**Power System Dynamics, Control and Monitoring**
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**Lecture – 23**
**Power System stability, Eigen properties of the state matrix (Contd.)**

So, in the previous lecture we ended up here that your Equation 18 right.

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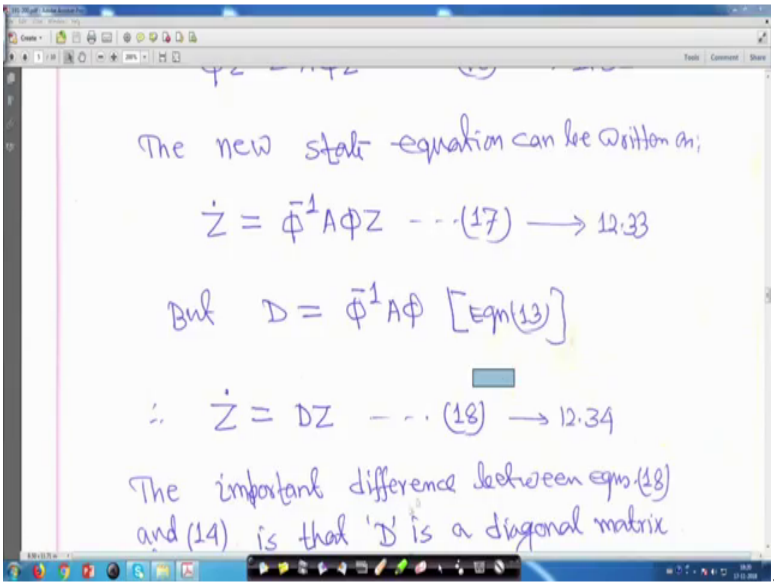


And, this is nothing this is for my own reference right. So, we got Z dot is equal to D Z and d is a diagonal matrix and each diagonal elements is nothing, but the Eigen values of the matrix A right. So, let me clear this.

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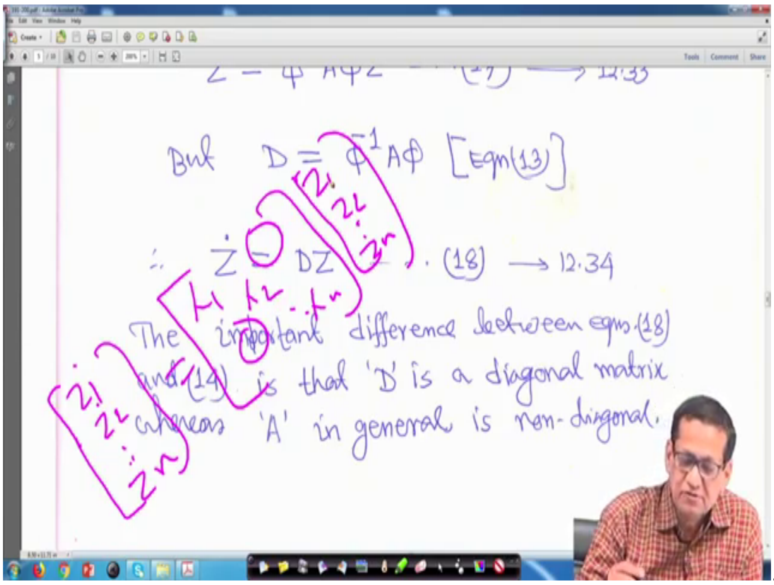


So, now equation 18 actually represent n uncoupled first order that is scalar equation right, because if you look into this equation, that your this equation Z dot is equal to D Z. Generally, if I am just over writing on it just see this is say Z 1 Z 2 up to Z n. Suppose matrix is n into n is equal to this D is actually diagonal matrix lambda 1, lambda 2, lambda n into your Z 1 Z 2 up to Z n right and all other elements are 0. Therefore, Z 1 will be is equal to lambda 1 your Z 1 dot Z 2 dot and Z n dot right.

So, Z 1 dot will be is equal to lambda 1 Z 1 Z 2 dot will be lambda 2 Z 2. So, it is only Z 1 dot is function of Z 1, there is no this thing no other ZS are appearing so, completely decoupled right. So, because of this then these equation that we can write then for your for i state variable say we can write Z i dot is equal to lambda i Z i right.

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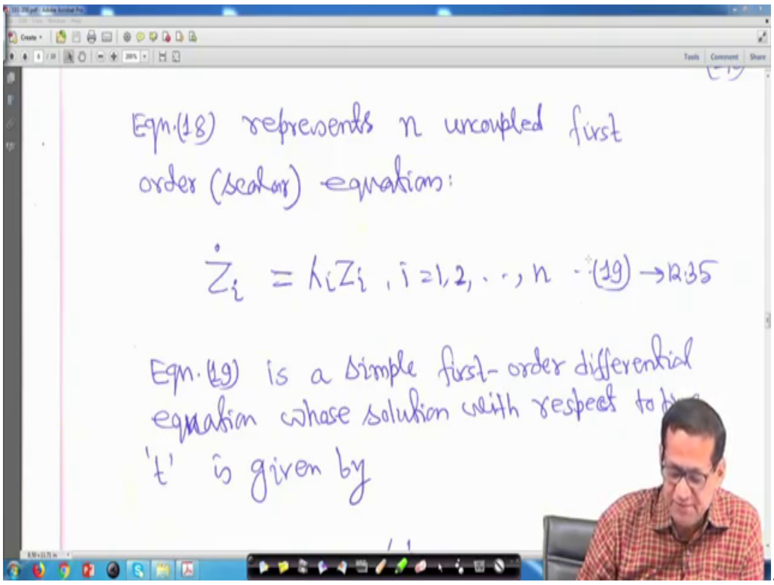


I told you before that actually your what you call lambda is like this, but my habit has become to write like this. So, I hope you will forgive me for that right.

So, we can write that Z i dot is equal to lambda i Z i for i is equal to 1 to n this is actually equation 19 right. Now, equation 19 is a simple first order differential equation whose solution with respect to time ‘t’ is given by right if we want to get the solution.

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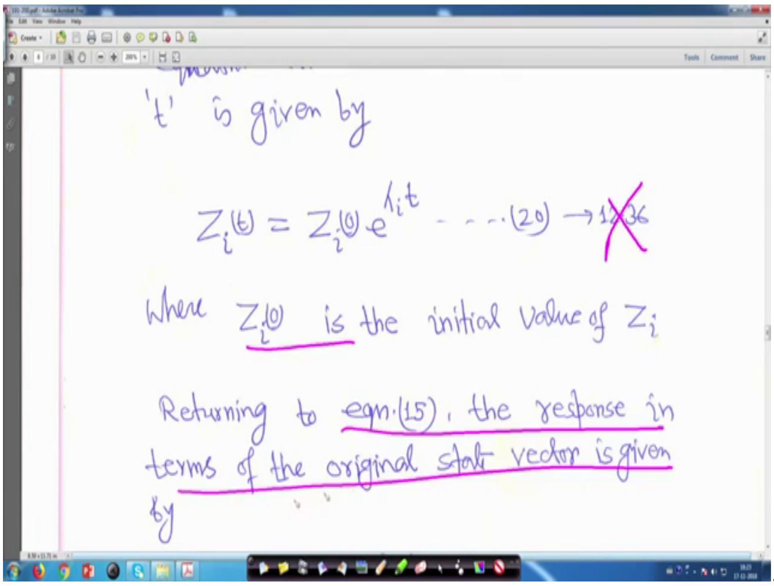

So, it is can be given as Z i t will be Z i t is equal to Z i 0 e to the power lambda i t, this is actually equation 20 right.

So, this is nothing, this is for my own reference. So, where Z i 0 actually the initial value of Z i right. And, now returning to equation 15 the response in terms of the original state vector is given by right, when we come back to equation 15 right you come back to equation 15 here right.

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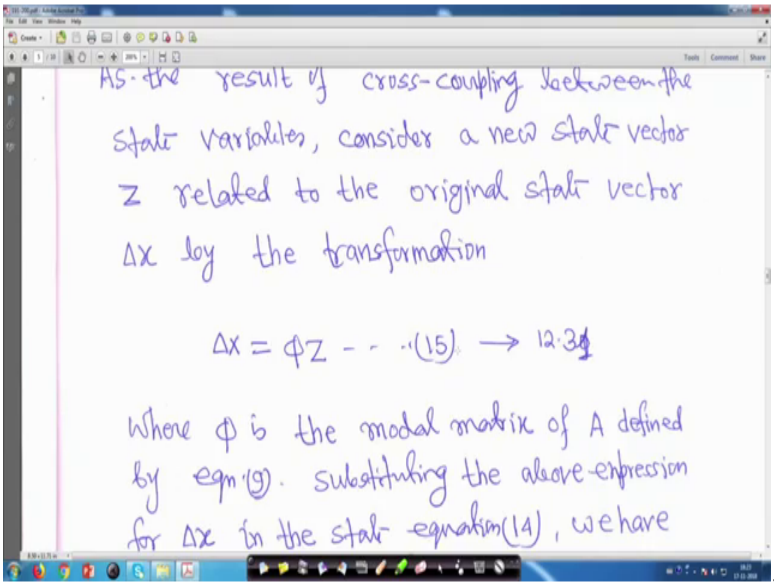


This is actually delta X is equal to phi Z right that is equation 15. So, and we have got the expression for Z.

