Power System Dynamics, Control and Monitoring
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Lecture - 10
Power System Stability (Contd.)

So, this is our i fd is equal to this expression, right.

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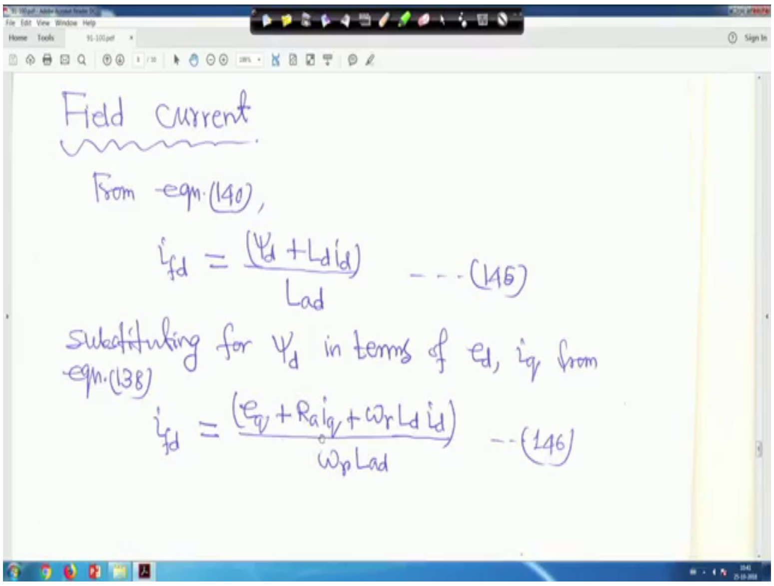

Now, this R a i q is the voltage drop and omega r L d i d, right. So, this we omega r L d i d, so generally L omega is reactance, so this form we can make X d i d, right. So, that is why this i fd this term equation can be written as e q plus R a i q plus X d i d and omega r lad L omega generally reactance, so omega r L ad, so this is X ad. This is equation 147, right.

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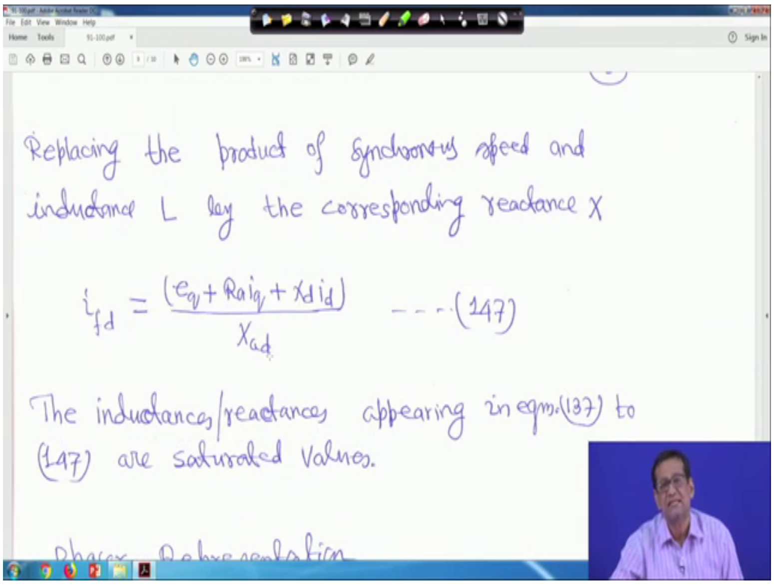

Now, the inductances or reactances appearing in equation 137 to 147 they are basically we are using saturated values, right.

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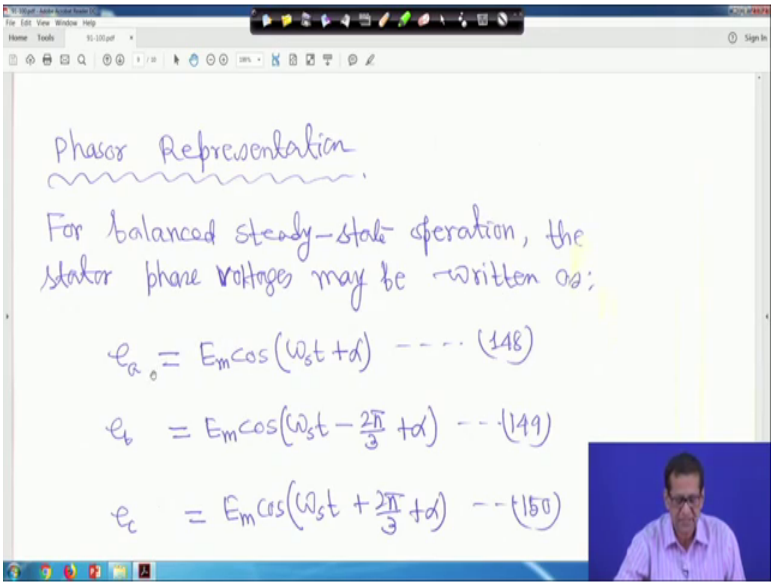

So, next is the phasor representation. For balance steady state operation the stator phase voltages may be written as, right that is e a is equal to E m cos omega s t plus alpha, right. And similarly e b is equal to we can write m cos omega s t minus 2 pi by 3 plus alpha 120 degree phase shift. Similarly e c is equal to e m cos omega s t plus 2 by 2 pi by 3 plus alpha. This is equation 150, right.

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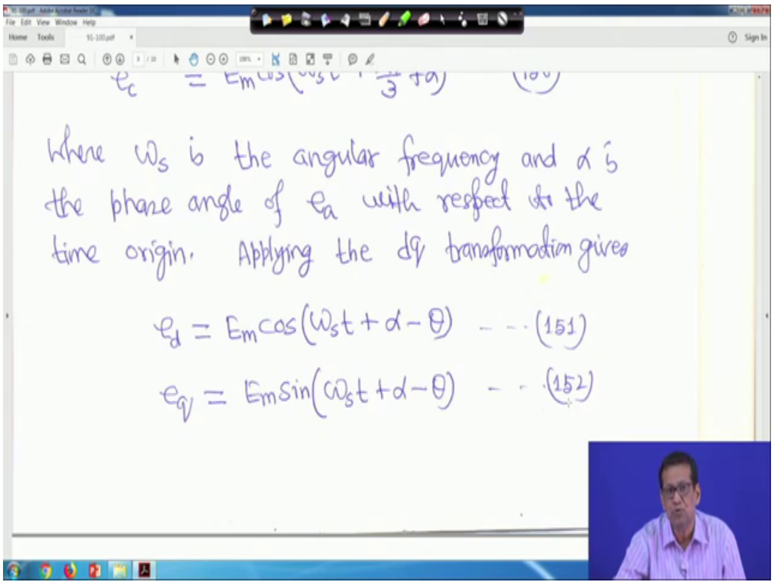

Now, where omega s is the angular frequency and alpha is the phase angle of e a with respect to the time origin. Now, applying the dq transformation for this thing earlier we have seen dq transformation if we apply the dq transformation it will give e d will give E m cos omega s t plus alpha minus theta, this is equation 151. And e q will give E m sine omega s t plus alpha minus theta, this is equation 152, right.

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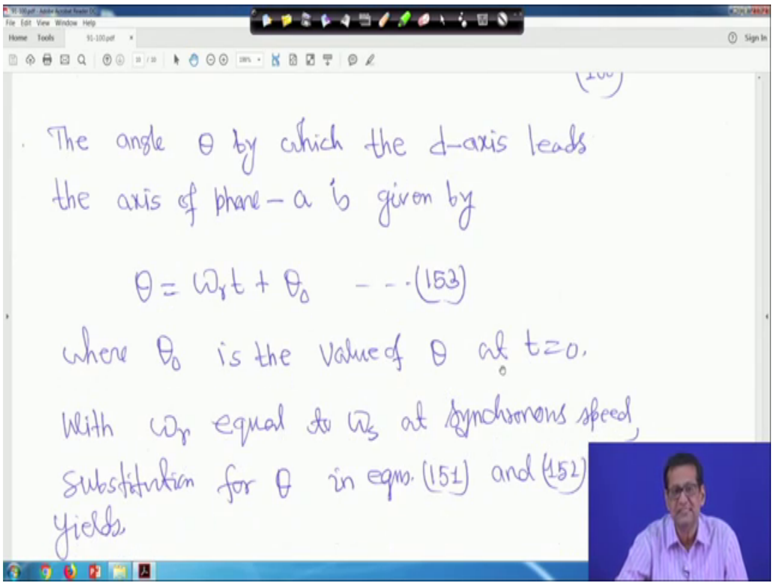

Therefore, now the angle theta we have seen that figure 9, we have seen. The angle theta by which that d-axis lead the axis of phase a is given by you can write theta is equal to omega r t plus theta 0 this is equation 153 and theta 0 is the value of theta at t is equal to 0, right.

Now, with omega r equal to omega s, right because it is run machine is running at synchronous speed say at synchronous speed substitution for theta in equation 151 and 152 it will give you e d is equal to E m cos alpha minus theta 0 that means, your this thing, your this one your here.

