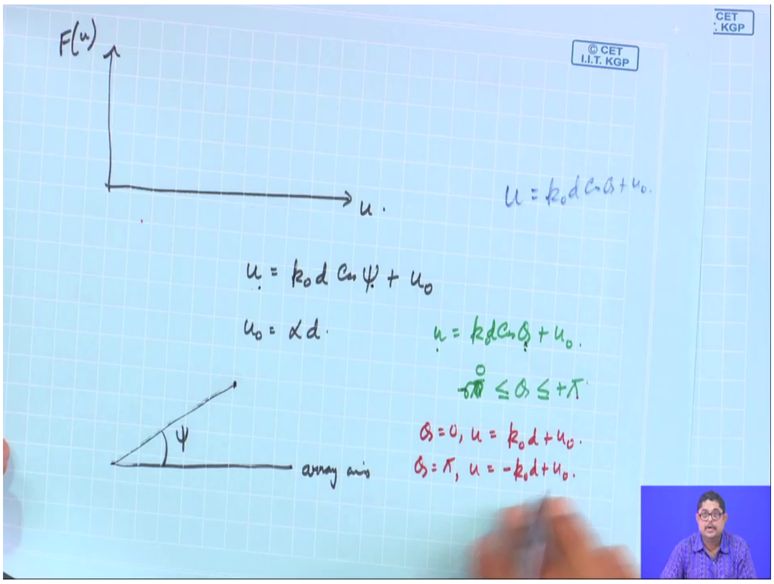

But, before going there, I will see if you see my earlier array antenna things also, there is one thing that I always express the array factor as F u by u. Now, u what is u, u we have defined as k naught d cos psi plus alpha d or I will plus u naught, where u naught is equal to alpha d or beta d as the case may be that means, the progressive phase shift etcetera.

Now, this is a constant in a particular case or anything. The point is you see this u, what is the physical significance of this u. The significance of psi, we have seen that if I have an array axis, this is my array axis, and my observation point is here, the solid angle that the observation point's radius vector makes with the array axis is psi angle psi that was our thing.

But, if I have the array axis as a particular x and y, then I know that what is the value of this psi in terms of theta phi. But, what is the physical significance of u, actually there is no physical significance of u, but it is very easy to understand this whole array theory in terms of this F u function, because this becomes very simple that sinc type of function it comes that is why we do it.

But now, suppose I got a pattern array pattern function, now can I go back to the physical significance that means, if I want to see that in theta plane or phi plane, how it looks like, so that means, we generally want radiation pattern of this array factor to be in the polar plot. So, how to do that? That we will now see.

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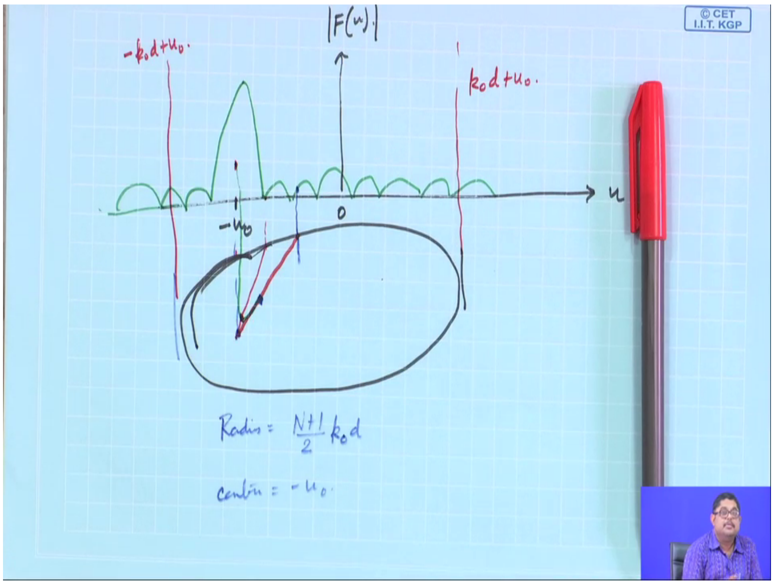

So, in this lecture, we will see suppose I have a F u something like this that this is my u. And this is my I am taking the mod, so there is no negative term, I am arbitrarily drawing it this arbitrary. So, this should be symmetric, so here. Suppose, this is my main beam, I am getting. So, obviously, main beam always we get at a point, where the sin u plus u naught that is should be 0 that means, at minus u naught, this is always true. If for a broad side array, this u naught is 0 that is why it comes here for an (Refer Time: 27:05). So, this is a general thing. So, this is my 0.

Now, the first task is to put the where is the let us say that we are talking of x-axis array, which we did for our non-linear array. So, first let us put that actually this relation you see u is equal to k naught d cos phi plus u naught. Now, this cos phi can be in case of any particular axis, this will be replaced by u is equal to k naught d cos theta let us say plus u naught.

