Analysis and Design Principles of Microwave Antennas Prof. Amitabha Bhattacharya Department of Electronics and Electrical Communication Engineering Indian Institute of Technology Kharagpur

Lecture - 40 Antenna Measurements

Welcome to this 40th lecture of NPTEL course on antenna. In this lecture, we will see the Antenna Measurements. For last 39 lectures, we have seen various analytical methods to analyze the antenna; we also took the help of various numerical techniques some numerical techniques to have the antennas analysis synthesis etcetera. Often many antennas because of their complex structural configuration and the excitation methods cannot be investigated analytically.

So, though that type of antennas are falling day by day in number because, various modern analysis electromagnetic analysis tools like CST microwave studio and others or based on moment method or finite element method etcetera or DTD or TTD. So they have been developed. So, that various structures do, but still in some cases like one example I can give that this UWB antenna with metamaterial lens. It cannot be at presence till now cannot be analyzed with any structure electromagnetic tool.

So, that type up always with the new developments. There will be some instances which cannot be done by the things. So, that is why always we need also an engineer's antenna, engineer should know that how to measure antennas various parameters important radiation parameters. And also it is required because by analysis what you are getting may be by the tool or by yourself whatever that needs to be validated with the measurements.

So, measuring antennas parameters is an important thing, but like any microwave measurement this antenna measurement also has certain drawbacks or certain chance of error and certain complexities. One of that is suppose I want to measure the pattern, now the distance of the antenna needs to be put at 2 d square by lambda.

Now, if I do not do that I will get something and if I am not aware that it is measurement has been taken in the near field; then I will get some wrong pattern. So, that is the thing that whether we are really in the far field or not that needs to be asserted that depends on what is the antenna, which antenna is radiating in size etcetera also what is the wavelength etcetera.

Also in many cases it is impractical to move the antenna from the operating environment to the measuring side. I can take the example suppose you want to measure a ship borne antenna. Now it is if you take it from ship and misery it will have some different radiation characteristic, then in it is operating environment because, of mutual coupling and proximity of other things etcetera. But ship you cannot come and put into an measurement things, so but people have welcome that how to do that, but still these are all challenges.

Then in outside today it is very difficult to get a clean environment because, there are mobile phones everywhere there are various communications everywhere. So, always you get some interfering signals in outdoor, so is the indoor safe problem is in indoor you do not have that much space to have really a far field range, then obviously microwave measurement experiments it is instrumentation there quite expensive.

So, all these poses problems but we will see that the people have developed some special techniques to overcome that, one of that is there is a technique that you measure some antenna in the near field, you can by signal processing you can get a near field to far field transform and you can predict what is the far field.

So, also so that is why also there is a thing that suppose you do not have a very large space, you can use reflectors and make plane waves because, in far field actually we assume that the waves that are coming on the antenna those are plane waves.

Now to create plane waves you would require a lot a lot of distance, but you can use reflectors like our parabola reflector, so that reflector very fine reflector very high quality reflector and in a short space you can put plane waves those are called compact ranges. Likewise people have done also the interference problem. People have solved there made anechoic chambers and so that no outside echo comes, no inside echo comes. So, these are various things so we will see some of these the first one where the measurements are takes place they are called Antenna Ranges.

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ANTENNA RANGES	L.I.T. KGP
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Antenna measurements are usually taken so they are called ranges. Now, in general there are 2 types of ranges one is reflection ranges, another is the free space ranges.

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We can see the diagram it is the reflection range you have a transmitting antenna usually lock periodic antennas are a good candidate, then at a height and in a suitably designed height you have the transmit receiving antenna and you see that the ground reflection from the range surface that is. So, designed that both the directory and the reflected ray they go here simultaneously; so there is a construction a thing constructive interference and by that you measure the pattern. So this is called reflection type ranges.

So, this actually creates a place here the by proper design, it creates a constructive interference in the region of the test antenna which is also called quite zone. So, there you the specular reflections from the ground are utilized here to do that.

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Now, there are another type of ranges which are called the free space ranges. So, in free space ranges you assume that there are plane waves coming. So one of the thing is you can have elevated ranges like this you see source antenna and test antenna and in between there should not be any obstruction and also you see the ground should be terrain should be smooth and from the terrain there should not be any reflection.