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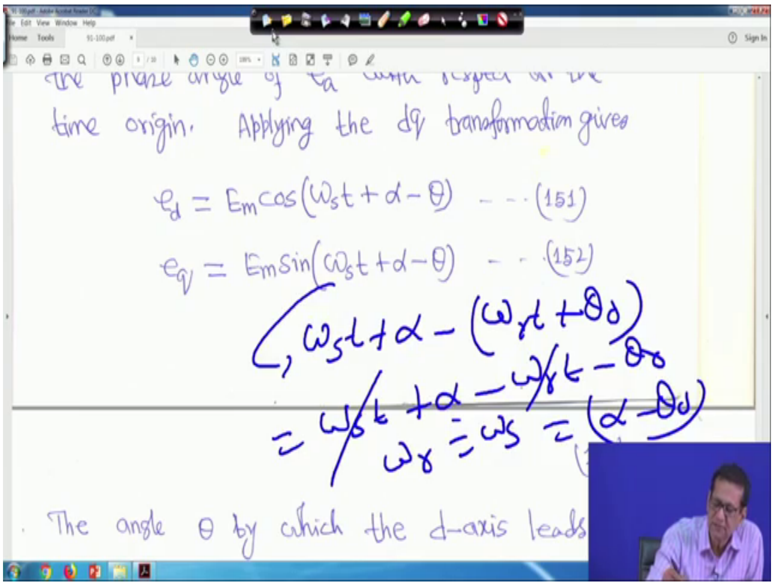

This only we are writing say only one term say this is omega s t plus alpha and minus theta is equal to omega r t plus your theta 0, right. So, this one actually omega s t plus alpha minus omega r t minus theta 0, but omega r is equal to omega s, right therefore, this term this term will be cancel. So, ultimately it will be say alpha minus this one this term will be alpha minus theta 0, right.

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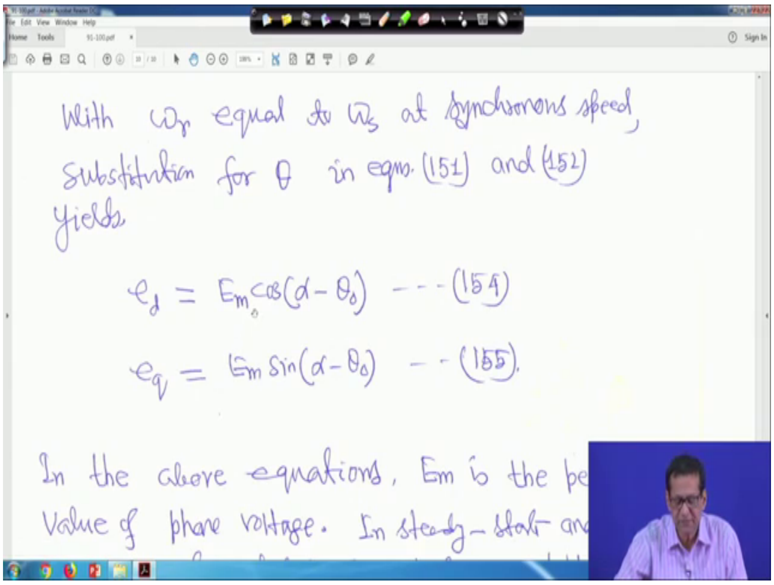

So, that is why here we are writing that e d is equal to e m cos alpha minus theta 0 this is equation 154. And e q is equal to e m sine alpha minus theta 0 that is equation 155, right. So, in the above equation that is E m is the peak value of phase voltage, right.

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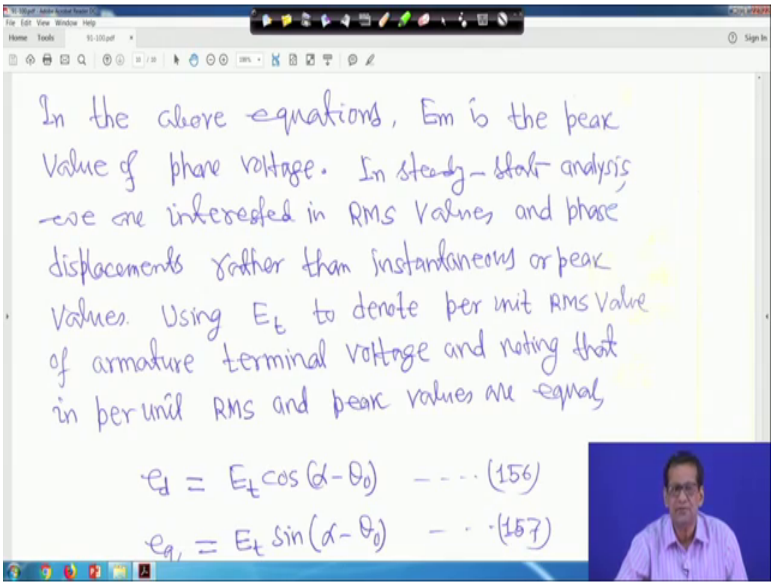

And in steady state analysis we are interested in RMS values and phase displacement rather than instantaneous or peak values. Now, using E t to denote per unit RMS values of armature terminal voltage and noting that per unit RMS and peak values are equal because when we converting it to the per unit values RMS value and peak value they will be equal. Therefore, we can write like this E t is equal to E t cosine alpha minus theta 0 this is equation 156 and e q will be E t sine alpha minus theta 0 this is equation 157, right.

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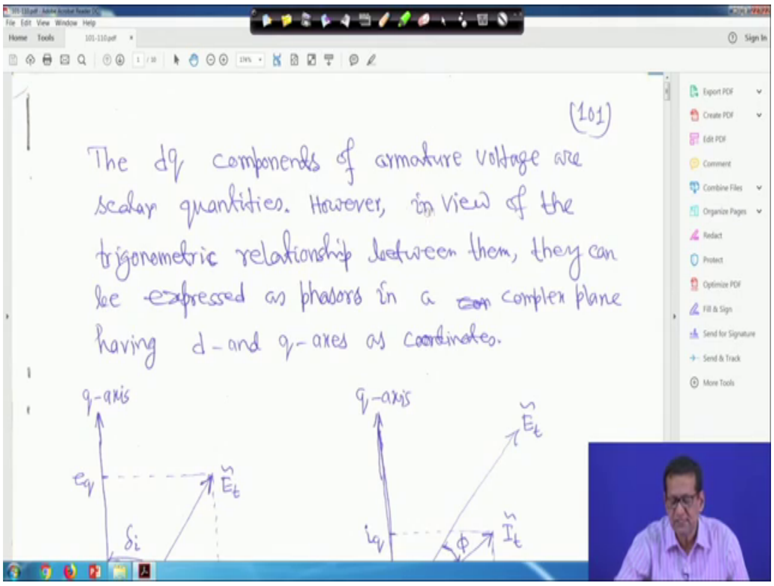

Now, just hold on now that dq components of armature voltage are scalar quantities this we have seen before. However, in view of the trigonometric relationship between them they can be expressed as phasors in a complex plane having d and q-axis as coordinates, right.

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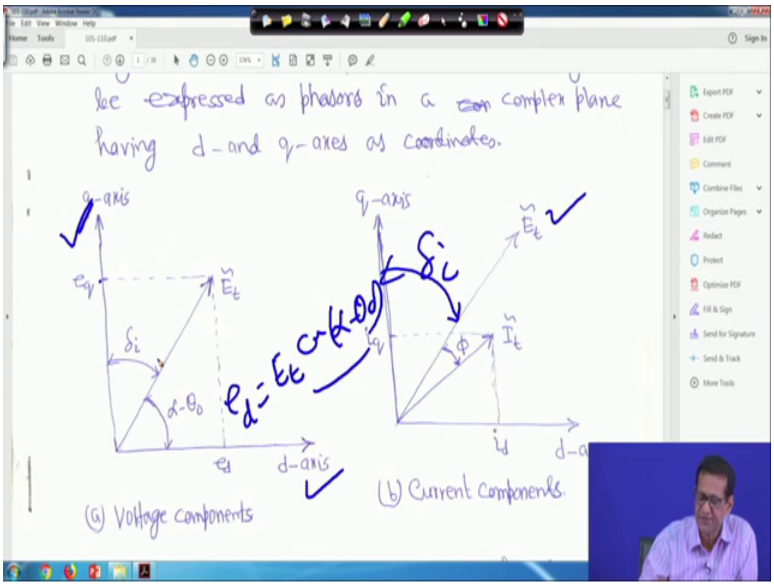

For example, this is my, your what you call this is my q-axis; this is my q-axis and this is my d-axis. So, E t tilde is the phasor quantity that, right and this angle is delta i and this angle your alpha minus your what you call theta 0, E t is equal to E t cosine alpha minus theta 0 and e q is equal to E t your what you call sine alpha minus theta 0 and this angle is delta i.

Similarly, here also q-axis I have not written here this angle actually delta i, right. So, similarly if you and this is say E t tilde, right and this E t is the current. Therefore, E t is lagging from your E t, I t tilde lagging from E t tilde by an angle phi, right. So, in this case for this case your what you call that e d is equal to your in general E t cos alpha minus theta 0 whatever, we have seen before. Similarly, e q is equal to E t your sine alpha minus theta 0 this side, right, but this angle delta i will come later, right and this angle is delta i, similarly for the current, right.

