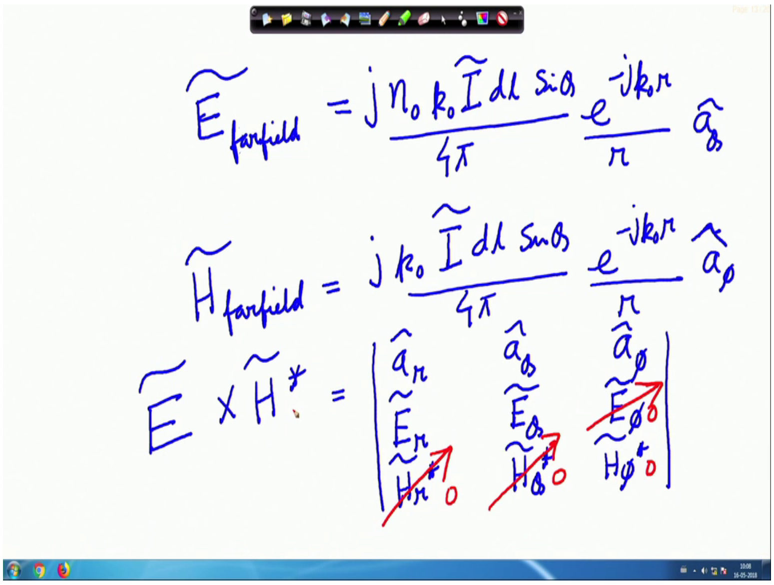
Analysis and Design Principles of Microwave Antennas
Prof. Amitabha Bhattacharya
Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology, Kharagpur

Lecture - 04
Specific Properties of the Radiated Fields from a Current Element

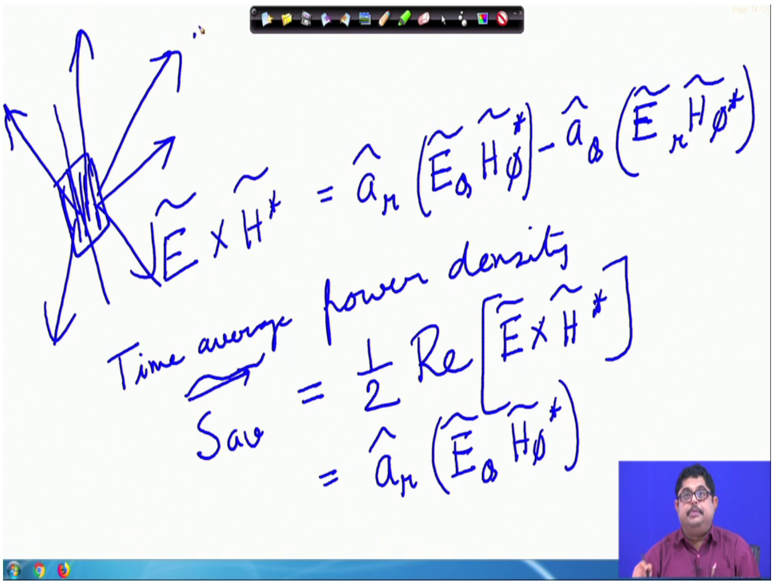
Welcome. So, we have analyze the nature of the fields electric field and magnetic fields radiated by a current element in the last class. Now, we will see that how; what is the proof that radiation is taking place.

(Refer Slide Time: 00:36)



So, we look at the expression of E far field and H far field at a distance from the antenna these are dominating. So, other fields are negligible there so, we write that now we will have to take the pointing vector pointing vector you know E cross H star. So, we will have to take this in spherical coordinate; that means, a r a theta a phi these all you know just I, if any of you forgot that is why I am writing E r, E theta, E phi and H r star, H theta star, H phi star. But you know that in our case, we do not have this, we do not have this, we do not have this. So, these are 0's here and you can find this expression.