So, then source antenna is illuminating and test antenna is getting. But nowadays with the high rise buildings etcetera it is very difficult to get these, previously there where in various places people used to have antenna ranges for this. But nowadays this thing has gone generally.

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So, people nowadays what they do, so they make the slant ranges. So, in slant ranges, you have the test antenna is put on a tower and source antenna is not on a tower it is on the ground it is put and by suitably designed also you remove the specular reflections. Now, slant ranges are better utilize the space because for them you do not require a very large tower as the case of elevation in this etectetera, then people have come up as I said with a compact range.

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In compact range what the, it is run the test antenna is put as a feed of an offset reflector and this feed is put to a range reflector. This is a very high quality high precision it is actually RMS error as we have seen when we are discussed paraboloide thing that RMS error is very small. So, after reflection to these this becomes a plane wave. So now you can put the test antenna it is here and you can make your things.

So, this is a feed antenna which is illuminating and within a very short space you can create plane waves by this. So, the test antenna gets illuminated here by this but these are quite costly, but modern this aviation industry then missile industry etcetra, then aircraft industry they have this type of ranges.

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Then you have anechoic chambers. Anechoic chambers means, you put absorbers throughout and then you have a source and internally you see from here from the side walls there is reflections. So, it is so designed that all reflections come together constructively, also there are anechoic chambers are sometimes called semi anechoic chamber.

Actually if you do not have the ground reflections considered then it is called semi anechoic. But if you have ground reflection that is a full anechoic chamber, it can have a rectangular shape and there is always a quite zone created where you get good constructive interference here. So, that quite zone typically 3 meter by 3 meter or 1 meter by 1 meter etcetera, so that actually is a characteristic of the thing. Now this absorbers

there RF absorbers, they are quite costly and their size is important actually they work on the principle that they absorb the weight they do not reflect at all and also outside there is a metallic enclosure, so that from outside no a thing comes.

Now, the source usually is an wideband source as we have saying tm horn that is a typical source for UWB applications. So, you can have this now one is these anechoic chambers are proliferated various small establishments also have this anechoic chambers.



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Also there is a tapered anechoic chamber, so here you see that side reflections do not come because of this tapering, so it also makes the space more efficiently utilized.

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Now, these are some these are observers.

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Actually this is also another thing GTEM cell actually in a small space, if you want to create a plane wave that GTEM cell gigahertz transverse electromagnetic cell they also give you plane waves or TM waves in a very short zone that is their quite 0, so this is also used for testing inside lab. So, otherwise the anechoic chambers they are put on a separate building or separate room is required for that.

Whereas, this is on a UC lab table you can also have a GTEM cell. So, this also if you have storage of space, you can easily this is simply metal thing the main thing is that design actually how the it is a transmission line based design, but gigahertz thing microwave things can be tested here. Inside some absorbers are put because in spite of all the design things, if any high frequency things come or non planar waves comes then that gets absorbed.

Now, this is an example you see a car getting tested, this is a in a one of the Indian labs you can have these that whole car so it can rotate also and also in many modern anechoic chambers you do not needed a rotation, actually the sensors are here throughout and they are sensing. So, what rotation was used to give that information you can get from the sensors. These sensors are again council so that no reflections etcetera come from there. So, they are sensing there is a source somewhere.

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So, you see some nose cone of a missile or some aircraft that is getting tested. So here is an rotor I mean is a stand which can rotate. (Refer Slide Time: 14:32)



So, this is a example of various a things.

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Now, we come that generally we have already seen that one of the measurement things in these chambers or antenna ranges is the first thing is we want to have a pattern measurement. So, for pattern measurement we have seen that spherical coordinate geometry is required and you can have basically it is a 3D pattern. But generally we are interested in the principal plane patterns, so we keep some fixed. Let us say phi as we move then that gives this is an example of an elevation pattern.

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So, here for various values of theta sorry various values of theta you can get this pattern, the other one if you fix some theta depending on which principal plane you are interested and if you vary it in the phi you get that this is this may be called a plane as a azimuth plane equator is one or you can have in other places also. So, this is geometry actually we have already discussed theta earlier.