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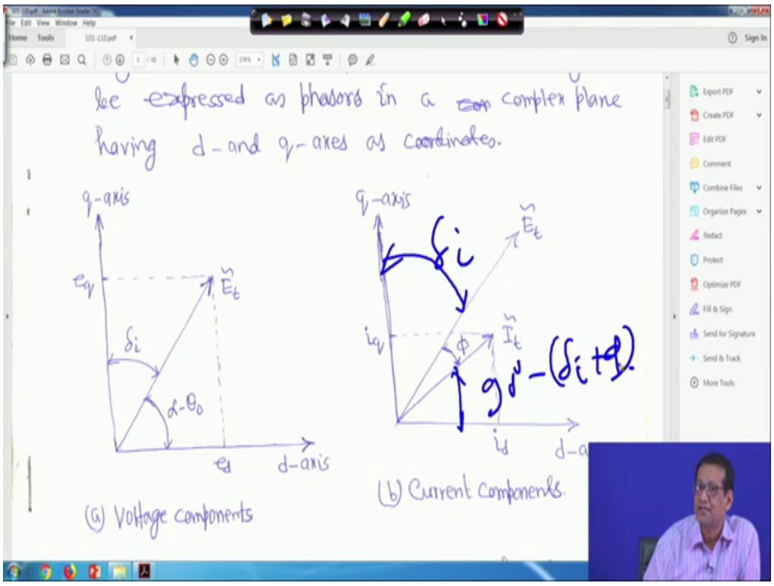

And this angle I told you this angle is delta i. So, similarly for if you just see this is my delta this is delta i and this is phi, right. So, this angle actually 90 degree minus delta i plus i, right. Therefore, id will be is equal to I t cos 90 degree minus delta i plus phi that is i d is equal to I t sine delta i plus phi.

Similarly, i q will be your I t cos delta i plus phi, right. So, this is voltage component and this is current components, right. And this is figure something marked 15 representation of dq components of armature voltage and current as phasor.

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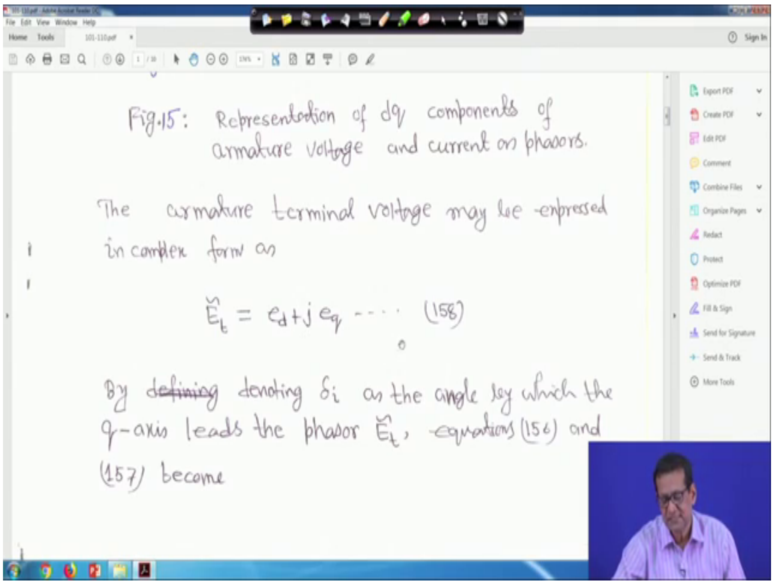

Now, the armature terminal voltage maybe now it can be written in complex form that e t tilde will be e d plus j e q. This is equation 158 because here you have this is e d this is e q. So, e t we can write at E t plus j e q. So, we are writing e d plus j e q this is 158.

Now, by denoting delta i as the angle by which the q-axis lead the phasor E t, right. Equation 156 and 157 it become that e d is equal to E t sine delta and e q is equal to E t cos delta i.

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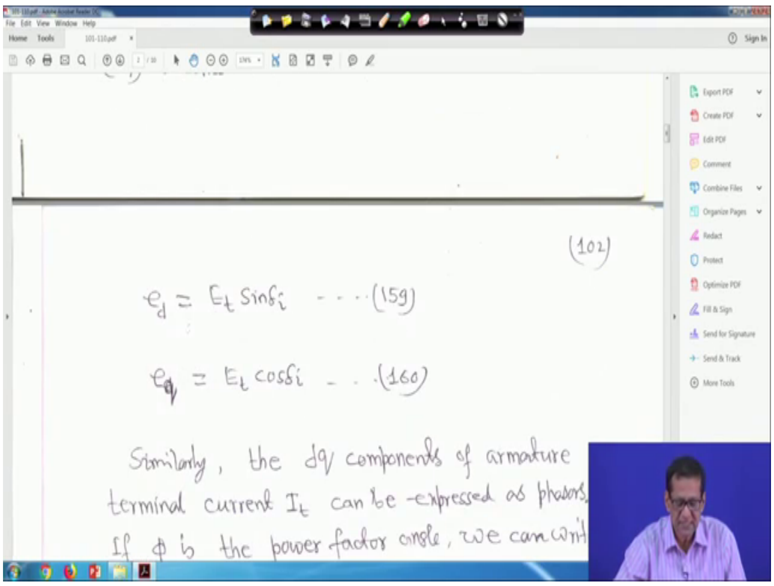

Now, if you look into this; if you look into this then your a once again that if you look into this your e q will be is equal to E t cos delta if you take delta i.

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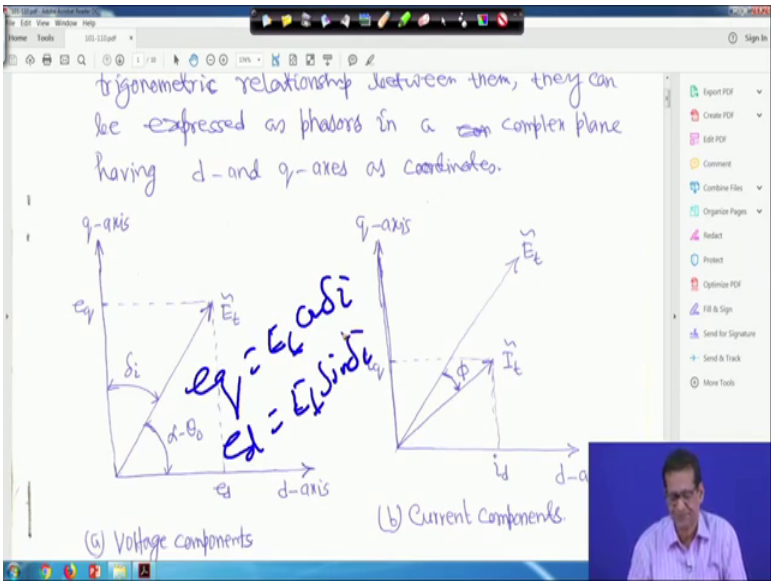

And similarly e d will be is equal to E t your sine delta i, right so that is what we are writing here.

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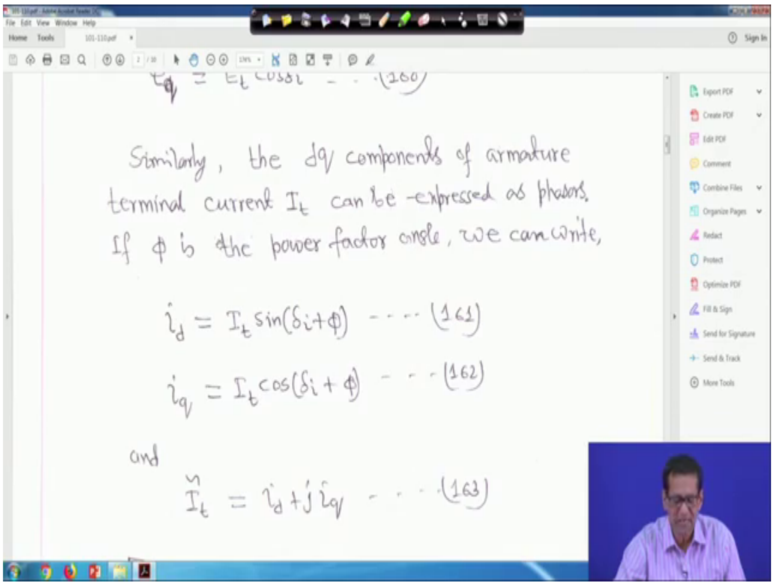

So, similarly for i d I told you similarly the dq component of the armature terminal current I t can be expressed as phasor if phi is the power factor angle we can write i d is equal to I t sine delta this I told you that this angle it is not marked here, but this angle same thing between q-axis and E t, so this angle is delta I, right and therefore, this angle is 90 degree minus delta i plus phi.