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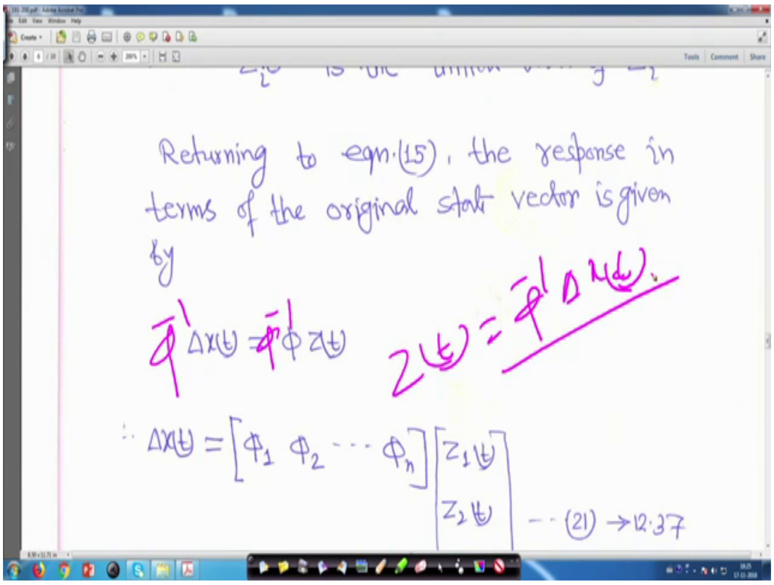


So, therefore, we can write that in terms of actually delta x is equal to phi Z that is true, but it is a time function. So, same equation we can write delta x t is equal to phi Z t, where delta x t is equal to phi 1 phi 2 up to phi n and this is Z 1 t Z 2 t up to Z n t this is equation 21. This way you can write that delta x t in terms of Z 1 Z 2 up to your, what you call Z n t right.

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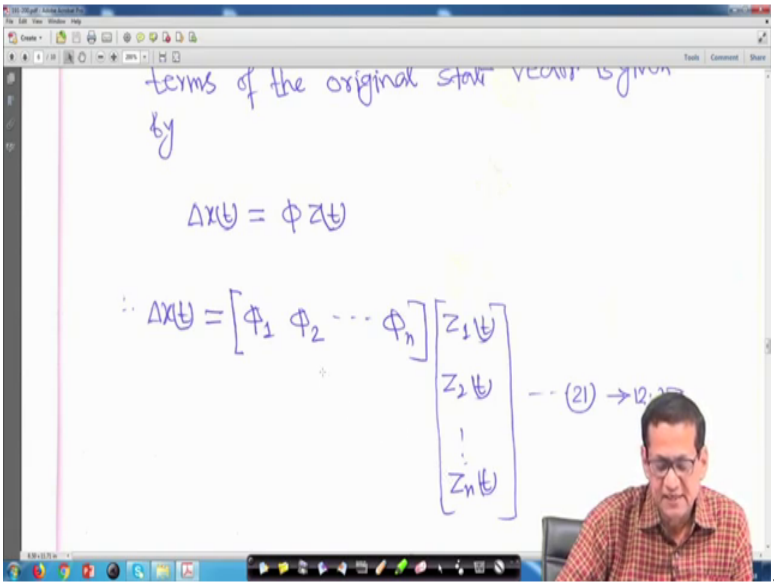

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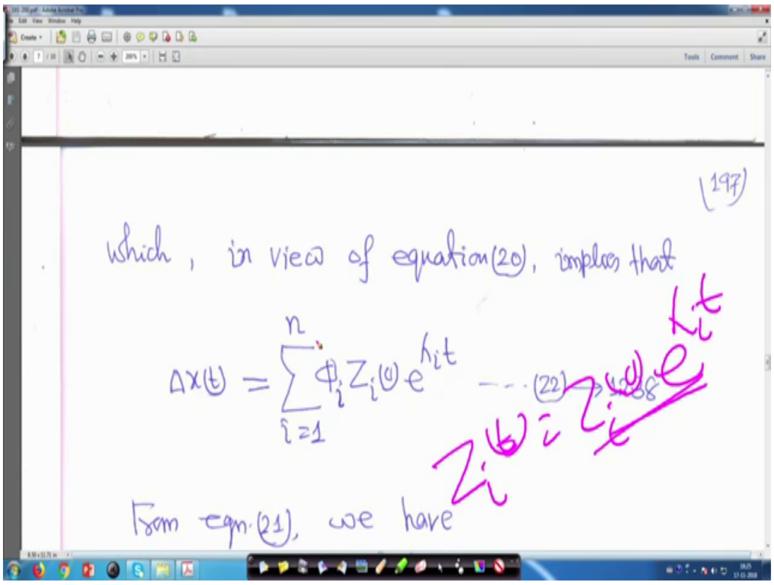


So, which in view of the equation 20, implies that delta x t will be is equal to sigma i is equal to 1 to n phi i Z i 0 e to the power lambda i t. Because the solution of your Z i t is equal to your Z i 0 e to the power lambda i t, in general and here you have phi 1 Z 1 plus phi 2 Z 2 and so on. Therefore, it will can be written as sigma i equal to 1 to n this phi i and your Z i t, I mean in general, I mean in general and here I am putting in general that your Z i t is equal to Z i 0 e to the power your what you call lambda i t right.

So, this Z i t we are replacing by Z i 0 e to the power lambda i t and this is i is equal to 1 to n the summation right.

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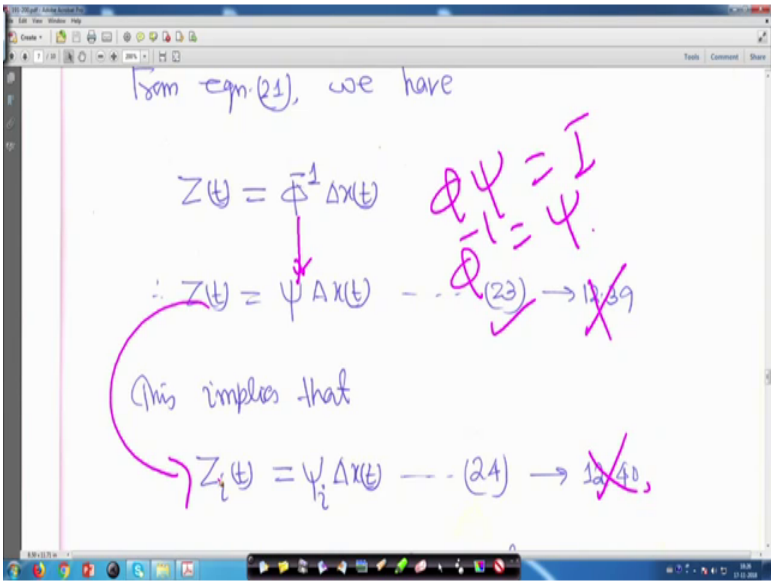


So, therefore, equation 21 I mean this that mean this equation 21 right, we have Z t is equal to phi inverse delta x t I mean from this equation 1, you can get Z t is equal to phi inverse delta x t I mean both side of this equation if you multiply by phi inverse, and then this is also phi inverse; phi inverse phi identity matrix; that means, my Z t is equal to phi inverse delta x t right.

So, that is what we are writing here right. So, Z t is equal to phi inverse delta x t, but we know that your phi i is equal to identity matrix. So, phi inverse is equal to psi right. So, here we have substituting that your phi inverse, we know that your phi psi is equal to I right. Therefore, phi inverse right is equal to psi that is your substituting here. So, Z t is equal to psi delta x t this is equation 2 3 this is nothing for you this is my own reference.

Therefore, this implies that the Z i t is equal to psi i delta x t, because your what you call? In general in this is Z t is equal to psi delta x t, but for i state variable say Z i t is equal to i mean this equation can we written as for i state variable Z i t is equal to psi i delta x t right. So, let me clear it.