Now instrumentation for a typical antenna range measuring system here, you see you have a test antenna then this thing is called test positioner, it is job is to rotate along the theta phi actually the distance is known. So, in that previous geometry you see the R is fixed. Now only variable needs to be theta and phi, so to be able to vary theta phi a positioner is needed.

So, positioner can be controlled by some servo mechanism, so there are positioners which can be either move in theta plane or move in phi plane also; this is the from the test antenna you are getting the receiver then position is getting control. So, test indicator is giving in the pattern is getting recorded here automatically this side is source antenna source signal etc, so this is a typical instrumentation. (Refer Slide Time: 16:49)



Now, as I saying that, this means azimuth over elevation positioner. So, here it can rotate theta rotational axis. This is phi rotational axis.

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So these are called azimuth over elevation positioner or if we want to move rlevation over azimuth that means, first you have move azimuth; then it will go to elevation. So you see this type of thing.

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So, here you can see that there are gears etcetera there and by that it can move in this positioner thing. So, these are nowadays available or you can make your own also if you were running short of money; particularly in academia, we make this type of things on our own.

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Now, apart from pattern; so from pattern we will see what are the interest actually in analysis techniques also, we have seen that from a radiation pattern. We try to find out the 3D beam width. If in 2 principal planes, we get beam width from that we can make.

But another thing is that another interesting thing is another important parameter is finding gain. Gain of an antenna gain we defined as a function theta phi, but the maximum radiation generally we call it gain. So, for gain measurement we assume this is the structure and there are various gain measurement options (Refer Time: 18:20) before that.

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So, we will be referring to this, but now we will see that how the ranging ranges measured we have seen Friis transmission equation. Now there are 2 types of gain measurement: one is called absolute gain; absolute gain measurement. There is also another technique which is called transfer gain measurement or gain comparison also that is called transfer gain. Now, we will discuss one by them because this is an very important thing everyone should know how to do it. So absolute gain first I will talk about.

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Now absolute gain measurement; now there are if the antenna whose un gain is unknown you want to measure, if you have 2 search identical antennas, then we take 2 identical antenna technique. So, in this technique we assume that polarization is matched, that means again referring to this diagram that polarization is matched and maximum directional transmission and radiation they are aligned with.

So, that time we have seen that from Friis transmission equation, ultimately keeps us that Pr by received power by the received antenna by the power transmitted, so they have this relationship 4 pi R whole square G ot G or. So, where this is the maximum gain, so this is all aligned. Now this is transmitting gain transmitting antennas gain this is received antennas gain.

Now, this equation if we convert to dB because gains etcetera generally converted to dB. So, we can write got dB plus G or dB is equal to 20 log 10 4 pi R by lambda plus 10 log 10 P r by P t ok. Now if 2 identical antenna means my G ot and Gor are same. So, if that is their so I will have G ot dB equal to Gor dB is equal to half of this 20 log 10 4 pi R by lambda plus 10 log 10 P r by P t.

So, in this method you see I can easily find out gain by measuring, what are the measurement things I need to measure one is R, what is the distance between the 2 then what is lambda. That means, what is the wavelength or what is the frequency and this ratio P r by P t so this can be done. That means, the requirement here is the unknown

antenna should be available in identical pairs. Now if that is not there, then we have 3 antenna methods.

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So, that means, suppose a is an antenna which is unknown, I need to have 2 more antennas let their name is b and another name is c, now I even do not need to know the gain of b and c but I require 3 antennas. So, actually 3 measurements need to be done one is the same measurement as we have done for a b first I will make a pair of ab, that means a is transmitting and b is receiving.

So, the equation I can write is G a dB G a is the gain of a antenna plus G b dB is equal to 20 log 10 4 pi R by lambda plus 10 log 10 P rb by P ra, then I will make a c pair this is transmitted this is received. So, I can make Ga dB plus Gc dB is equal to 20 log 10 4 pi R by lambda plus 10 log 10 P rc by P ra and then I will make bc pair. So, Gb dB plus Gc dB is 20 log 10 4 pi R by lambda plus 10 log 10 P rc by P rb.