(Refer Slide Time: 02:06)

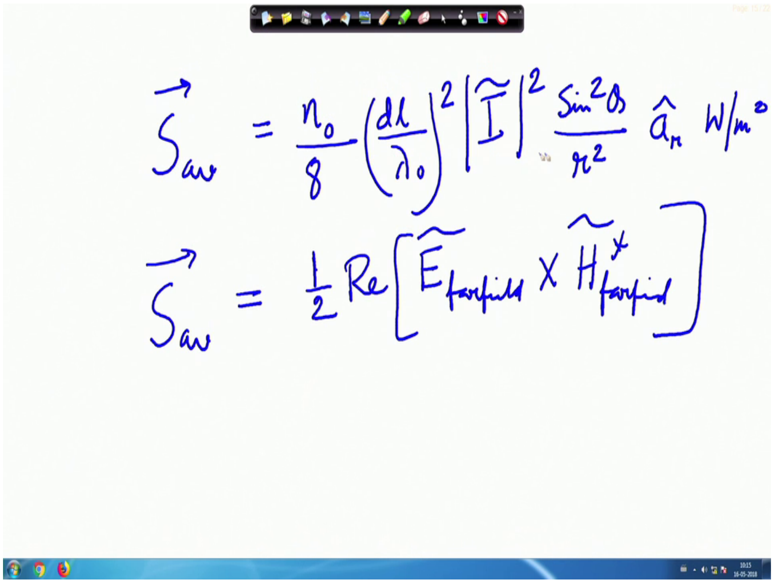


So, you can find that E cross H star that will be having a radial component. So, this will be the expression, now again if you look at you have all this expressions just look at them E r H phi star, that actually is a purely imaginary term. So, in the pointing vector it is present, but we know the time average time average power density is given by sorry yes phasor S av that is half real E cross H star.

So, that ohm given E time average power flow that is only local power there; So, that is nothing, but only a r so, it has only a radial component that means, time average power is flowing only in the radial direction. It is not flowing in any other direction; that means, I have the current element, I have the current element here, this is my current element in this direction it is going in these direction. So, all radial directions it is flowing out, but in no other direction it is flowing. So, if I look from here it is flowing here, if I look from here, it is flowing here, but do not assume that just because I have drawn arrows means it is equally flowing.

I have said that no antenna no real antenna can radiate equally, similarly all direction that will see just now that there is a shape of that, but it is having radiation power is flowing out. So, this is the proof that it is getting radiated.

(Refer Slide Time: 04:49)



Now, let us complete that because, we know the fields so, we can write what is the expression for that, if you do that you will be able to write it in this eta naught by 8 d l by lambda naught whole square, I square sin square theta by r square a r watts per meter square. So, this is the expression so the power flow; that means, the radiation from this antenna that depends on this intrinsic impedance you know it is see 377 ohm or 125.

So, in that terms it can be also said that 15 pi etcetera this it depends on the size of the current element, infinite decimal by lambda naught. Now, in practical cases if I d l by lambda naught a suppose I have a one centimeter antenna, but my wave length is let us say 100 meters etcetera. So, that we can consider as infinite decimal so, current elements are used, then it depends; obviously, on the source strength how much current I am giving.

Then it has a distribution power distribution varies as sin square theta that will see and, also it is inversely proportional to r square, because S average is nothing, but E cross H, E in the far field depends on 1 by r H also in the far field depend in 1 by r. So, their product that is natural it is dependent on r square, that is why we say that power these are all inverse squeal of power, but remember that fields they are 1 by r variation.

And so, this clearly says that this radiation is taking place. Now, instead of going here, you can do an exercise that, I have found it from the actual expressions derived, but this its average power can also be obtained from only far field components. So, if I find half real E far field, cross H far field star that also would give me this same value.

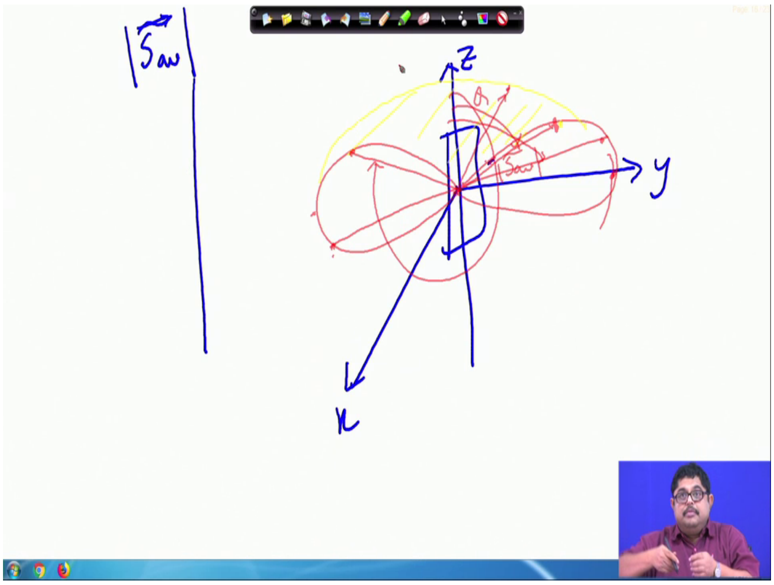
So, it also shows that actually far fields are important, because they are carrying power. Other components was induction fields electro static fields, they are effective locally, but they are locally they do not represent radiation of power, they actually represent local storage, because they are if you do this E cross H star business there, then you will see that they actually give raise to reactive power, reactive power does not flow.