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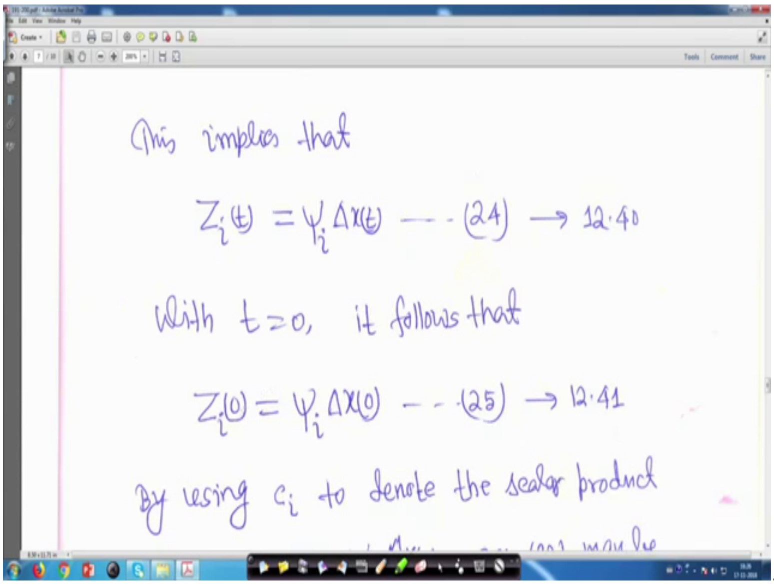


So, with t is equal to 0 it follows that Z i 0 is equal to psi i delta x 0, because initial condition. So, here if we put t is equal to 0 Z i 0 will be psi i delta x 0 right.

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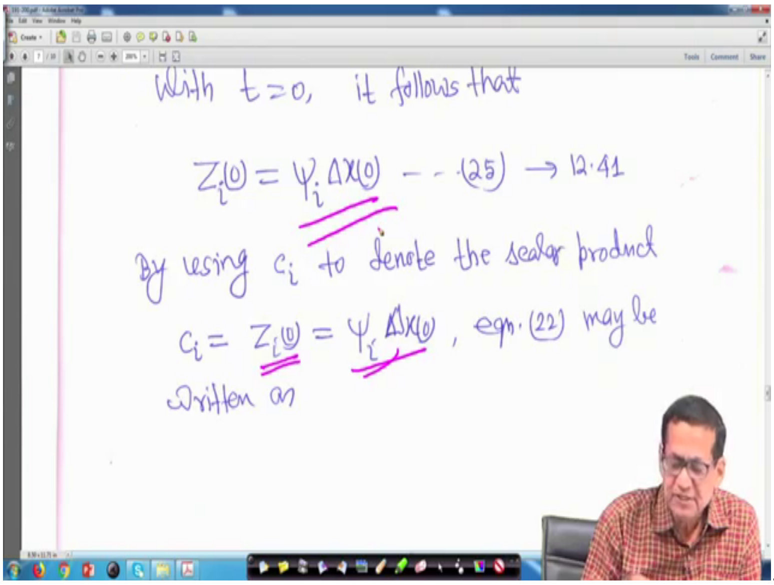


By using C i to denote that scalar product C i is equal to say Z i 0 is equal to psi i delta x 0 then equation 22 may be written as; that means, this we are assuming that this is Z i 0 you are assuming C i is equal to Z i 0 is equal to this one psi i delta x 0. Therefore, equation 22 it can be written as right, it can be written as your I mean here it is.

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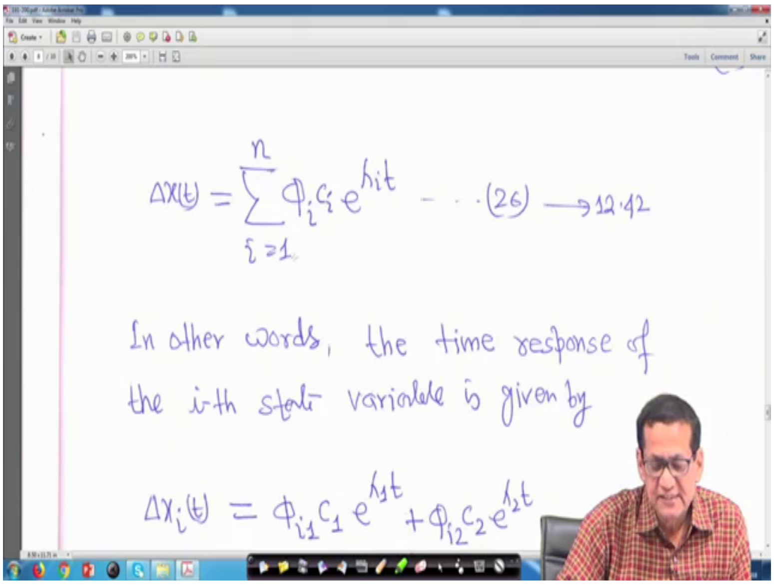


This equation 22 it can be written as your delta x t is equal to sigma i equal to 1 to n and then phi i C i e to the power lambda i t right. So, it is Z i your because c i is equal to your Z i 0 is equal to psi i delta x 0.

So, this is actually delta x t is equal to i is equal to 1 to n phi i C i e to the power lambda i t this is equation 26. In other words the time response of the i-th state variable is given by right.

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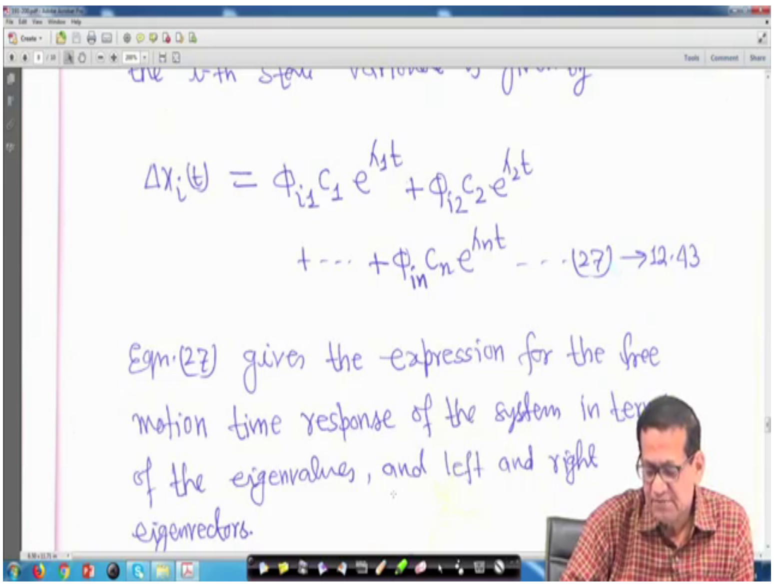

I mean if you expand this one it will be delta X i t is equal to phi i 1 C 1 e to the power lambda 1 t plus phi i 2 C 2 e to the power lambda 2 t up to you come plus phi i n C n e to the power lambda n t this is equation 27 right.

Let me response actually it depends on the all the eigenvalues function of all the Eigen values right. Now, equation 27 actually gives the expression for the free motion time response of the system in terms of the Eigen values and left and right Eigen vector right.

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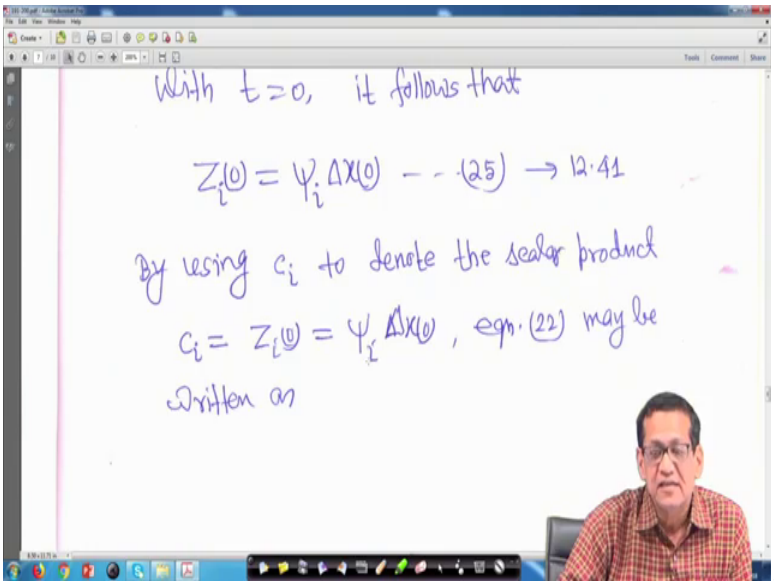


So, 1 is that your, phi is the left Eigen vector and C 1 if you come here that C constant also function of psi i right. So, this response actually function of both left eigenvector and the right eigenvector right.