Now, you measure R lambda and these 3 ratios P rb P ra P rc P ra and P rc P rb, you see that if I do that everything is known this is 3 equations in 3 unknowns so this can be solved. So, you can get all G a G b G c, so an unknown antenna thing only thing you will need to have this power ratio measurement and capability then you can do. Now, I come to another more popular method is called that gain transfer method of antenna measurement or gain comparison.

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Actually this requires only one unknown antenna any antennas gain can be measured; only you require a non gain antenna. So, actually there are standard antennas particularly in standard antennas are generally made of 2 types, one is either a dipole a half wavelength dipole it is gain is 2.1 dB or horn antenna you have seen that any required gain can be found.

So, measuring the dimension of the horn you can always find so these are mainly standard antenna. So, you need to have a standard antenna for measuring these. So, what is done it is a very simple experiment so standard antenna is required. So, you know it is gain well let us call that Gs is the standard antennas gain.

Now, to the antenna unknown antenna here I am calling test antenna and it is gain let us say G test. So, you do 2 measurements in this here only required to measurements you do not have to because, here you see in previous cases you need to measure R lambda and these ratios here only 2 measurements; one is you actually have that same thing instruments that have a very reviving transmitting antenna.

Now, in the receiving antenna first you put here, so the first measurement time you put the test antenna as receiver receiving antenna. There should be a transmitting antenna you should not ask that there is an arrangement that there is a distance all those things.

Now, here you measure received power actually even if you do not have a power meter, you just take the power from the received antenna and measure it even through a VSWR meter actually in you can do it use a VSWR meter and that is connected to a match load, so that there is no reflection from that. So, there you put something actually instead of a if you have a power meter you can use otherwise you may VSWR meter also you can put and note where it is giving the thing. So, measure the received power let me call it P test into a matched load.

Now, nothing should be changed only change the test antenna and replace it with a standard antenna. So, standard antenna as receiver then again you measure same thing. So, P I am calling P s

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(Gt) dB + (GI text) dB = 20 Log10 (4TR) + 10 Log10 (Ptext: Pt) $(Gt)_{AB} + (G_{0})_{AB} = 20 \log_{10} \left(\frac{4\pi R}{n}\right) + 10 \log_{10} \left(\frac{P_{0}}{P_{4}}\right)$ $(G_{Abul})_{AB} = (G_{0})_{AB} + 10 \log_{10} \left(\frac{P_{Abul}}{P_{0}}\right)$

Now, you see what is the those 2 equations become, so I can write G t dB plus G test dB is 20 log 10 4 pi R by lambda plus 10 log 10 P test by Pt and G t dB plus Gs dB is 20 log 10 4 pi R by lambda plus 10 log 10 Ps by Pt. So, you just after this equation what you get is G test dB is Gs dB plus 10 log 10 P test by Ps that is all so know everything.

So, you can find out the unknown thing so this is gain transfer method, so I think all of you will be able to measure gain. Next the again directivity you can always measure you measure the if.

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 $JZ_{A} = \mathbf{F}(BW)^{e}_{3dBE} \times (BW)_{3dBH}^{e}$

So, next is measurement of directivity because gain measurement directivity also needs to be measured. So, directivity you know that approximately directivity what we can do you take 2 principal, if principal E plane and principal H plane pattern and from this pattern you can easily find out the I will write b beam width 3 dB E. Let us say it is in radian similarly I can find out beam width 3 dB H in radian.

Now, you can always say that the beam solid angle, that means the beams main beam that is nothing but that is generally called antennas. So, this is product of these 2 actually for horn we have used these into radian, so what is directivity 4 pi by sigma A. So, you can easily find. Now if instead of radian you have degree then this you can easily convert those are thing. Now, next is once I know this, then a very important characteristic of any antenna is radiation efficiency.

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I find that in our country people are very unaware of this and how to measure these people do not know, so I will say this that what is radiation efficiency we have already defined it. Now this is nothing but gained by directivity where in the direction of maximum radiation. So, we know how to measure gain we know how to measure directivity so we can find that.

Also we have seen I think the next, we have seen this while discussing basic parameters that antennas radiation resistance and antennas loss resistance, they for simple antennas they come in series in complicated things they are not. So, there you can measure in those complicated cases otherwise these radiation efficiency also, if you know the radiation resistance then that time also we have derived this R r that this R l plus R r.