But real power flow that is contributed by far field that is why from the very beginning we have separated them; that means, at a certain distance from the antenna, whatever fields dominate, they are actually important they carry power others do not carry any power. So, far fields are the vehicle for transportation of energy. So, now, we can say that why inductive fields are called inductive fields.

Initially I could not tell it, but now I can say that if you take inductive field terms E and H inductive field terms, electric field and magnetic fields do this pointing vector business, then you will find that they give rise to a j component. Now, that is plus j so, that is why it is called inductive and if it turns out to be minus j it becomes negative usually it is a plus j component of power that is why they are called inductive fields as, if in an inductor there is a locally storage of energy that energy does not flow in a typical circuit also.

So, here also near the antenna there is a energy storage, but that energy is not flowing on the after certain distance that energy starts flowing ok. And, now it is we can also find out you see all these are constants. So, at a particular distance r the radiated power that has a sin square theta type variation.

(Refer Slide Time: 09:53)



So, we can plot S this magnitude of this if we plot. So, suppose what will do that we have x y z coordinate. So, this is our current element this now let us say this is our y axis, this is our x axis, this is our z axis and in this three dimension we will plot this average. So, that thing is so, suppose this thing actually is a three dimensional thing, I have put a cart across this red line. So, it looks like this; that means, cross section of this looks like this.

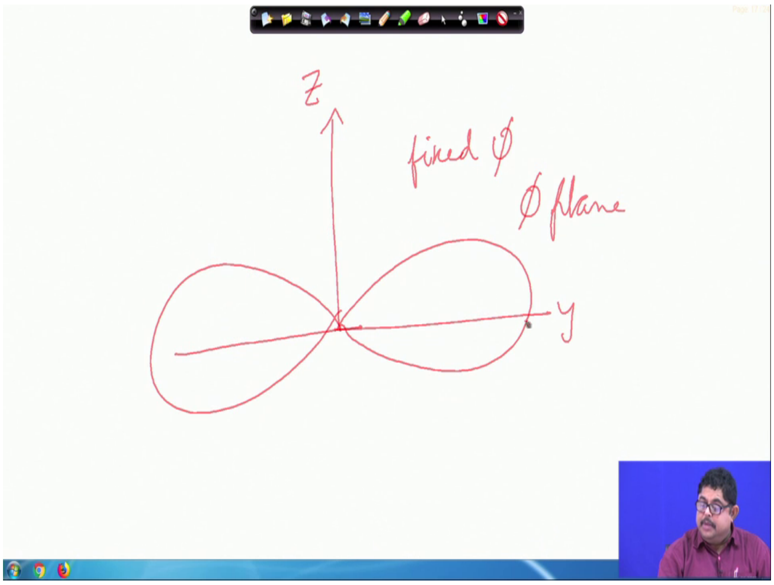
So, what is S average here, actually the S average values we have put it putting here, what is theta suppose this point. Suppose, I am talking of this point, what is the value of S average this length is S average. So, you see at various point this is taking different values. Similarly here what is S average these distance is S average here, what is S average this distance is S average, here what is S average this distance is this distance is the S average. Now, the question is that where it will be minimum and what is theta, you see in this coordinate system this is theta, when I am talking of this point this is my theta, when I am talking of this point this is my theta ok.

So; obviously, it is having a maximum at theta is equal to 90 degree, because this point is the theta is equal to 90 degree and it is having a minimum at 0. So that means, it shows that along the direction of the current element, where the current is flowing there is no radiation you see and, it is having a maximum at a brought side to this antenna.

That means, this antenna is the line 90 degree to that this side this side every side on this line, there is a; that means, if this is a line here is my maximum here is maximum here is maximum and, also you can see that this whole thing actually, if we look at this expression S average does not have any variation with phi that means, that along azimuthal direction there is no variation. So, that is why it is uniformed this shape is called doughnut shape doughnut.