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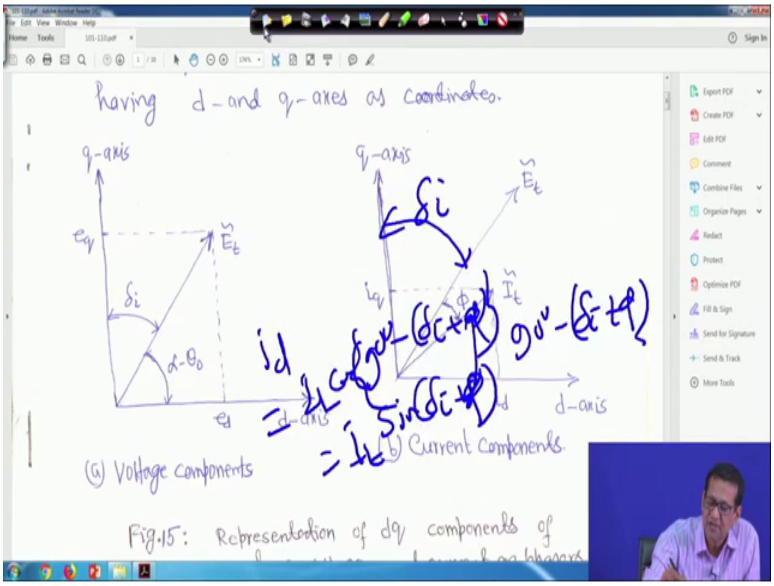

Therefore, I here I am writing therefore, i d is equal to your I t then this one will be cos 90 degree minus delta i plus phi, right. So, that is nothing but your I t it will be sine delta i plus phi, right that is i d. Similarly, i q will be is equal to I t cos delta i plus phi. So, that is what it has been written here, right. So, this is i d is equal to I t sine delta i plus phi and i q is equal to I t cos delta i plus phi this is equation 161 and this is 162.

Now, and I t tilde is i d plus j i q this is equation 163 because it is complex now, right. So, with this way you can represent I t is equal to I t tilde the phasor i d plus j i q this way you can represent.

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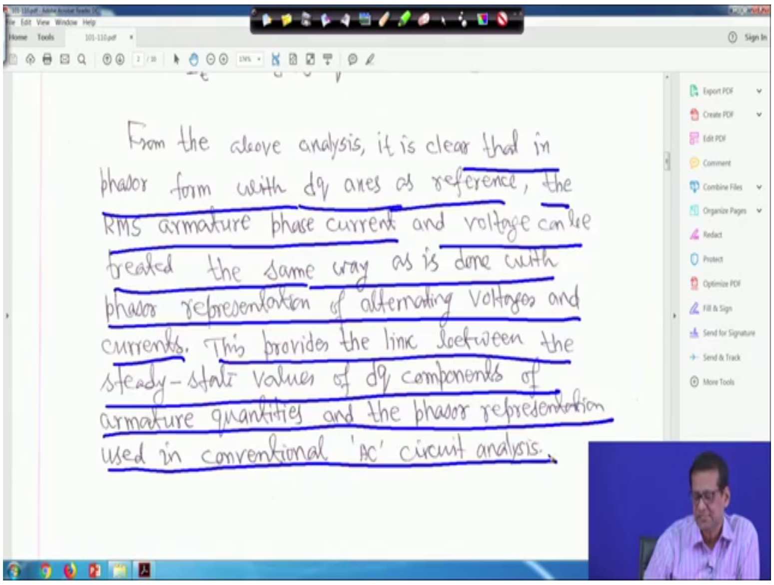

From the above analysis it is clear that in phasor form, right that is in the your what you call in the phasor form with dq axis as reference the RMS armature phase current and voltage can be treated the same way as is done with phasor representation of alternating voltage and current. So, this way we can make it the way we represent the phasor quantities in you know in that your what you call in AC. This provides the link between the steady state values of dq components of armature quantities and the phasor representation used in conventional AC circuit analysis, right. So, this actually dq your conventional AC circuit analysis and in the dq component, this actually there is a link, right, this actually provides the link.

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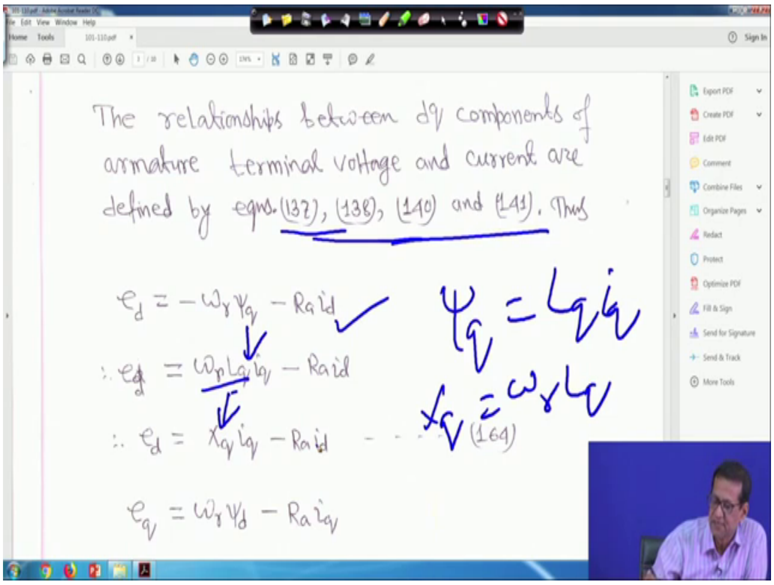

So, that means, the relationship between dq component of armature terminal voltage and currents are defined by your this equation 137, 138, 140 and 141. That means, my e d can be written as minus omega r psi q minus raid, right.

Similarly, or psi q we know from psi q is equal to L q i q. So, we can write omega r that is your psi q is equal to L q i q that that has been substituted here. So, omega r L q i q minus raid and omega r L q actually is nothing, but sq. So, sq actually X q is equal to omega r L q. So, basically e d which actually X q i q minus raid, right, so this is actually equation 164, right.

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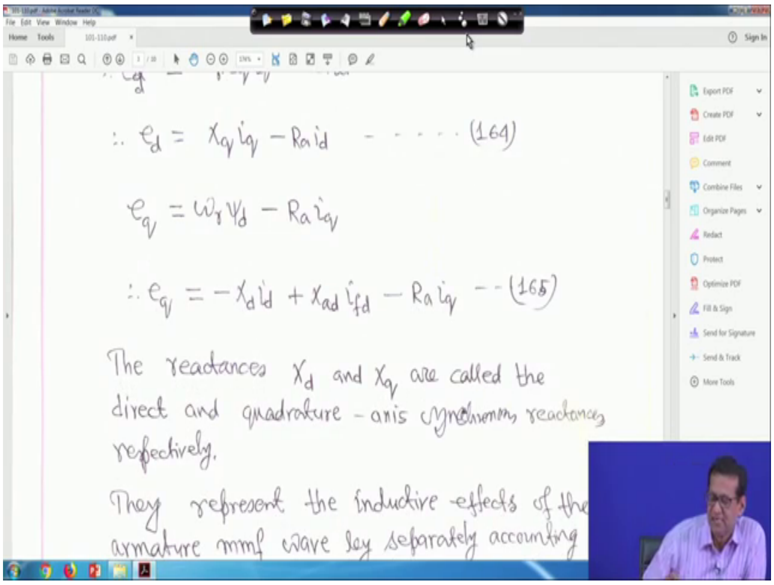

Similarly, your e q is equal to omega r psi d, right. So, omega r psi d minus R a i q. Now, from the psi d expression if you substitute here you will get e q is equal to your minus X d i d then plus X ad i fd minus R a i q. You put that previously we have derived the psi d expression just you put it here and you will get this one. And this is my equation this is our equation 165, right.

Now, the reactances X d and X q are called the direct and quadrature axis synchronous reactances respectively. So, X d is the direct axis and X q is the quadrature axis synchronous reactances, right. Little bit exercise you do, just put this i d expression previously derived and you will get this one.

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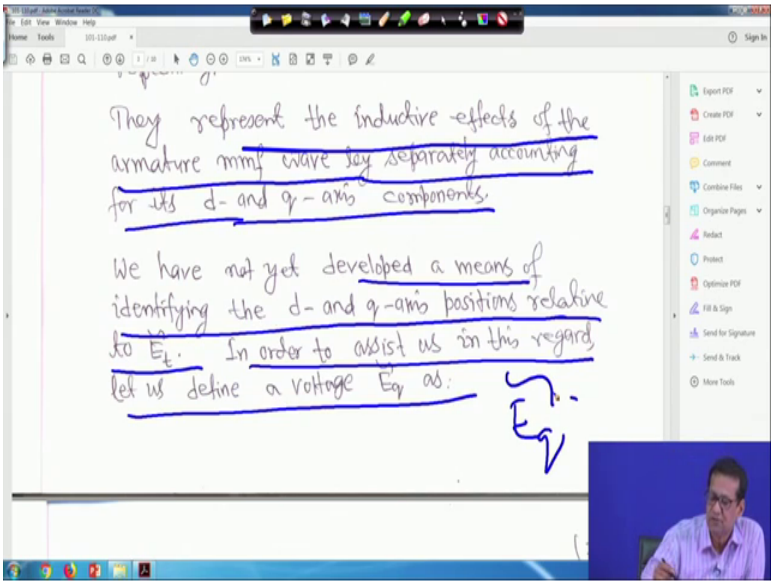

Now, they represent the inductive effects, right. They represent the inductive effects of the armature mmf wave by separately your accounting for its d and q-axis components, right.

Now, this is what is X d and X q. Now, we have not yet developed a means of identifying the d and q-axis a positions relative to E t tilde, right. In order to assist us in this regard let us define a voltage E q tilde that means, we were defining a voltage say E q tilde, right.