Now, the question is measuring this R l is the not so easy job in either in many cases, it particularly for dipole etcanalytically people have found out that serves good things. But otherwise if you want to measure that then there is a technique particularly nowadays with planar antennas this R l finding is difficult, because planar antennas they have dielectric as well as conductor, so there how to model that R l.

So, for simple antennas there is a method actually all the simulation software nowadays this radiation efficiency you can find out, that is one thing actually I will see that in various conferences people present the paper, but they are unaware that this can be measure. But this is a actual proof that radiation efficiency is the proof whether antenna is radiating, not the radiation pattern not even the gain a non radiating thing also may have some special gain.

But now I will say a very important a think method that was suggested by wheeler for this planar antennas. Small antennas it is it is an approximate method but very useful method it is called wheeler cap method. There are also other techniques some waveguide techniques etcetera.

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I am not going that way so the idea is that the losses this R l is a function of the currents that gets generated on the antenna, currents in dielectric or in conductor that radiating on the radiating surface of the antenna what are currents are getting generated that is the thing for loss.

Now, what wheeler suggested that small antenna, suppose I have a antenna now you put a metallic shield over the antenna he called it cap because, if you have a planar structure it will the cap will be something like a sphere hemisphere. You put a cap on that so what will happen the radiation will get stopped, but his assumption is the loss will remain same. Now what is assuming that this is sufficiently large this is a metal that there is no interaction between the antennas currents and this metal that is the assumption.

Now, that is not actually true but for small antennas it is you may people have found out that it is not even one percent error. So, what is says that now you measure that means

you measure the radiation input impedance radiation part once with this metal cap. So, when it is with metal cap you are getting the radiation, this is input impedance will consist of sonly R l and when you are removing this metal and measure it again you will find it to be actually both radiation resistance as well as loss. So, you can find out the loss that is the idea.

So, I will now tell you that originally wheeler recommended that the radius of this hemisphere that should be lambda by 2 pi. Now later people have so this is called wheeler cap later people have modified this thing also. But I will try to give an example of this method, so what is done that you see the any antenna there is a feed then after that we are assuming that there is a loss resistance, this is the antenna started from here after feed line antenna started.

So, what is the antenna model it is a resonating structure, so it has LCR and this is the loss resistance loss c. So, this R is comprising both radiation resistance as well as the loss resistance.



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Now, measure the a what is that the return loss, so you will get you see with the cap you will get something like this and without the cap these. So, from here you can find out what is the loss in without the cap that is calling L r and these you see that at a non resonant point actually antenna is not radiating. So, this is actually the loss resistance L r

ok, here you see obviously these 2 frequencies are getting shifted but that does not change the (Refer Time: 41:21).

So, what is doing in from this measurement from the measurement of this diagram and this L r, so that means he now knows what is L r ; far L r far means actually due to the loss only and L r this means that this is the with without cap, that means with radiation resistance. So, what is finding that we can always write what is the Rs that was shown in the circuit, that means the total resistive part of the input impedance that can be always written as Z c 1 plus rho res by 1 minus rho res where Z c is the characteristic impedance of the feed line. The feeding structures transmission line these are from the end, what is the value of this because actually we have measured L r. So, what is the relation of the magnitude part of the reflection coefficient this can be this you can easily find out that this is L r minus L r far by 20.

So, Rs is the what is Rs resistance of the series resonant circuit, already you have shown that Z c characteristic impedance of the feed line in ohm, rho Rs voltage reflection coefficient of the resonant circuit. Dimensionless L r res return loss at the resonant frequency return loss at the resonant frequency that is why it is called L r res and L r far return loss far from the resonance. That means, far from the resonance there is no radiation resistance, so it is basically due to the loss far from the resonance in dB this is also in dB.

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So, now that what will be the radiation efficiency eta r is you can easily derive this from whatever you have seen minus Ls by 10 R s2. So, I have got 2 Rs because of 2 measurements so R s2 minus R s1 by R s2, these Rs I am doing 2 experiments one is with cap without cap. So, R s2 is without the cap R s2 is without cap and R s1 is with cap no.