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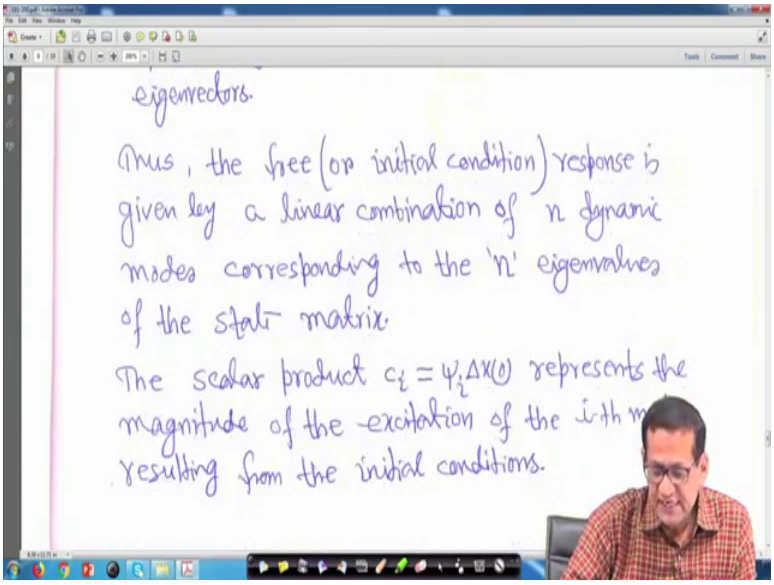


So, thus the free or initial condition response is given by a linear combination of n dynamic modes corresponding to the 'n' Eigen values of the state matrix right. Now, the scalar product C i is equal to C i delta x 0. It represent the magnitude of the excitation of the ith mode resulting from the initial condition. This thing should be in your mind that the scalar product that is C i is equal to psi i delta x 0 actually, it represent the magnitude of the excitation of the ith mode resulting from the initial condition right.

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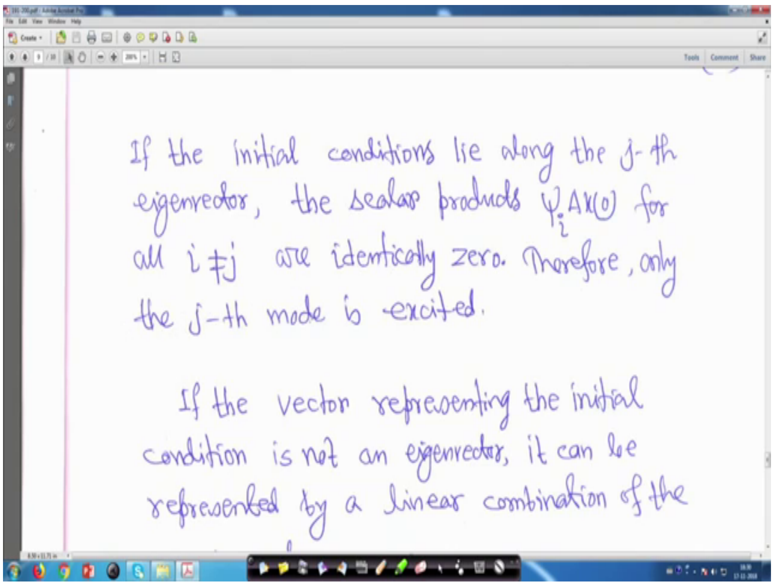

Therefore, if the initial conditions lie along the j-th eigenvector right, the scalar product psi i delta x 0 for all i not equal to j are identically 0 right. I mean if the initial conditions lie along the j-th Eigen vector the scalar product psi i delta x 0 for all i not is equal to j are identically 0. Therefore, only the j-th mode is excited. Now, if the vector representing the initial condition is not an eigenvector it can be represented by linear combination of the n Eigen vector right.

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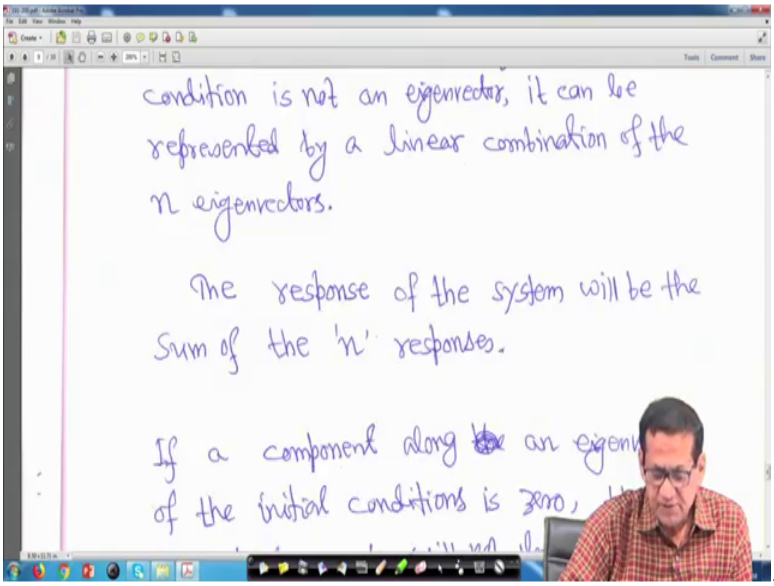


The response of the system will be the sum of the n responses.

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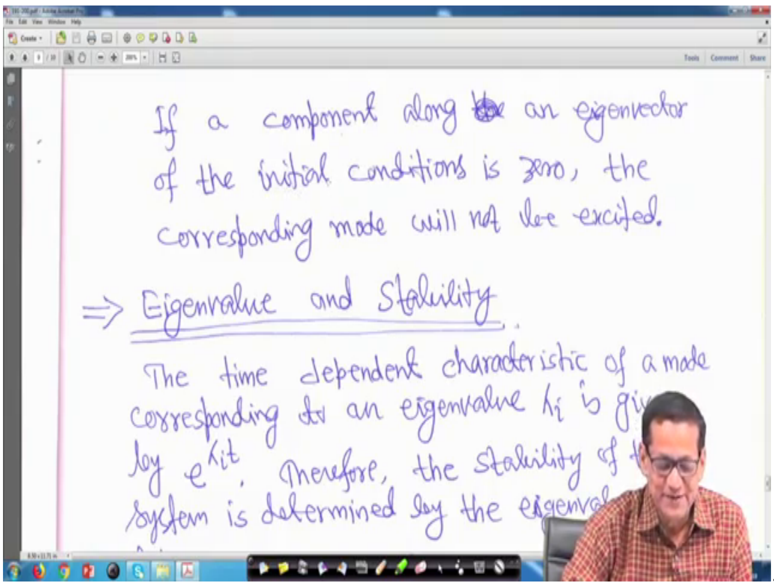

Therefore, if a component along an Eigen vector of the initial condition is 0, the corresponding mode will not be excited. So, little bit of you know little bit of understanding a little bit of practice is necessary we will take all together I believe 3 or 4 examples we will take later right. Next is that eigenvalue and stability. The time dependent characteristic of a mode corresponding to an Eigen value say lambda i is given by e to the power lambda i t this we have seen the response is function of that.

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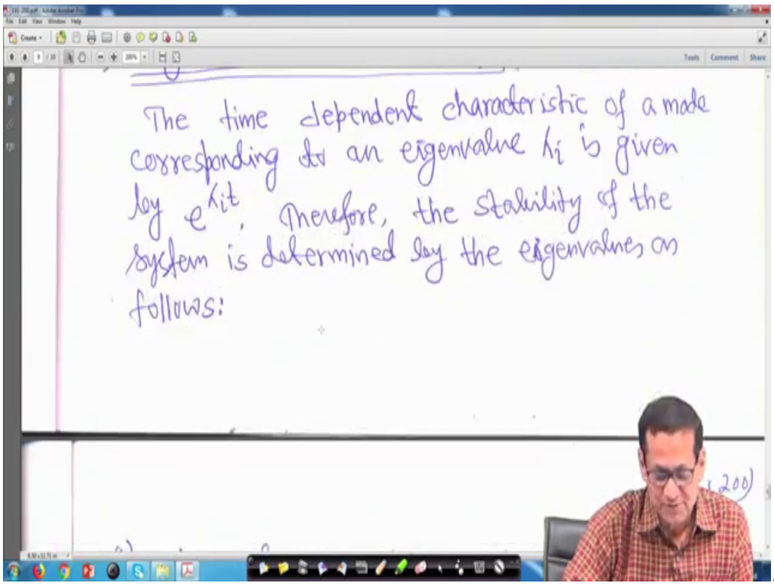


Therefore, the stability of the system is determined by the Eigen values as follows.