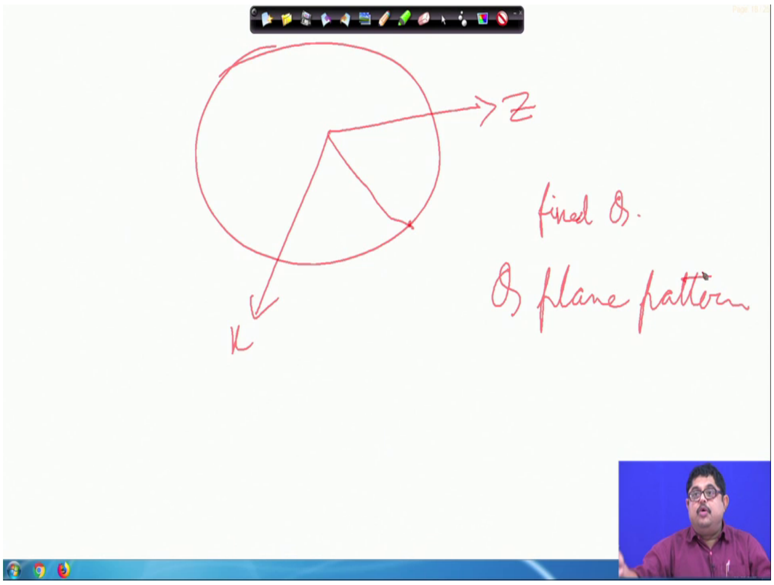
So, you have that there is a this thing if you revolve three 60 degree you get a doughnut shape, but it is difficult to visualize so, what we do generally this type of pattern three dimensional pattern we plot it as two dimensional patterns. So, what is the two dimensional pattern.

(Refer Slide Time: 14:19)



Let us say that I am taking only the z y if I take only the z y plane so, there so what is z y plane definitely I have put some value of phi so, this is called fixed phi that is why this is also called phi plane pattern. So, a phi plane pattern is dumbbell, current elements phi plane pattern is a dumbbell maximum here, it is a sin squared theta variation.

(Refer Slide Time: 15:11)



Similarly another two dimensional thing will be if I fix the theta so, fix theta means you consider that, if I fix the theta it will be a x z plane fix theta and they are the pattern will be something like a circle. So, at any point if I want to find what is the value of S average, this is the value of S average you see always it is same.

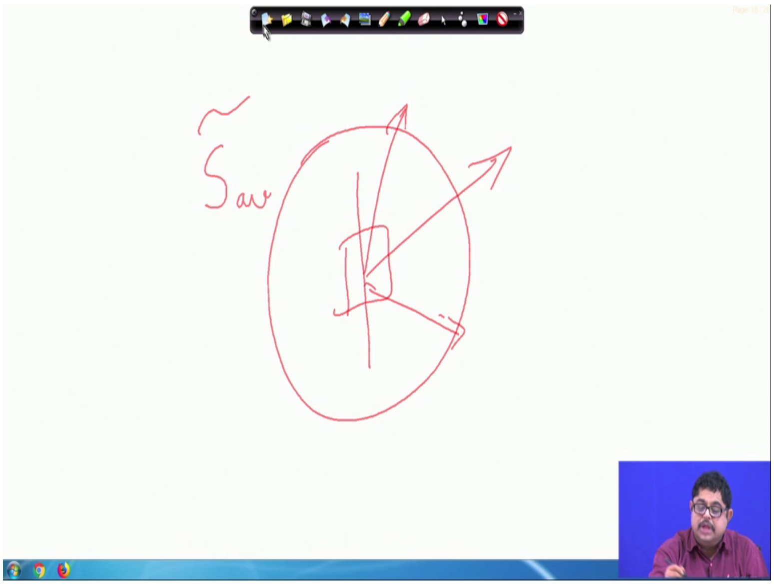
So, this is a theta plane pattern so, power patterns of antennas are like this, it shows that in the azimuthal plane a current element does not have any variation, it is behaves like an Omni directional antenna, but in a phi plane it behaves like a dumbbell. So, it has in this direction it is giving more power than other direction and at the current element line, or axis it is having no radiation.

So, now can you explain why this is so, because the E field and h field both have a sin theta variation. So, E field and h field does not exists along this line actually that is why vector potential exists along this line, because vector potential is cross product of magnetic field ok. So, this power pattern is a simple visualization of how effectively antenna puts it is radiation in a particular direction.

That means, I have say that the beginning of the class that antennas two things, one is how much power it is able to put that means, how good impedance matching it is doing with the free space how much power it is putting and, another is how directed it can make its beam. Now, this pattern power pattern shows us visualization of how directed it is and, we will do the next calculation for how much energy it is putting how efficient it is. So, both these are terms efficiency one is how directed, it is and how much power it can put.