So, and actually you can say that when only Ls is present we can say that Ls is equal to L r far by 2, because that time only L r is there. So, half of the voltage will be there half of the loss will be in the Ls part so this is wheeler cap thing.

So, by this you can find the radiation efficiency and there is one more thing that I want to say that is the impedance measurement. So, I will just I think all of you we have in earlier classes also, in the basic building blocks we have said how to measure VSWR by this what how to measure impedance of an antenna.

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This is a final thing impedance measurement; again I refer to that I have a generator. This diagram we have seen earlier then we have a transmission line that is going and then there is a it is connected 2 antenna that was our a b point and this is our antenna. So, the impedance that is seen from here this is the transmission line. This will I should say the transmission line.

So, here there will be a reflection coefficient and let me say that P incident is the power going here and obviously this antenna is as will be load. So, depending on the obviously the antenna has a reactive part, so characteristic impedance is a real thing for transmission line. So, there will be a reflection this is P reflector so that is all. So, I can always write P reflected by P incident is this is a voltage reflection coefficient.

So, this is equal to z antenna minus Z c square these are all known expressions just to recap this and this can be always related to the VSWR minus 1 by VSWR plus 1 whole square this is all of you know. So, basically you can always find out what is the VSWR from these transmission line techniques and finally we want to know that this is the magnitude we are getting.

But to find the reflection coefficient I need the apart from the magnitude there should be also a phase of the reflection coefficient, so this is the voltage reflection coefficient at the antenna terminal. Now VSWR is the at the antenna terminal by input now generally in the a time actually you see that if I know VSWR I can find the antenna because the Z c is known.

So, impedance measurement means I will have to I will have to measure VSWR, but VSWR measurement will give me only the real part of the impedance to have the complex part I also need to measure this phase. So, I will now tell you that how one by one you can.

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1. Meanne VSWR. 2. Daterrin [M] 3. Daterrin the phase of 100 M -> dist: of min^b free anter ten k_n . $\mathcal{V} = 2\beta K_n \pm (2n-1)\mathcal{K}$ $= \frac{4\mathcal{K}}{2g}K_n \pm (2n-1)\mathcal{K}$. n=1,2,3... $\widehat{Ag} = \mathbb{Z}_e\left[\frac{1+\Gamma}{1-\Gamma}\right] = \mathbb{Z}_e\left[\frac{1+|\Gamma|\cdot e^{-j\mathcal{K}}}{1-|\Gamma|\cdot e^{-j\mathcal{K}}}\right]$

So, first step is you measure VSWR that you know that you can connect a thing and VSWR at the antenna terminal you measure. Then from there you determine the magnitude of the reflection coefficient by this formula, then you determine the phase of tau how you will do a phase of not a tau, how?

So, for that you in the transmission line you now see where is the next voltage maxima or minima, generally we prefer minima for the known reasons I have already explained earlier in earlier NPTEL lectures. So, you measure the nearest minima. Now if that is too small then you take some integral multiple of the nearest minima and so let us say that that distance is distance of minima from antenna terminal is x n.

So, then we know that this gamma will be 2 beta x n plus minus 2 n minus 1 pi, where here I am assuming that nth minima I am finding. So, this is nothing but 4 pi by lambda g x n plus minus 2 n minus 1 pi and where n is equal to 1 2 3 etcetera whatever value we obtain. So, now you have got this; that means, you have got basically this you know you know completely this complex quantity term.

So, how to find impedance? So next step is j the input impedance of antenna which previously many times you are calling Z a, now I am calling Z antenna is Z c 1 plus tau by 1 minus tau. So, that is Z c 1 plus tau may matured e to the power j gamma by 1 minus tau mature e to the power j gamma that is all. So, by the you will find antenna impedance.

So, these are fairly the main parameters that we measure. So, anyone can measure with any given antenna, what are it is impedance what are it is gain what is it is directivity what is it is radiation efficiency and what is it is pattern what is the DVB beam width.

So, I think with that you should be confident that with this knowledge, you can now try to have analyzed various antennas known antennas unknown antennas then and then also you can synthesize various antennas. Also you can measure various antennas hope that will this knowledge will help you whenever antenna comes in your professional life, you would not fear it rather you will love to explore that antennas either analysis synthesis measurement etcetera.

Thank you.