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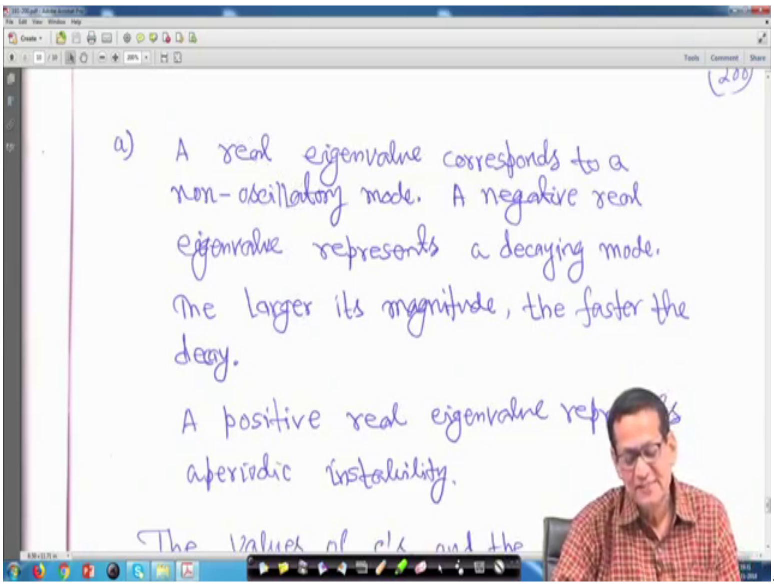

This you know thata real eigenvalue corresponds to a non-oscillatory mode right. A negative real eigenvalue represented decaying mode. So, a real eigenvalue corresponding to a non-oscillatory mode, real eigenvalue; that means, if it is real part is positive right. And, a negative real eigenvalue represent a decaying mode the larger it is magnitude the faster the decay. A positive real eigenvalue represent a periodic instability right.

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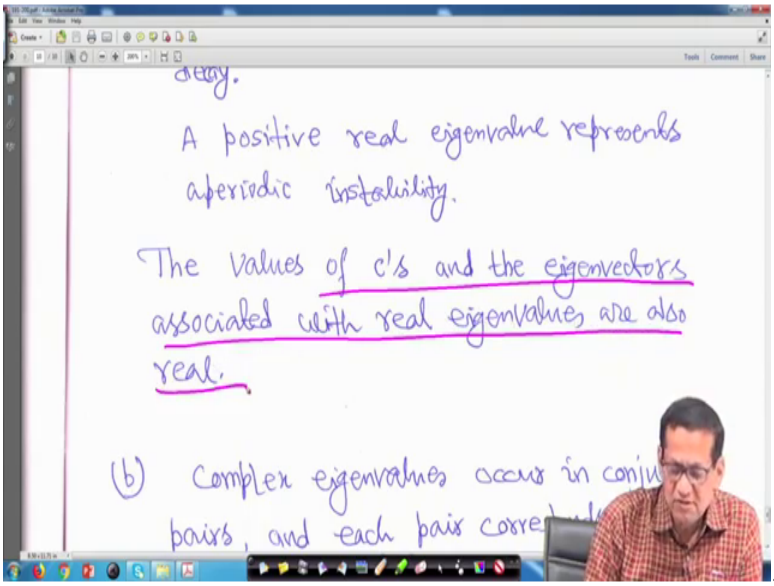


The values of c constant the c's and the eigenvector associated with the real Eigen values are also real. So, I mean 1 or 2 things you have to keep it in your mind the values of c's and the eigenvectors associated with the real eigenvalues are also real right. Now, the complex eigenvalues occur in conjugate that is true right.

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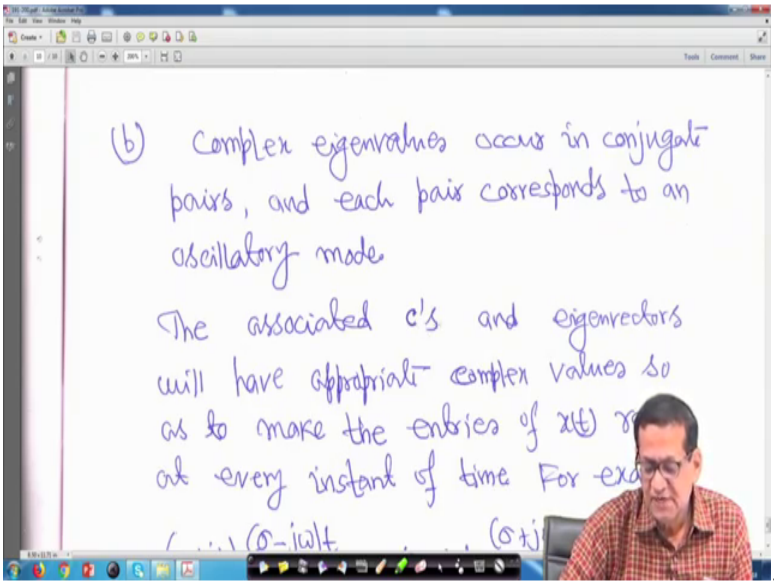


So, in conjugate pairs and each pair correspond to an oscillatory mode. Now, the associated c's values and eigenvectors we will have appropriate complex values so, as to make the entries of x t real at every instant of time.

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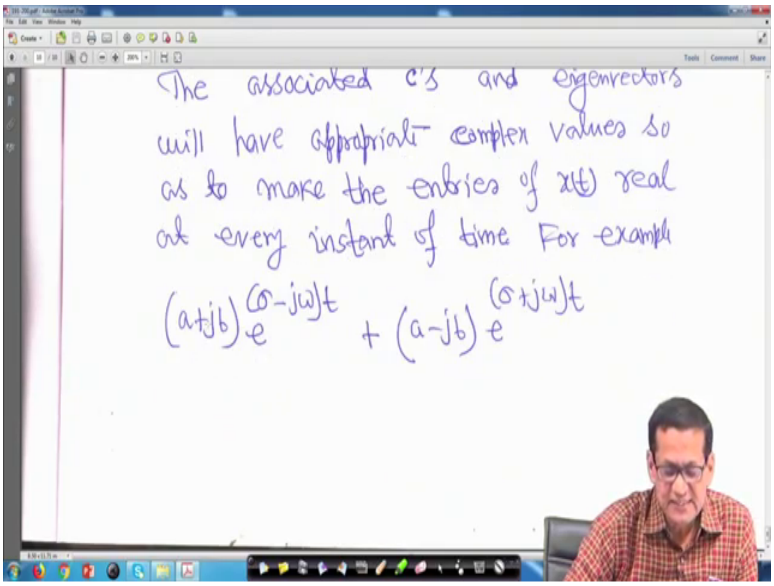


For example, it will be something like this a plus j b into e to the power sigma, minus j omega t plus a minus j b e to the power in bracket sigma plus j omega t right. So, this is actually it will come in this form. Now, next is your, just hold on.

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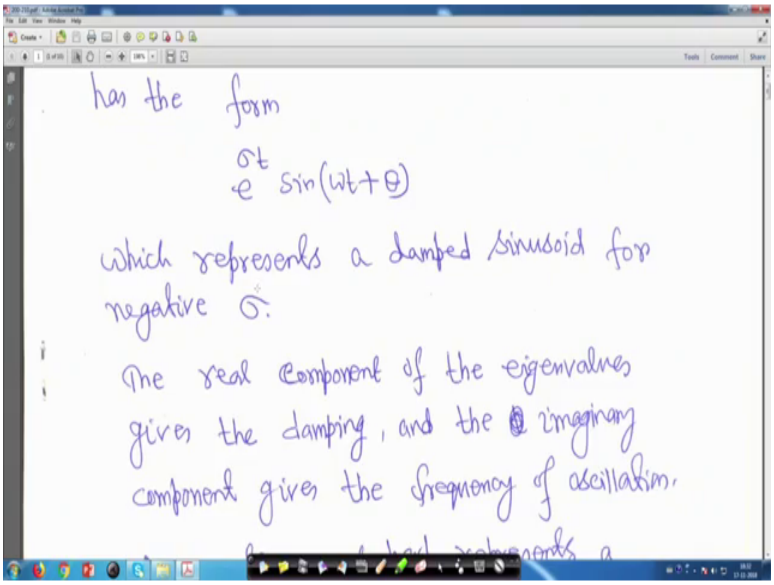


So, therefore, that equation that your a plus j b e to the power your sigma plus j omega t, then into plus a minus j b e to the power sigma minus j omega t that it can be written in this form e to the power sigma t into sin omega t plus theta right, which represent a damped sinusoid for negative sigma. The real component of the eigenvalues give the damping and the imaginary components give the frequency of oscillation this you know from your third year control system studied right.