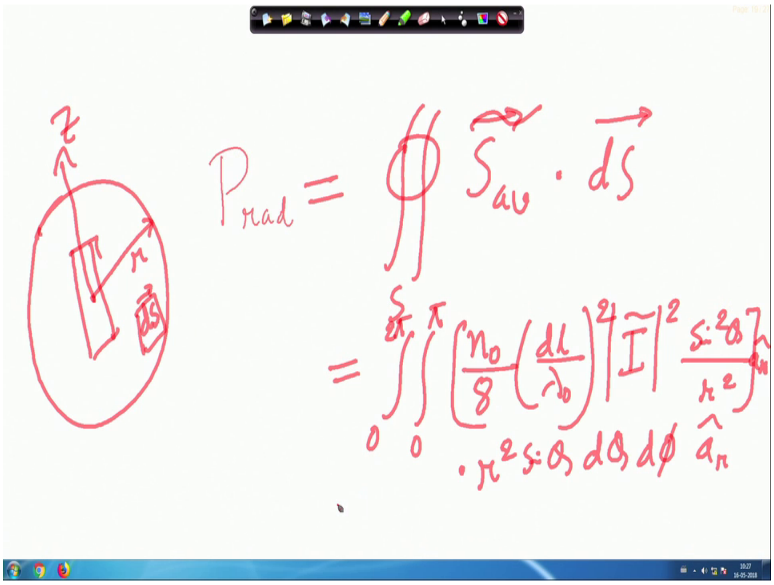
So, next we will do that what is the total power that, this current element is putting to free space; that means, what is the total amount of radiation taking place. So, for that what we can do we have already seen that S average, these the time average power density that is only directed in radial direction.

(Refer Slide Time: 18:17)



You have seen that if we have a current element here, the radiation is taking place only in radial direction. So, what is the total of this radiation taking place, it is we can think that we will have a spherical surface, enclosing these say closed spherical surface if we enclose, enclosing this current element and, then if we integrate this S average there because it will come out of this area.

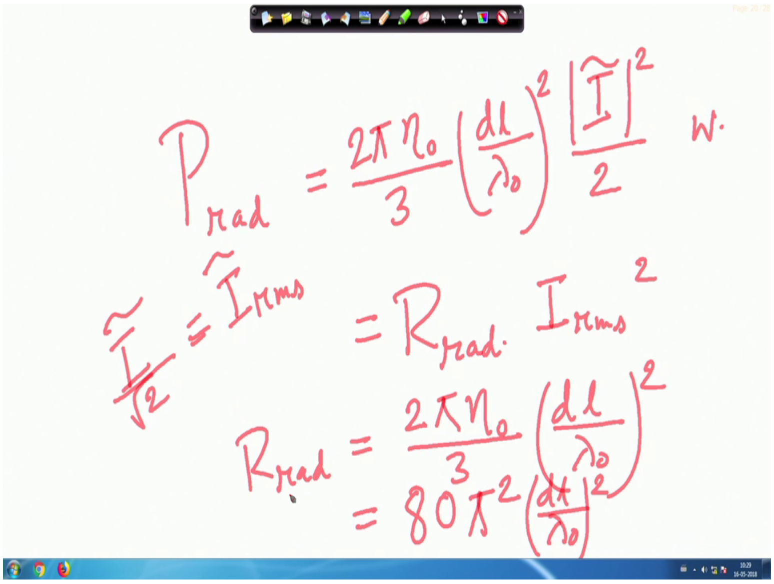
(Refer Slide Time: 18:53)



So, over the whole surface area of this sphere, if we can integrate will get the total power exactly we will do that. So, we can say that total power radiated that will be a closed surface S. So, you can see that this drawing is something like this I have the current element here and this is z axis and, I am having n radius so, that is r and on this there is a d S vector, we know these value of ds vector in case of spherical coordinate, it is radially directed and its value is r square sin theta d theta d phi.

So, we can put that do this integration and it so, I can write this that S average will be already, you have found eta naught by 8 d l by lambda naught square I square sin square theta by r square. This is a r vector dot, this d S vector is r square sin theta d theta d phi a r and, how I wrote d theta means 0 to pi integration d phi means 0 to 2 pi integration.