Now, what is actually this is the transformation relation that means, I want it in place of theta u, you see I have written these, because cos psi actually it is k naught sorry k naught d cos psi. Now, that psi in a particular case, we will take the form of either sin theta cos phi or cos theta something. Now, so this is a physical thing. Now, theta can suppose we are looking at that we want the thing that in the phi a fixed phi plane. So, theta, theta is a physical thing, it is the elevation angle that means, with the axis you see.

Now, what is the theta, theta can vary from minus pi to plus pi. So, what will be the visible space, visible space is in the u plane or in the I mean u variable, the visible space will be from minus k naught d to plus k naught d. So, first in this pattern, I will have to look at that. So, let us say that arbitrarily this is my arbitrarily I am saying, because this pattern I have drawn here. So, first I will have to locate the thing, maybe it will so, this value is then k naught d theta can actually in our coordinate system theta is 0 to pi and phi is 0 to 2 pi sorry.

So, if it is 0 to pi, then what is the value. When theta is equal to 0, then u is equal to k naught d, first let me write theta is equal to 0, u is equal to k naught d plus u 0. And when theta is equal to pi, u is equal to minus k naught d plus u 0. So, basically I will have to locate that this is my k naught d plus u 0, this is my let us say minus k naught d plus u 0 ok. So, this is my visible space I want.

So, then if you look at this expression, u is equal to k naught d cos theta plus u naught. Can I say that in the theta plane or in terms of theta, this is an equation of a circle u minus u naught is equal to k naught d cos theta, it is a non-linear relation, but it is an equation of a circle. So, the next thing I will have to do, I will have to draw a circle I will have to draw a circle, where I will draw the circle, the circle will be at centered at u naught the circle will be centered at u naught.

And the circles radius the radius of the circle that will be N plus 1 by 2, so radius is N plus 1 by 2 k what I will calling k naught d, this is the radius, center is minus u naught. So, this is a circle. And this circle, you will see extends this whole visible space, because I have taken this k naught d you see minus k naught so, this is represented. Now, any point you want to put there what you do, you see that.

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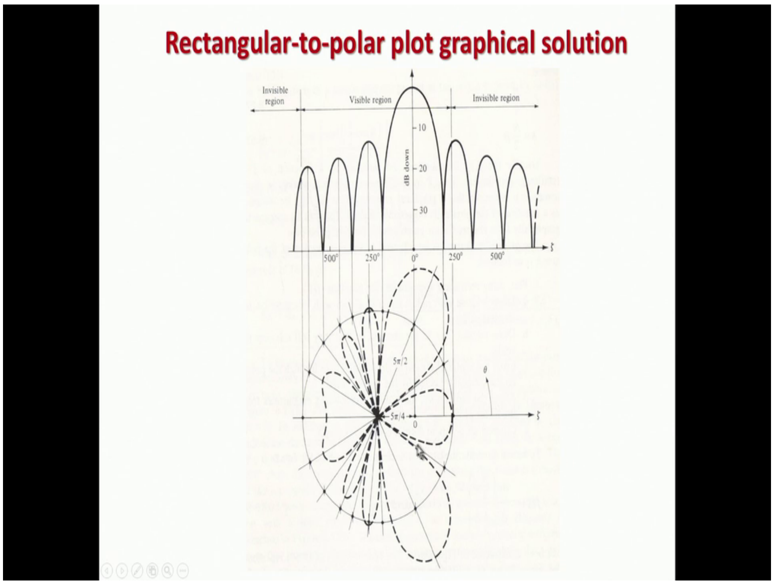

Suppose, this is u plane, so you draw the visible always this circle will be covering the visible region. Now, you suppose actually let us again come here. Suppose, I want what is the value of this point, so what I will have to do, from here I will have to put a vertical line that line will intersect some point like this circle. Now, from the center, you draw and connect this intersection point. And now, from this plot, you find out what is this value in this scale. Now, in this line you plot that maybe it is let us say 2 dB, so it will come here. So, this is a point on the final polar plot.

Similarly, this point, suppose it will come here, so this point may be here, this point let us say here, so it will be somewhere here much lower. So, you see that finally, when I take this plot, it will be something like this. So, every point now you made, and then take a locus of that, that will give you the polar plot as this fellow is doing.

You see that main beam is here in the u plane, it looks that it is in the center, but actually if you put you see the beam is tilted, actually the beam is like this. So, this point will correspond here, so this intersect and then from here, the value we put, and that point is here that point is here. Similarly, this point is put here, but the value is actually here. So, this point he has connected like this. So, finally, this locus is the polar plot.

So, this is the technique that should be used, because this transformation actually demands these. So, by that, now you get a physical insight that ok, where is the beam etcetera. So, with this, I end this discussion. The next lecture, we will go to that what is the synthesis, how to do the attempt the synthesis procedure that we will discuss.

Thank you.