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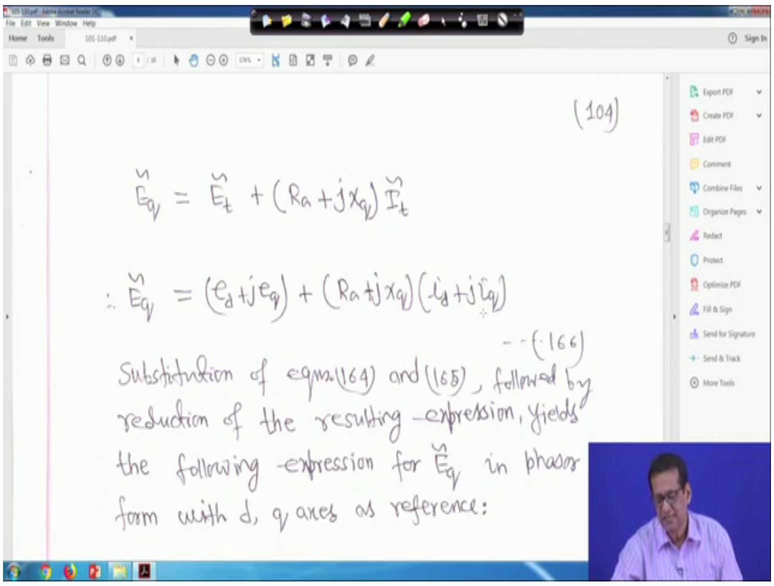

That means we are defining say e q tilde is equal to E t tilde plus r a plus j x q into I t tilde this you are defining say. Now, E t is equal to E t tilde is equal e d plus j eq therefore, e q tilde is equal to e d plus j e q plus R a plus j x q into i d plus j i q this is equation 166, right.

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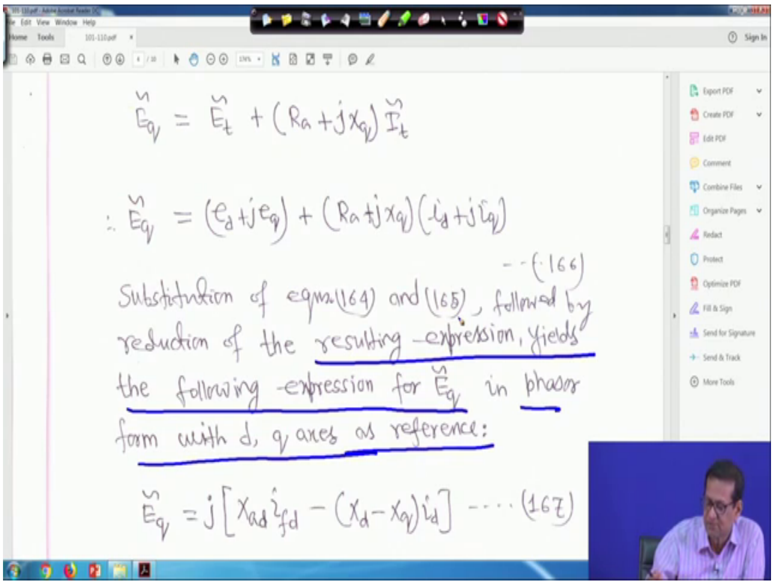

So, now substituting equation 164 and 165 followed by reduction of the your just hold on followed by the your reduction of the resulting expression it gives actually following expression for E q tilde in phasor form with dq axis as reference, right. Therefore, in equation 164 and 165 you substitute, right equation of 164 and 165 that means this one, that means, this one that is your equation 164 e d expression and e q expression and then you simplify you substitute and you simplify.

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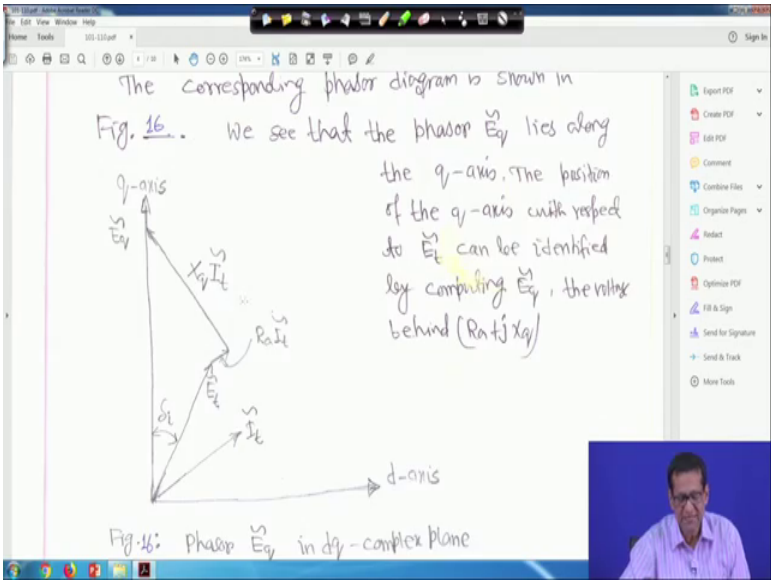

If you simplify, right then you will get E q tilde will be j into X ad capital X ad i fd minus in bracket X d minus X q into id this is equation 167. That means, it is a pure complex quantities coming that this E q tilde will lie on the q-axis, right. Therefore the corresponding phasor diagram is shown in figure 16. So, this is e q tilde because it will lie on the q-axis because only j no real part is involved, right. So, it is j. So, this is E q tilde is equal to E t plus R a I t plus j X q I t, right. So, whatever we have assume here E q tilde is equal to E t tilde plus R a plus j x q I t tilde.

So, this is the phasor diagram. And I t is lagging from E t by an angle phi not shown here this is E t and this is I t this is d-axis and this is q-axis, right. Therefore, the corresponding phasor diagram is shown in figure 16, this is figure 16, right. So, we see that the phasor E q tilde lies along the q-axis. The position of the q-axis with respect to E t tilde can be identified by computing E q tilde the voltage behind that is R a plus j x q, right so that means, your this one R a plus j x q, right.

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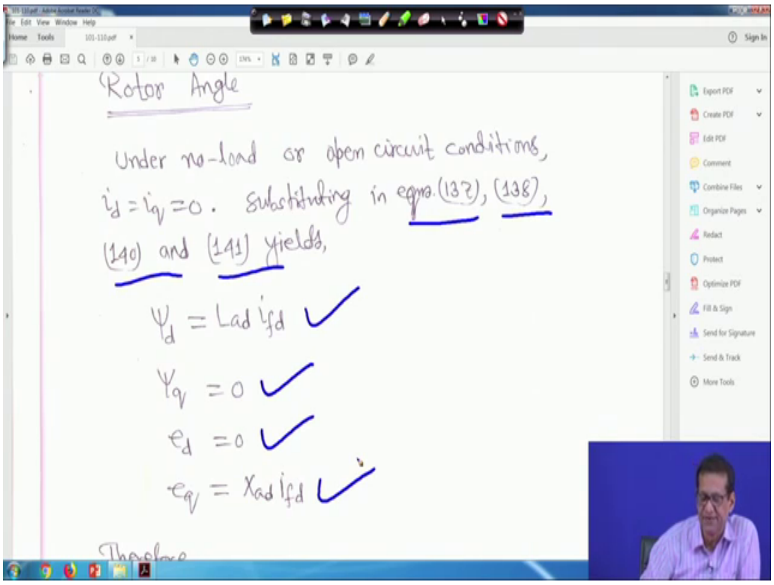

Next is rotor angle under no load or open circuit conditions i d is equal to i q is equal to 0, right therefore, substituting equation 137, 138, 140 and 141. This will this it will give because under no load or open circuit condition i d and i q both are 0 you substitute that in those equation in your in equation your 137, 138, 140 and 141 you substitute that. If you do so, psi d will be is equal to L ad into i fd, psi q will be 0, e d will be 0, and e q will be X ad i fd, right, this will get.

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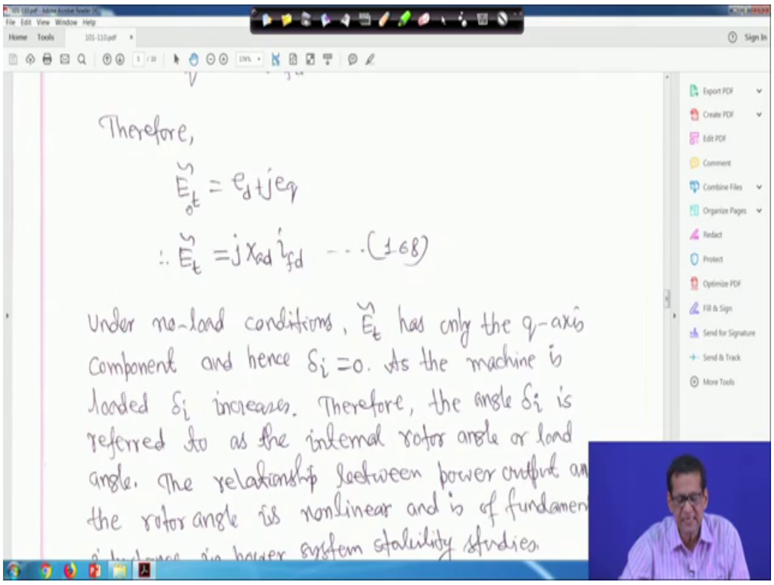

Therefore E t tilde we know e d plus j e q, but e d is equal to 0 therefore, E t tilde will become j X ad into i fd, this is equation 168, right. So, under no load conditions e t tilde has only the q-axis component and hence delta is 0, right. So, just hold on, right. As the machine is loaded delta i increases. Therefore, the angle dealt is referred to as the internal rotor angle or load angle of the machine that your studied in synchronous machine, right. So, as the machine is loaded delta i increases therefore, as the angle delta i refer to as the internal rotor angle or load angle of the machine. The relationship between power output and the rotor angle is non-linear and is of fundamental importance in power system stability studies, right.