(Refer Slide Time: 20:40)



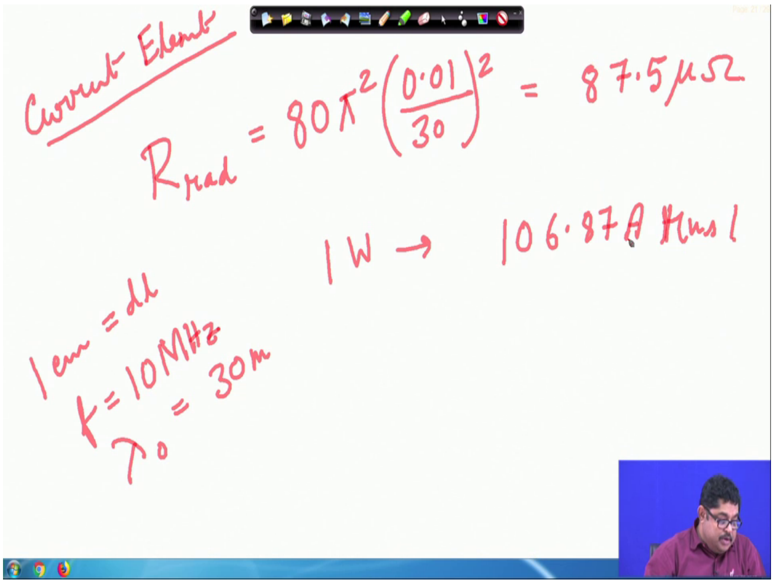
So, if I do this then let me go to the next page. I can find that this will be 2 pi eta naught by 3 d l by lambda naught square watt. So, radiation intensity integrated so, total radiated power is this. Now so, from here actually if I call that I by root 2 I will call it as I rms because it is a sinusoidal power. So, in that terms I can write it as I rms square and the whole rest thing you see power is equal to I square so, who is missing r so, that is called a radiation resistance. So, radiation resistance of an antenna in this case this value is nothing, but 2 pi eta naught by 3 d l by lambda naught square.

So, if you manipulate because eta naught value we know 120 phi so, from that it comes 80 pi square d l by lambda naught whole square. Now, what is radiation resistance; that means, if I want more radiation if I want a more efficient antenna means more period, more period means whatever I am giving is in my hand here, but how much it is sending that is given by this. So, more R rad means more efficient antenna so, this R rad is called radiation resistance. Now, actually we can say that as if instead of an antenna you have a fix fictitious resistance, I am giving I rms current and power is dissipated here in case of antenna power is radiated.

So, it is radiation resistance is a fictitious resistance that dissipates the same amount of power, as that radiated by the Hertzian dipole or any antenna, when both carry the same value of rms current. So, more R rad better remember this is not a circuit theory, where we do not want resistance to be high, because that is the dissipation of energy we that is not useful, but in these case it is radiation so, more radiation we want from antenna, because it is job is to pump more and more power as much it can do whatever I am giving I want 100 percent should be given.

So, more it can give that is better so, radiation resistance higher is better in this case, but if I so this is a performance measure of an antenna for that how much power it can radiate. And already I have seen the directed nature, that was given by power pattern radiation resistance give the how much, it can give and how this Hertzian dipole performs it is very poor in its performance.

(Refer Slide Time: 24:24)

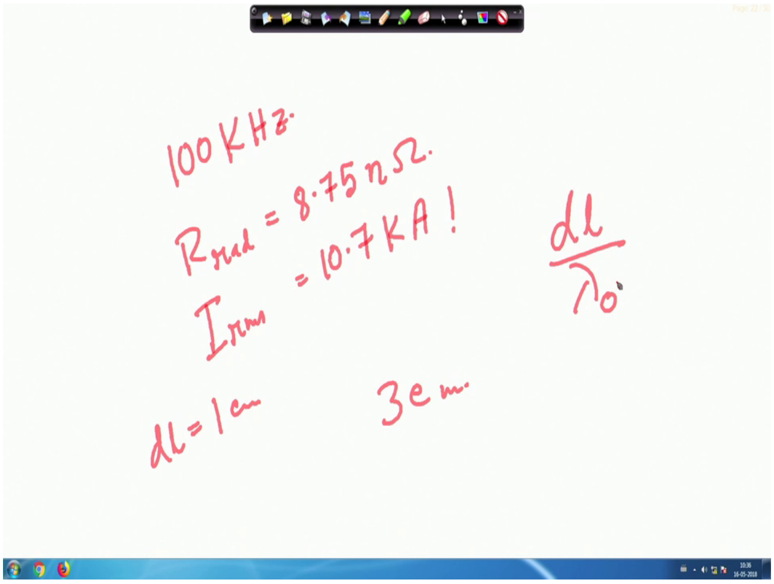


Because if we just calculate the radiation resistance of things suppose I have so, for current element or Hertzian dipole; Let us calculate suppose I have a one centimeter Hertzian dipole; that means, my d l is 1 centimeter and I want to radiate, let us say 1 ohm watt ohm power at which frequency that is important, let us say f is 10 megahertz.