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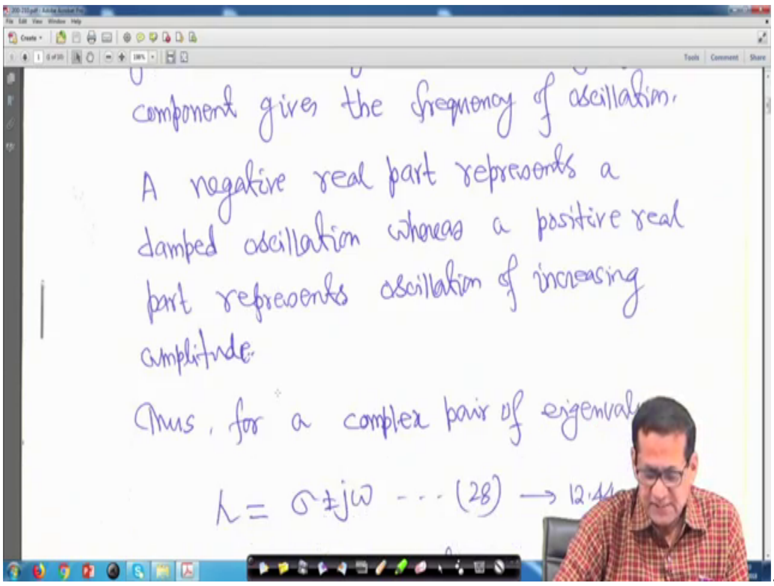

So, a negative real part represented damped oscillation whereas, a positive real part represent oscillations of increasing amplitude; that means, it makes the system unstable. A real part of the eigenvalues negative means in general it is stable, but a real part is positive then it is unstable right. Thus for a complex pair of Eigen values we can write say lambda is equal to sigma plus minus j omega. This is equation 20 right hand side all these thing this is for my own reference.

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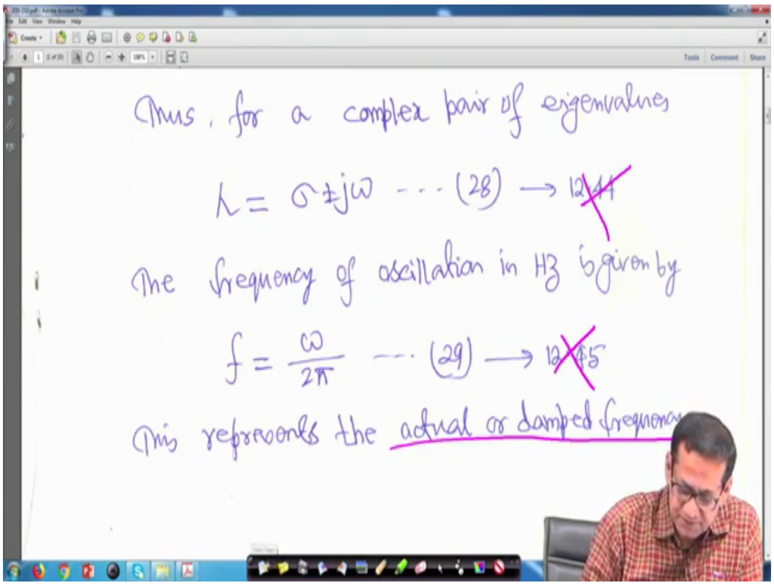


So, the frequency of oscillation in hertz then can given i f is equal to omega by 2 pi. This is actually equation 2 sorry 29, this is not for you this is for my own reference. The frequency of oscillation in hertz is given by this you know that omega is equal 2 pi f. So, f is equal to this much omega by 2 phi. So, this actually represent the actual or damped frequency right.

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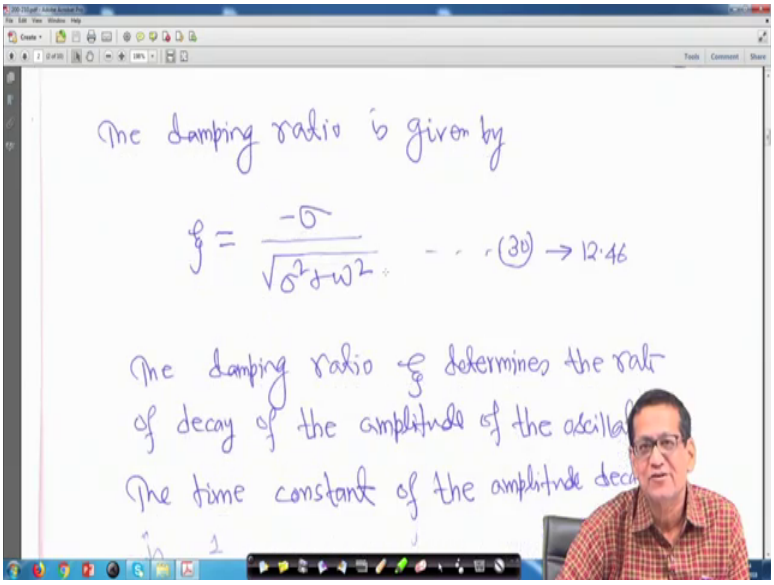


Next is that the damping ratio that also you know from your third year control system, that is xi is equal to minus sigma upon root over sigma square plus omega square this is equation 30 right.

Now, the damping ratio xi determine the rate of decay of the amplitude of the oscillation right. So, time constant of the amplitude decay is one upon mod sigma this also you have studied right. So, in your third year control system course undergraduate course, or in some other if you are studied linear up to linear control theory they are also you have studied this.

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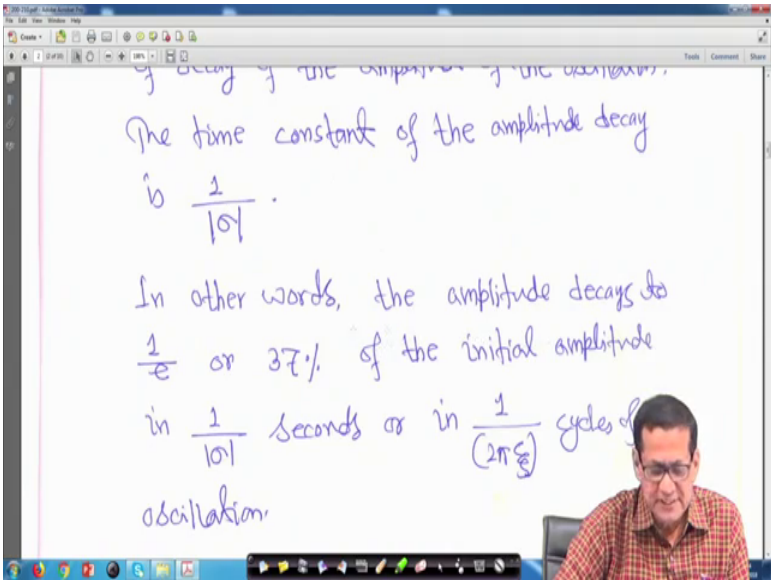


In other words the amplitude decays to 1 upon e or 37 percent this also you know right of the initial amplitude, in 1 upon mod sigma second or in 1 upon 2 pi xi cycles of oscillation right. This also you have studied, but just a your what you call just to your, what you call our just recall all those things right.

So, revisit have to we have to revisit certain things right. So, you can recall it from your memory and just have a look in the your what you call, that your third year control system that that chapter right, overshoot undershoot all sort of things where it is given any book any standard book you can follow right.

So, now this is an up to this right. So, this is not for you I did not bring this one I thought there is no need, this is actually this is not for you. So, this is not for you right. I did not bring those diagram I thought there is no need right. So, next is so this no question of figure 1. So, no figure is given. So, I just deleted that one.

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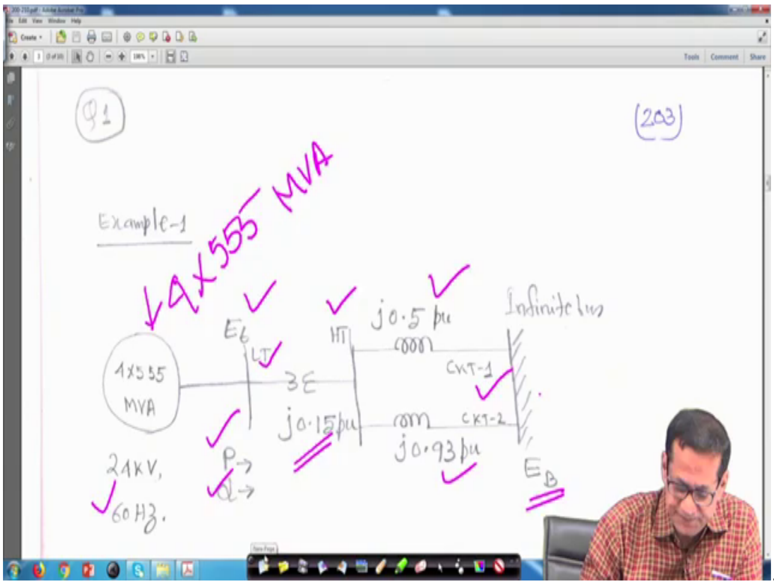


Next it let us take one example. So, this example actually if you look into that it is single machine infinite bus system, it is a there are 4 generating units each having 555 MVA. So, that is why it is 4 into 555 MVA right and term 224 KV and frequency is 60 hertz. This is the terminal voltage E t this is low tension side, there is LT HT is the high tension side.