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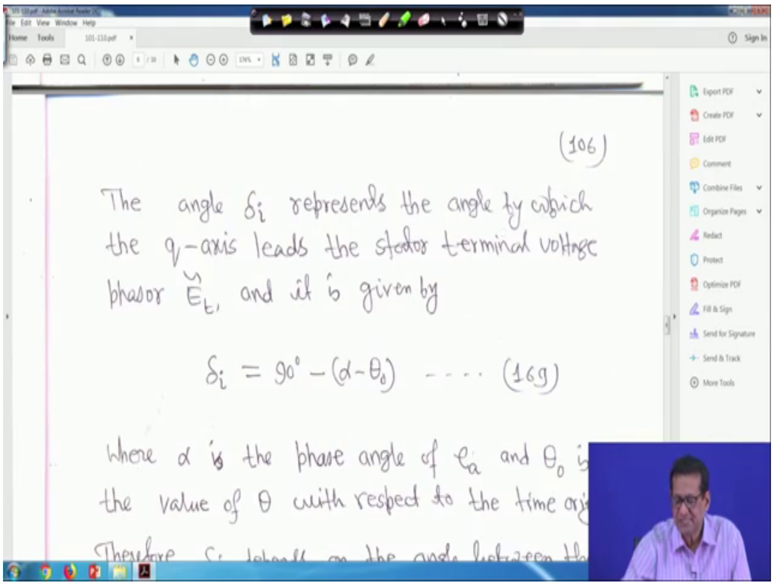

The angle delta i actually represent the angle by which the q-axis lead the stator terminal voltage phasor E t tilde and it is given by that I told you earlier the delta i will be 90 degree minus alpha minus your theta 0, right. So, this is actually your what you call that your delta i. So, if you go back to that previous figure 16 you see the delta is equal to 90 degree minus alpha minus theta 0. This is equation 169, right.

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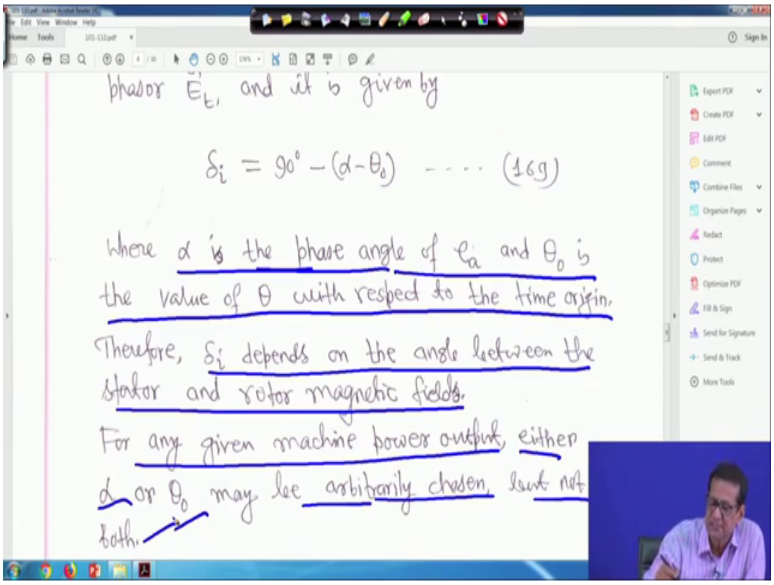

Now, where alpha, where alpha is the phase angle of e a and theta 0 I told you before is the value of theta with respect to the time origin set is equal to 0, right. Therefore, delta i depends on the angle between the stator and rotor magnetic fields, right.

For any given machine power output either alpha or theta 0 may be arbitrarily chosen, but not both your project either alpha beta to 0 for any main forum for any given power output, but not both you have to choose either alpha or theta 0 for any for for any given power output, but not both, right. So, that is the condition.

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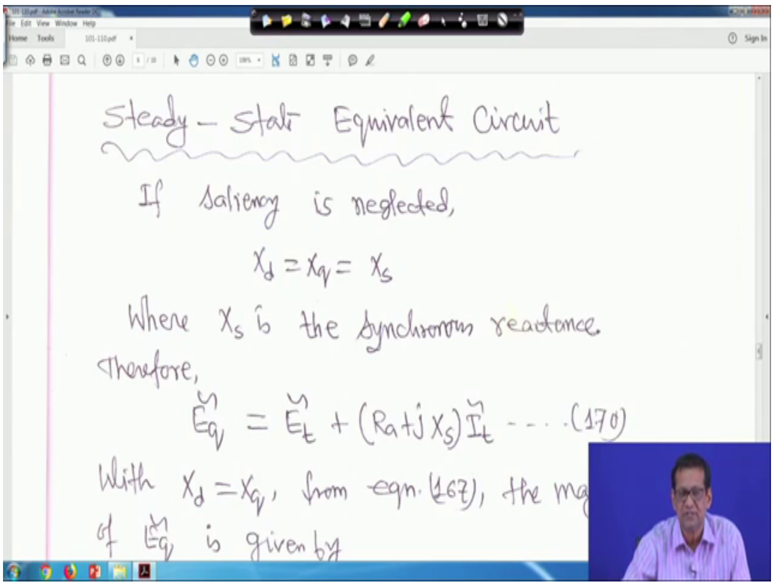

Now, steady state equation a steady state equivalent circuit. If saliency is neglected then we can assume X d is equal to X q is equal to X s, right if you neglect the saliency then X d is equal to X q is equal to X s, right, where X s is the synchronous reactants we are assuming both are same.

Therefore we can write e q tilde we know E t tilde plus R a plus j your it your X q, so now, X q is equal to X s. So, j X s into I t tilde this is equation 170, right where X t is equal to X q. So, from equation 167 the magnitude of E q is given by.

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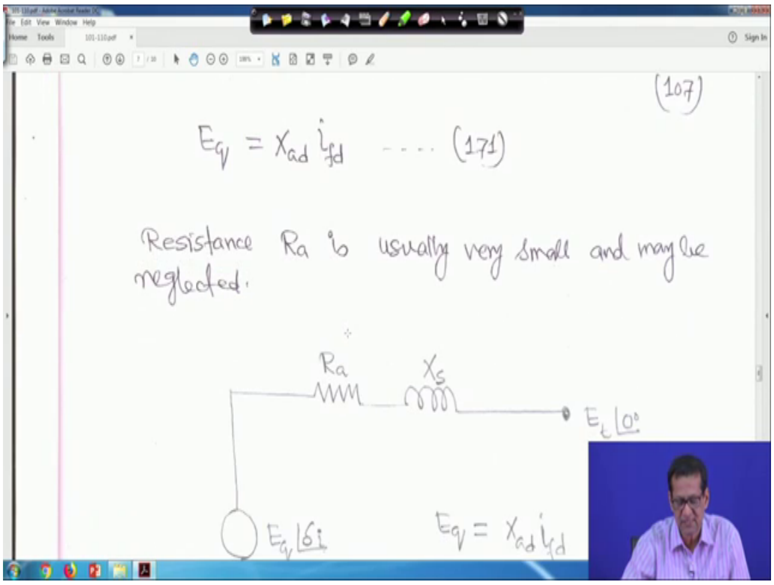

So, this is then E q will be X ad i fd. So, that is from equation 167 I mean here.

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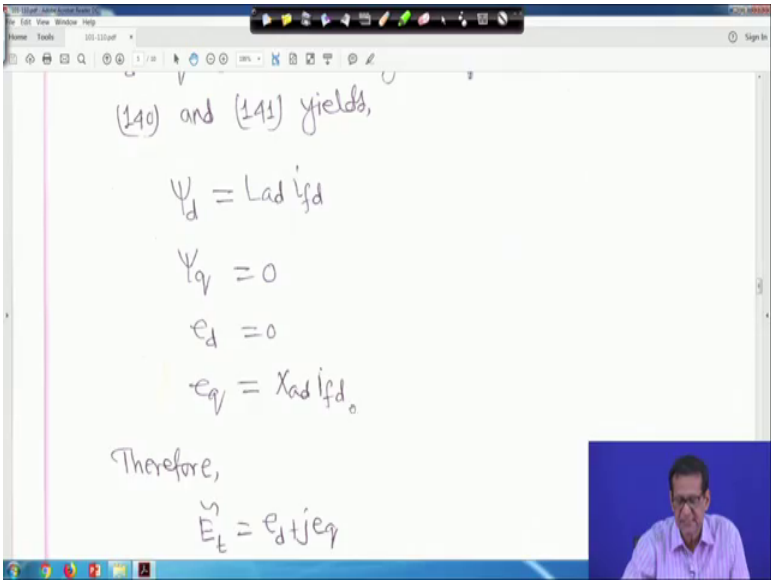

This is your E q is equal to X ad i fd, right this is this is your just hold on 167, right because X d is equal to X q, here X d is equal to X q. So, is equal to X s, so X d it is this term will be not, will not be there. So, it will be only j into X ad i fd, but if you take the magnitude then E q will be only is equal to X ad into i fd only magnitude. So, that means, your this is my capital E q is equal to X ad into i fd this is equation 171, right.