So, I want to transmit 1 watt 1 watt of power is sizable r f power so, at 10 megahertz 1 watt power, if I give actually the you know in cellular base station etcetera, they even 1 watt of power also they do not give they give 200 300 mill watt power. But I am considering suppose I use an current element and in a cellular base station I want to give 1 watt of power at 10 megahertz. Let us calculate 10 megahertz so, it is lambda naught that will be 30 meter. So, R rad that will be 80 pi square is roughly 10 you can say and 0.01 meter and it is 30 square.

So, that will come to be 87.5 micro ohm it very poor that. So, if I want to radiate 1 watt of power, how much current I will have to give the current, I will have to give if we calculate just by I square r so, what is the rms current, I will have to give 106.87 ampere rms current you see, it is a huge r f current 106 ampere, we cannot think actually the current element may burn also from (Refer Time: 26:40).

(Refer Slide Time: 26:47)



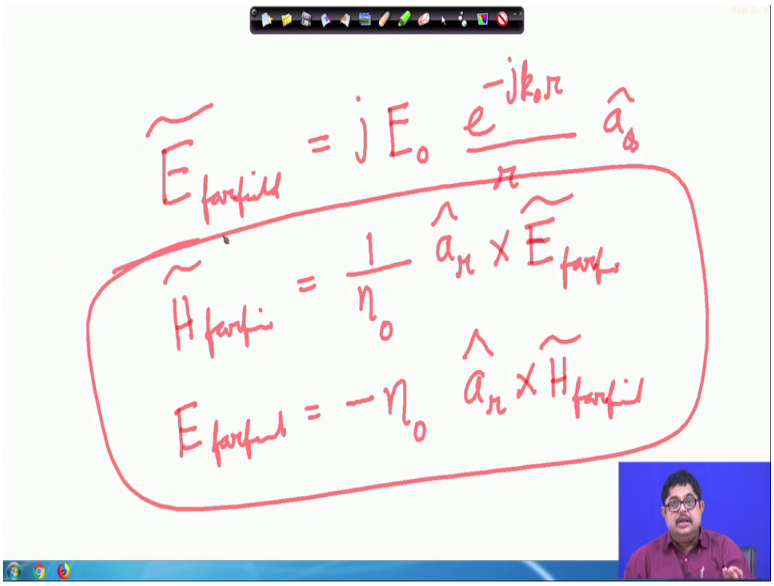
Now, instead of this suppose i want to re do this the same 1, but I change that frequency to 100 kilo hertz, the same conditions one centimeter current element 1 watt of power I want to transmit so, find out what is the radiation resistance radiation resistance will turn out to be 8.75 previously it was micro ohm, now it is nano ohm and I rms required will be 10.7 kilo ampere.

Previously it was 100 some ampere now 10.7 kilo ampere so, you see is practically there is no use for this, but this also this exercise showed us one more, thing you see that why in the next case; that means, in previous case in previous case I have this much it was also in efficient, but next case it became much more in efficient just I have decrease the frequency; that means, I increase the lambda.

If I increase the lambda then the electrical length of the current element, got decreased in the this case and it became much more in efficient so; that means, the more electrical length of an antenna is it is more efficient. So, this is an important thing two of all antennas, that more electrical length of the antenna what is electrical length the length by lambda naught.

So, if I can have more and more of this, then the antenna becomes efficient. So, the same current element, if I operate at 1 gigahertz it will be efficient, but then it would not be a current element remember that, suppose I operate it at 10 gigahertz 10 gigahertz means 3 centimeter the current element itself its d l is 1 centimeter. So, the size of current element is lambda naught by 3 that we cannot call a current element, we will see that that becomes a familiar very efficient antenna near to that, that is a dipole monopole etcetera. So, this is an important thing that dl by lambda naught, this size being small current element is a very inefficient antenna.

(Refer Slide Time: 29:46)



But then the question is that also let me see another thing of this that, if we again look at the far field expressions of the current element and, let me write E far field as j some constant let us say E naught, this is the shape this is the field. So, you see this is some constant, but this is coming from as we have seen that this is solution of wave equation. So, e to the power minus j k naught r by r, this comes whenever I have any AC current. So, actually this is the genesis of spherical waves so, in a where if I have a current varying current, I will get the far field will be of this and what is H far field.