This is power flowing this is real power this is reactive power data will be given. And, this is the transformer only reactance is considered j 0.15 per unit. This is a double circuit line a your what you call, then this is your j 0.5 per unit reactance, this is circuit 1 and this is j 0.93 per unit circuit 2 and this is infinite bus and voltage here it is E B right. So, now the problem is given like this; now let us see.

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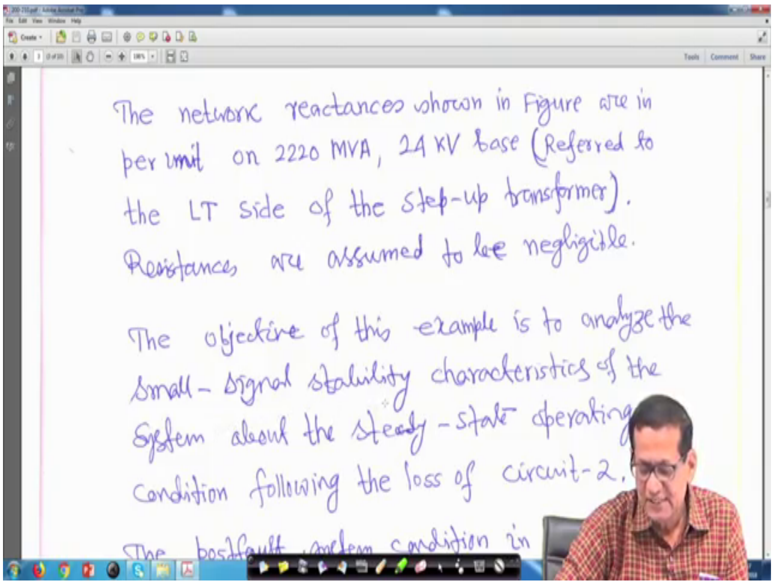


Now, the network reactance’s shown in figure are in per unit on 2220 MVA 24 KV base, because 555 into 4 is 2220 MVA, referred to the LT side of the step up transformer there is no tension side right. Resistances are assumed to be negligible, we have not considered resistance we have neglected it for the easy calculation right.

And, the objective of this example is to analyze the small signal stability characteristic of the system about the steady state operating condition following the loss of circuit 2; that means, the objective of this example is to analyze the small signal stability characteristics of the system about the steady-state operating condition following the loss of circuit-2. That means, suppose there is a fault and circuit 2 is totally disconnected right.

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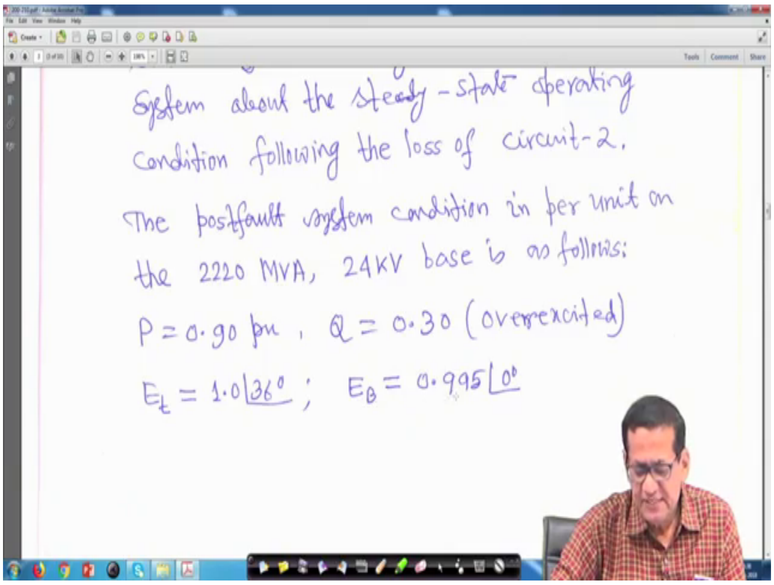


And, the post fault system condition in per unit on the 2220 MVA 24 KV base is as follows P is 0.90 per unit, and Q is 0.30 per unit, that is overexcited that is your per unit, that is overexcited condition right.

So, and this terminal voltage E t is given 1 angle 36 degree and E B is given 0.995 angle 0 degree all per unit; that means, that circuit 2 there is a loss of your what you call circuit 2 or circuit 1 just let me have a look, that circuit 2 right. So, this circuit suppose there is a fault and this circuit is out, this circuit will not be there. So, finally, what will happen the basically circuit is like this circuit is like this and this is infinite work this circuit is out. Therefore, total reactance of the circuit will be 0.5 plus 0.15 so j 0.65 right. So, this equivalent diagram is given.

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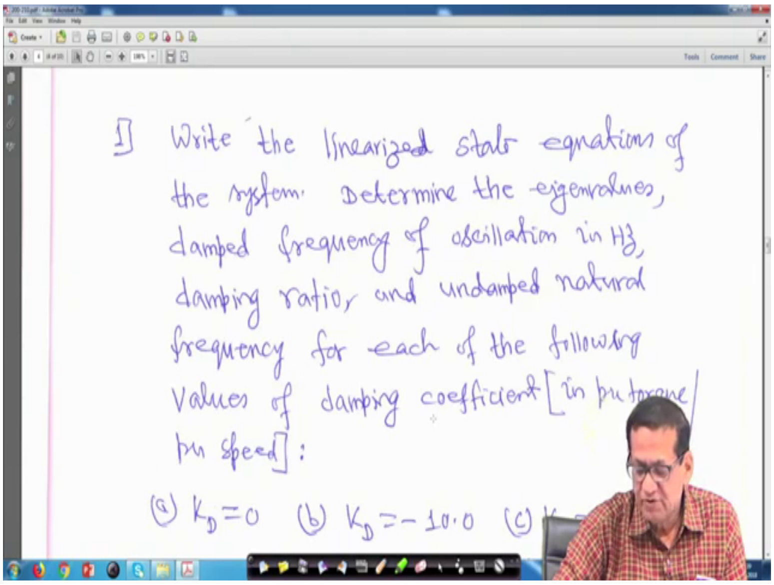


So, in this case in this case your now, you have to find out write the linear I will come to that circuit write, the linear linearized equation of the system determine the eigenvalues damped frequency of oscillation in hertz, damping ratio xi and un damped natural frequency. That is your omega n for each of the following values of the damping coefficient in per unit torque per second right, per unit speed that is one is case a will be a D is equal to 0, case b will take K D is equal to minus 10.

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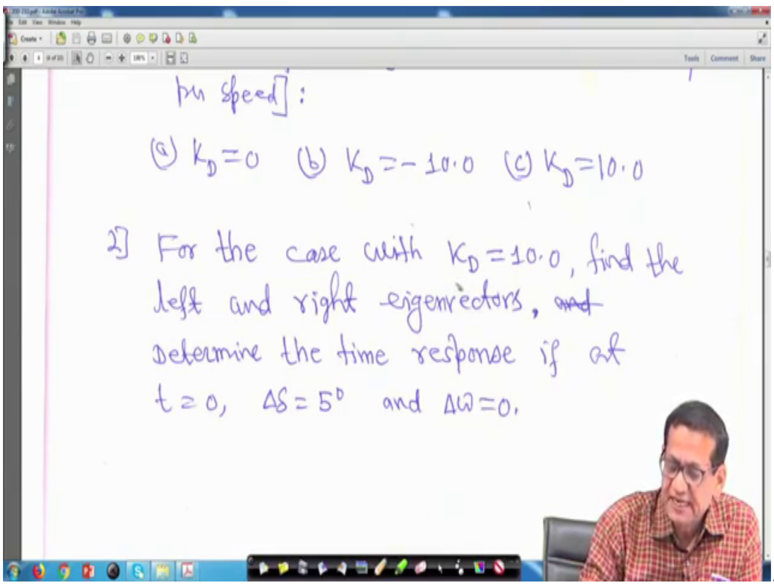


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And, case c will be k d is equal to 10 right these 3 conditions. Now, for the case for the case with K D is equal to your 10 you have to find out the left and right eigenvectors determine the time response, if at t is equal to 0 delta t is equal to 5 degree and delta omega is equal to 0.