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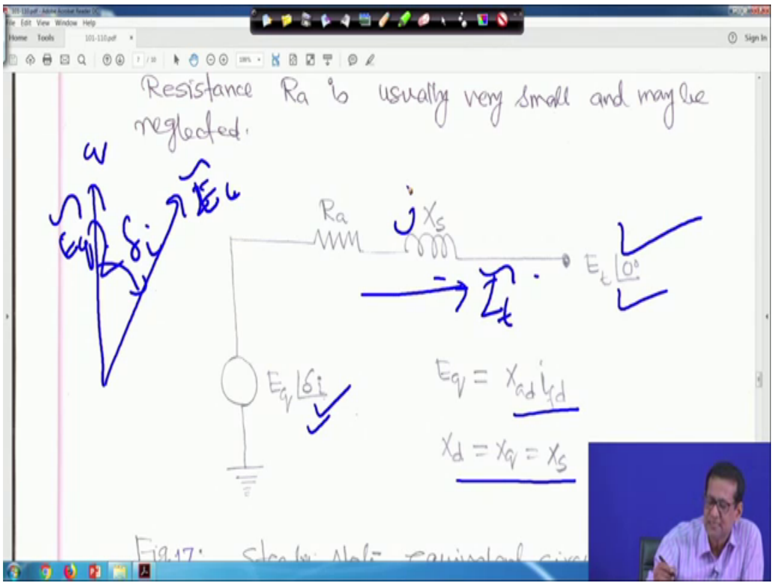

Therefore resistance R a is usually very small and maybe neglected therefore, this equivalent circuit we can make that this is my E q angle delta i this is R a X s and this is

E t 0. Only one thing I have to make it that this is the current then this is a I t tilde, right. Therefore, E q is equal to X ad i fd magnitude one and X d is equal to X q and X s this is E t angle 0 and this is we have taken E q angle delta i that means, I mean if you take this one as reference. So, E q leading E t by an angle delta i because this was this was your q-axis and this is your E q, and this was your E t, and this angle was your delta i, right. So, basically E q leading E t by an angle delta i, if E t has a reference. So, E t angle 0, right that is why we have written E t angle 0 and E q angle delta i and R a axis, right.

If you want to put j you can put j here, right. But generally for machine R a is very small can be neglected, but this is the equivalent steady state equivalent circuit, right.

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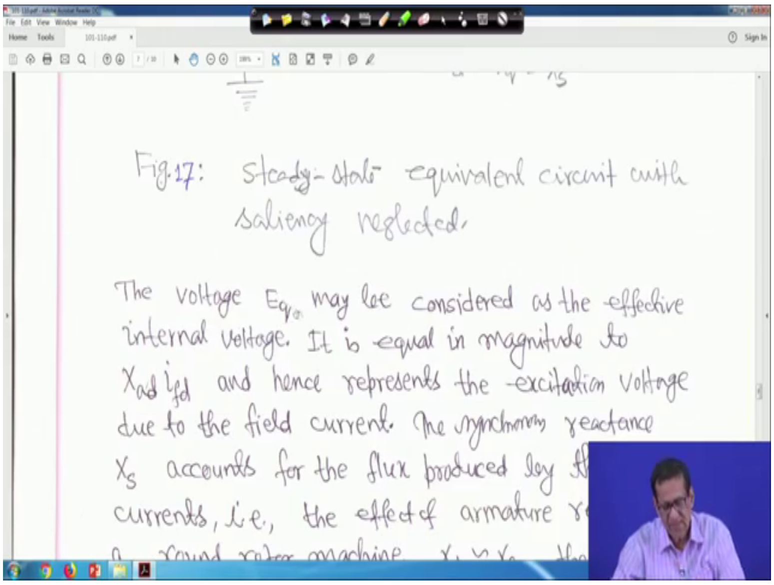

So, the voltage E q may be considered as the effective internal voltage, right.

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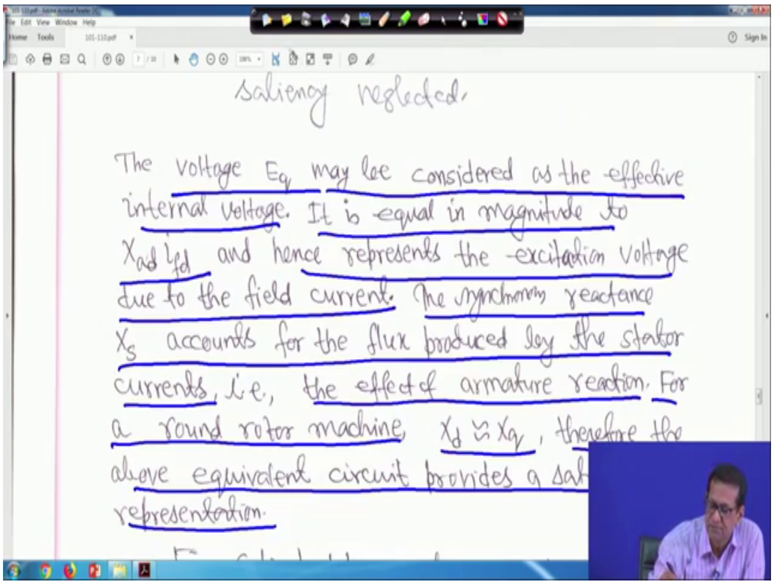

So, the voltage E q may be considered as the effective internal voltage, right. It is equal in magnitude to X ad i fd that we have seen just now. And hence represent the excitation voltage due to the field current because it is X ad into i fd i fd is the field current, right.

The synchronous reactance X s accounts for the flux produced by the stator currents that is the effect of armature reaction, right for a round rotor machine, right X d approximately is equal to X q is equal to X s we have taken. Therefore, the above equivalent circuit provides a satisfactory representation, right.

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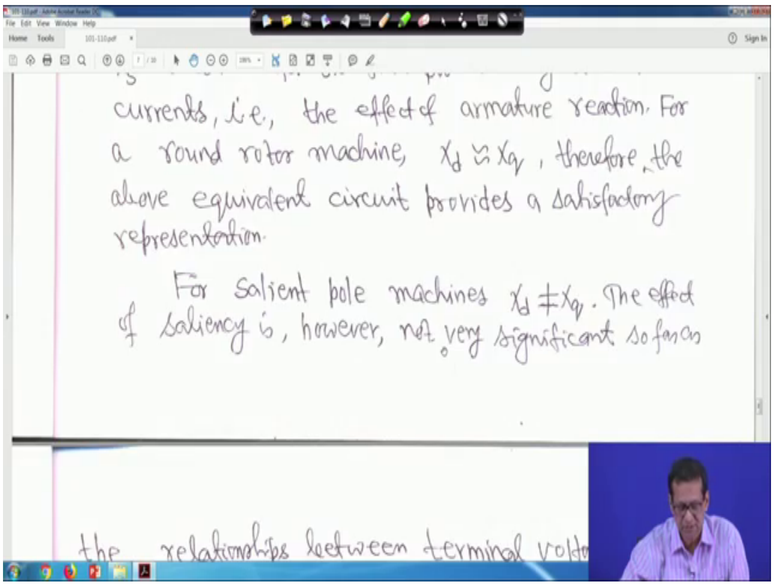

For salient pole machine of course, X d not is equal to X q, the effect of saliency is however, not very significant, right, so far as the relationship between terminal voltage armature current power and excitation over the normal operating range are concerned, right.

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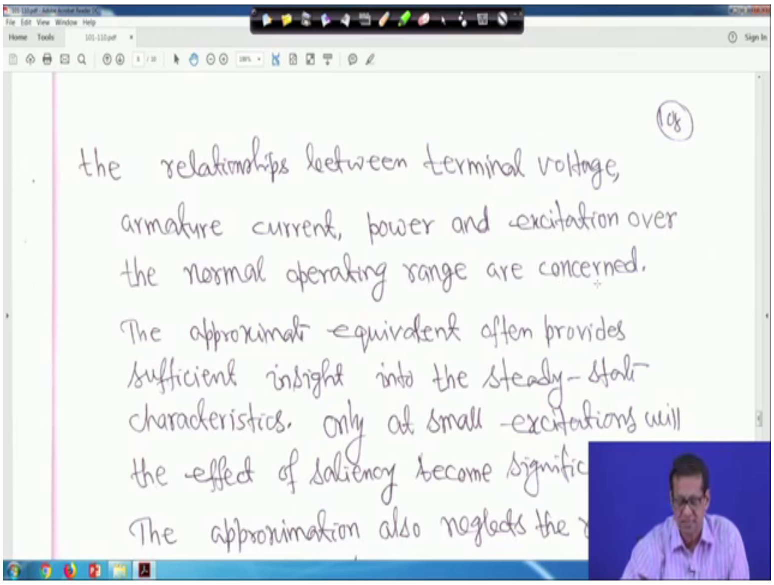

The approximate equivalent your what you call often provides sufficient insight into the steady state characteristics. Only at small excitations will the, that is will the effect of saliency becomes significant. The approximation also regards the your what you call neglects the your reluctant torques due to saliency, right.