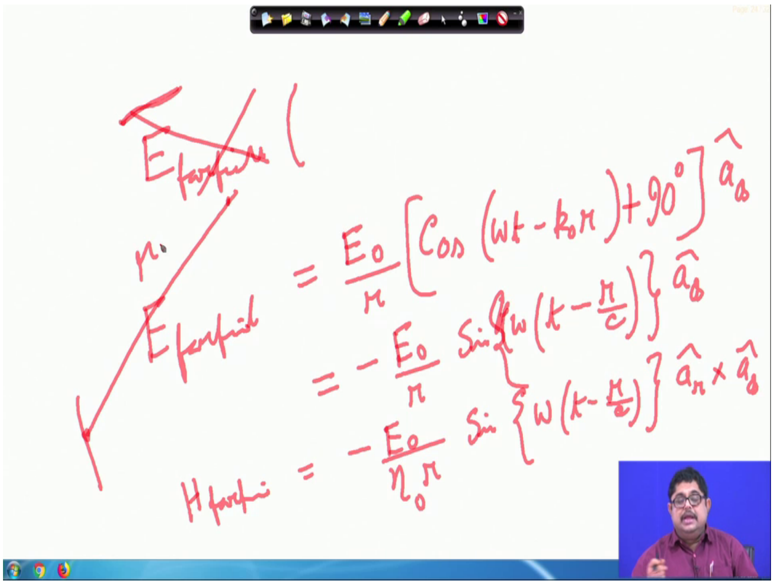
Now, previously I have written now I just want to say, that I can also write it in terms of E far field, you look at the expression and I can write it as E far field, you see that takes care of this direction, because it is a theta that will be a phi. So, a r cross a theta that will give me a phi, but this is a general relation. Similarly this E far field also now I can derive from the H far field, that E far field will be minus eta naught this minus is coming, because of the coordinate system a r cross H far field.

So, this is another way that H far field is a r cross E far field, E far field is a r cross H H far field. This shows that E far field H far field and a r, a r means the direction of power flow direction of wave, they are at right handed t plate now; that means, something like plane wave, in plane wave also we have seen the same thing the wave propagation direction E field and H field they are orthogonal.

It shows that in the far field of an antenna the electric field vector, magnetic field vector and the direction of wave propagation, or direction of power flow, they are also forming a t plane. So, they are locally plane wave, but they are not strictly speaking plane wave, because if they are plane wave this e to the power minus j k naught r by r term would not come.

So, it is this divided by r that gives it a plane wave form, but at a sufficient distance locally over certain distance we can consider it as a plane wave. Now, this is true of all the antennas though we have derived it only for these, but people have derived it generally that far fields of all the antennas have this type of structure. So, this is an important evaluation and from this we can also just find the far time domain expression of the far field.

(Refer Slide Time: 33:15)



So, what will be the time domain value of far field, if I say instead of phasor sorry far field that I can write just by I will have to multiply E to the power j omega t take the real part. So, by doing that I can write it as cos omega t minus k naught r k naught r is coming from e to the power minus j k naught r plus, there is a j term that is why it is j and then a theta and so, this can be written as minus E naught by r sin.

If we put k naught value we can write sin omega t minus r by c a theta and H far field as minus E naught by eta naught r sin omega t minus r by c a r cross a theta. So, this also shows that in time domain the fields, they are delayed by a delay of r by c, that is obvious if I have a current here, then it require some time to go to this point the fields does not reach immediately. So, this is the delay that delay corresponds to the distance. So, from here r and it propagates by velocity of light; so, wave that has a finite propagation time to reach a point instantaneously.

So, this shows that actually the analysis of this current element as reviled many of the properties of the antenna which are not simple current elements, they are very complicated some antennas exotic performances. But the general structure of the fields are same far fields are same, they always show there is a radial flow of power, there is a triplet locally they are plane waves in the far field. They also have this time delay they have the space delay, according to the wave that created them e to the power minus j k r k naught r by r. And, they have these all properties that is why we studied current element, current element is the very inefficient radiator, we will try to find that.

But before that we have not answered one question that is; what is far field? Because, we have said it is far from the antenna that starts dominating one thing, we have derived that magnitude wise the far field, becomes comparable to the induction field at a distance roughly r by c I lambda naught by 6, but is that the far field. So, in the next class we will say that for all antennas, what is the criteria of far field and what is the exact far field.

Thank you.