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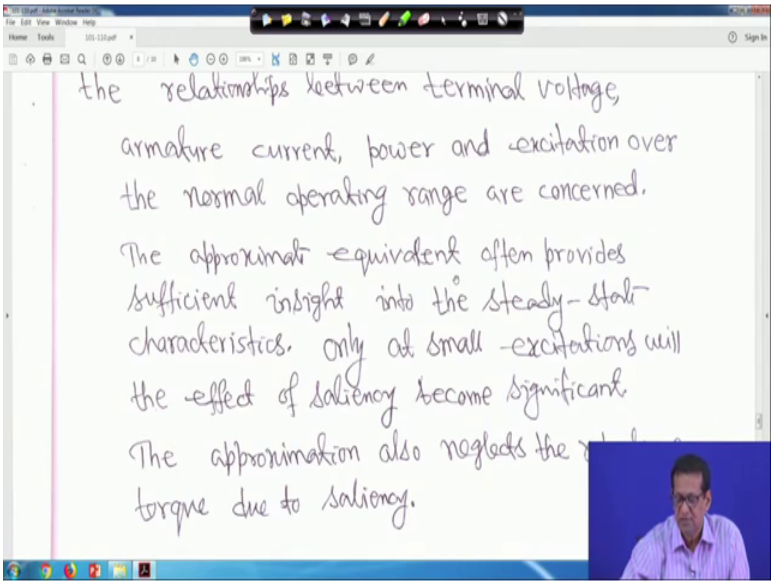

Now, next is active and reactive power.

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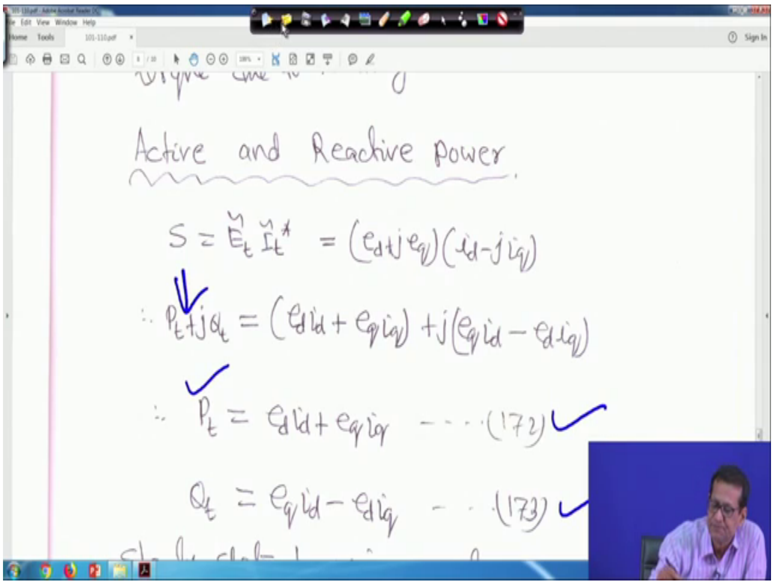

Now, we know S is equal to we know in general from load flow studies we know that either when you, right using general, right that P minus j Q is equal to b conjugate i or P plus j q is equal to b i conjugate, right. So similarly active and reactive power, right, S the apparent power E t tilde I t tilde conjugate. So, E t tilde is equal to e d plus j q and I t is conjugate I t is equal to tilde is equal to e d plus j i q. So, I t tilde conjugate is e d minus j i q that is e d minus j i q, right sorry i d minus j i q. So, e d plus j i q into i d minus j i q therefore, at S is equal to your, S is actually pt plus j Q t because it is e i conjugate, right. Therefore, E t you can if you multiply it will be e d i d plus e q i q plus j e q i d minus e d i q.

Now, we separate real and imaginary part. So, pt will be e d i d plus e q i q and Q t will be e q i d minus e d i q. This is equation 172, this is equation 173, right.

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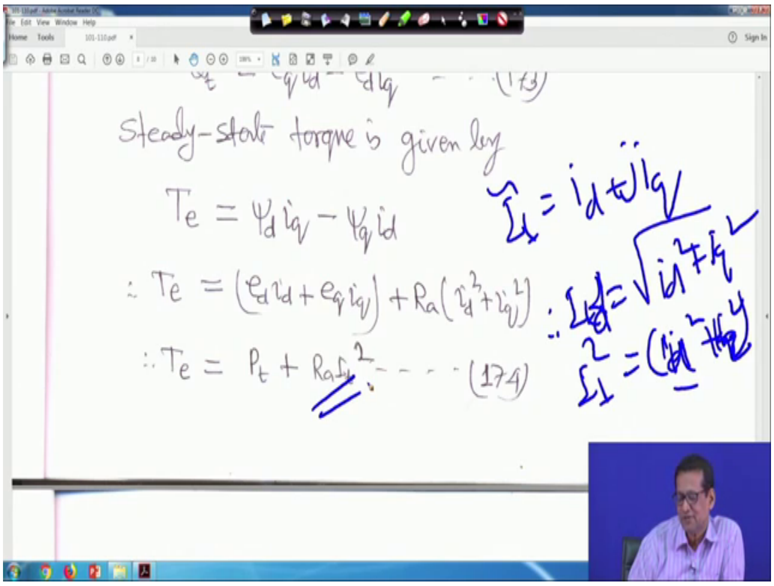

Now, steady state torque is given by T is equal to this we have derived that psi d i q minus psi q i d this we have derived or simply this one if you just put those psi d psi q all these expression you will get T e is equal to this one little bit I ask you to derive of your own, right.

So, just put those expression you will get T is equal to e d i d plus e q i q plus R a into i d square plus i q square or T is equal to this is my power. Just now we have seen e d id plus e q i q this is my this is my power pt, right just now we have seen and this is R a I t square because your I t tilde is equal to e sorry just hold on I t tilde is equal to i d plus j i q. Therefore, your magnitude if I take magnitude of this one is equal to this one, right therefore, my I t square is equal to your i d square plus i q square, right that is what is written here R a I t square. This is your equation 174.

So, now question is that your what you call little bit more steady state computation will go in the next class, next lecture. So, now, if you look into that that T is equal to equal to

P t plus R a I t square.

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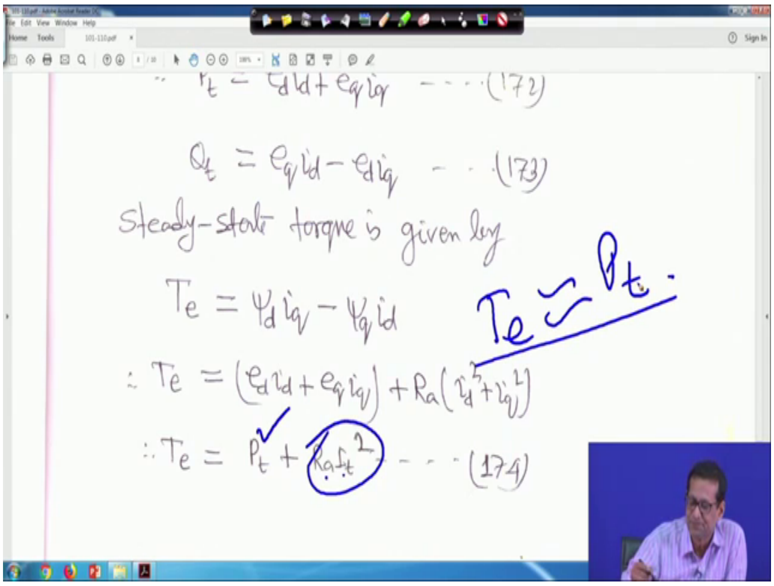

Now, in general if you look into this that this, this part, this part R a I t square actually very small, right. Now, if you do so, that means, if you do so that this is actually torque will be approximately equal to the power output P t, right. Therefore, in per unit system when you convert into per unit system, if I generally if you neglect the loss you will find in per unit torque and power both are same because this term R a I t square is very small, right. So, many-many in our analysis many cases we assume it is a you assume that per unit torque is equal to per unit power, if you convert it to this thing because this term is very small. Therefore, torque and whatever we have made it this torque is equal to this one P t plus R a I t square everything is in per unit and this term is very small compared to this term therefore, torque is approximately in power in per unit, right.

So, only thing is that that after this I mean after this we have one more thing that is your just computational procedure, and we will take one example little bit more derivation is there. And after that we will go for the your what you call that your dynamics that swing equation step by step we will follow, right and slowly and slowly we will go you know you know into much deeper analysis particularly in that your Laplace domain and we will derive all these all these your mathematical model block diagram representation.

And later we will see that stabiliser or system stabiliser your (Refer Time: 27:48) stabiliser we will consider and slowly an Eigen value analysis and participation factor all these things will your what you call will examine, but that is in you know slowly and slowly we move into that. But after this we will go little bit of computational analysis and then we will come to the swing equation.

Thank you very much. We will back again.