These are the initial condition is given, that at t is equal to 0 delta delta is equal to 5 degree and delta omega is equal to 0 and for the case, for K D the damping coefficient K D is equal to 10 we have to find this right. So, let us see the equivalent circuit, now the solution.

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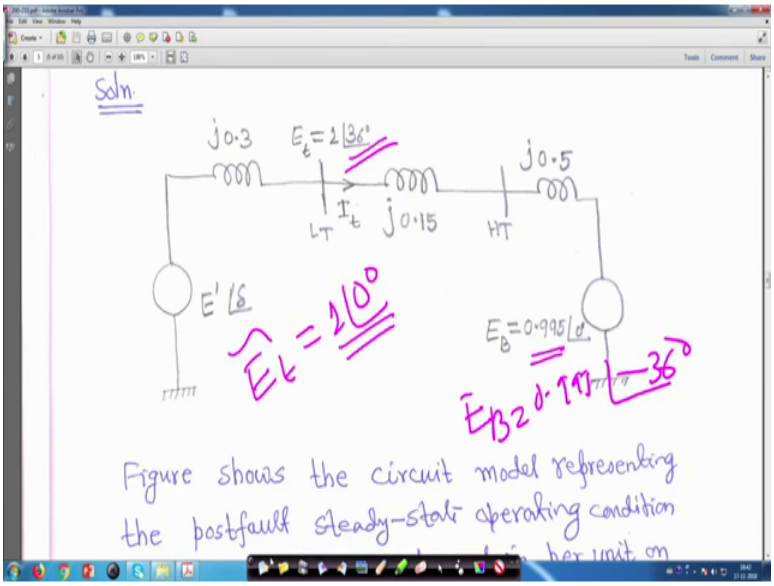


So, these parameters were given right. These parameters were given that if you look into that X d dash was given 0.3 per unit. So, this is actually your X d dash j 0.3 E t was given 1 angle 36 degree per unit and this transformer also given j 0.15 because circuit 2 is out right. And, this is the line reactance j 0.5 and this is actually E B is equal to 0.995 angle 0 degree this is infinite bus, and this is your E dash delta right voltage behind x D dash this is E dash delta.

So, our object will first you have to find out a delta. Now to solve this problem that of course, every meaning is same. For example, to solve this problem for easy calculation this is actually given that E t is equal to 136 degree. And, E B is given 0.995 angle 0 degree right.

So, now, if you take my E t as a reference E t is equal to say if I take 1 angle 0 degree right, then this E B will be your what you call 0.995 then angle minus 36 degree right.

Because, that ultimately either E t is leading E B by 36 degree or E B lags from your E t your 36 degree now right. So, meaning is same both are same if I take this is 1 angle 0, then it will be minus 36 degree. Now, if you draw for your further understanding if you draw the phasor diagram, suppose this is my reference line this is my reference line right.

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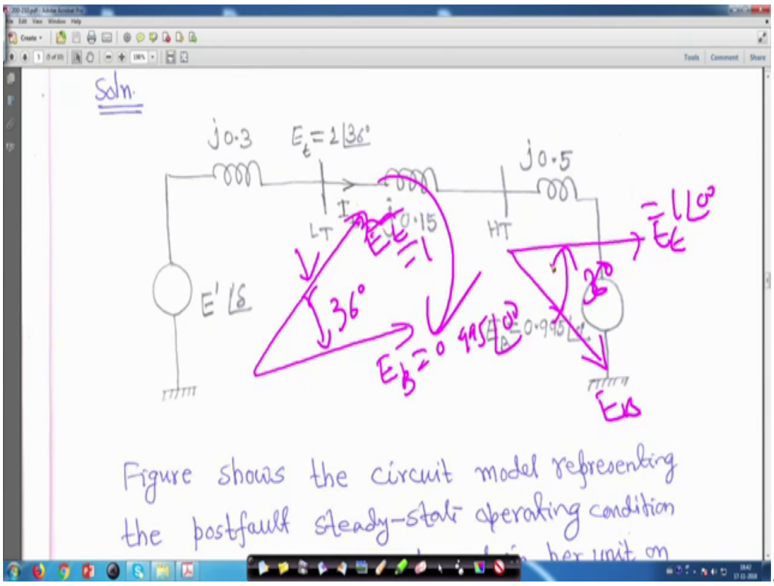


And, this is your E B is equal to your 0.995 angle 0 degree. Therefore, this is my E t this is my E t right and this angle is 36 degree right. This is E B and this is your this E t is equal to 1 right; this E t is equal to 1.

Now, if you rotate the phasor I mean if you bring this one as a reference rotate the phasor like this and bring a difference. So, it will go down. So, if this is my reference E t if this is my reference, if you take 1 angle 0 degree, then this one will become automatically this will be by E B and this angle will be 36 degree. So, meaning is same, these actually help us for your easy computation right. So, that is why we will we will follow that one right. So, figure shows this one.

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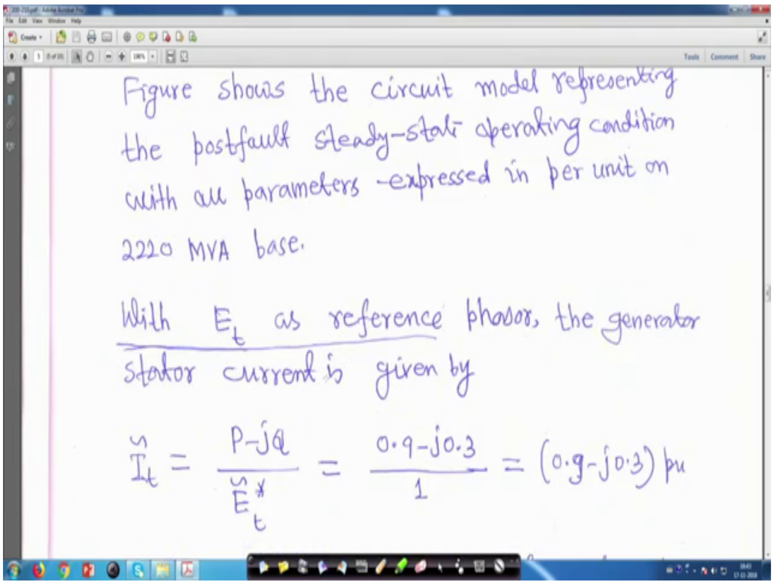


Now, with E t as reference that that is why with E t as reference right, this E t we have taken as a reference just now I told you, the stator current is given by P minus i Q is equal to E t E t tilde conjugate is equal to I t; that means, this is actually this is my come back to that, they come back to this your what you call this figure this is my E t right.

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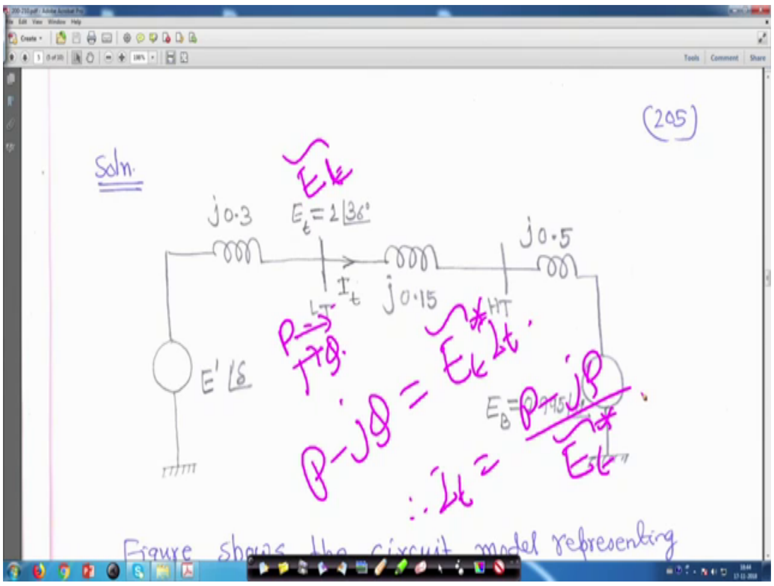


I take E t tilde as a and power was given P and P and Q both were given here; this is P and this is Q were given.

So, you know from load flow studies right that power injection you know P minus j Q we can write that E t conjugate and current flowing through this is I t. So, this is actually I t right; that means, my I t will be P minus j Q divided by E t the conjugate right and P i think it is given 0.9 and Q is equal to 0.3 right. So, if you come back to this current equation then that I t tilde your I t tilde is equal to P minus J Q upon E t tilde conjugate.

So, it is 0.93 minus j 0.31. So, it is nothing, but your what you call we have taken we have taken your E t as a reference.

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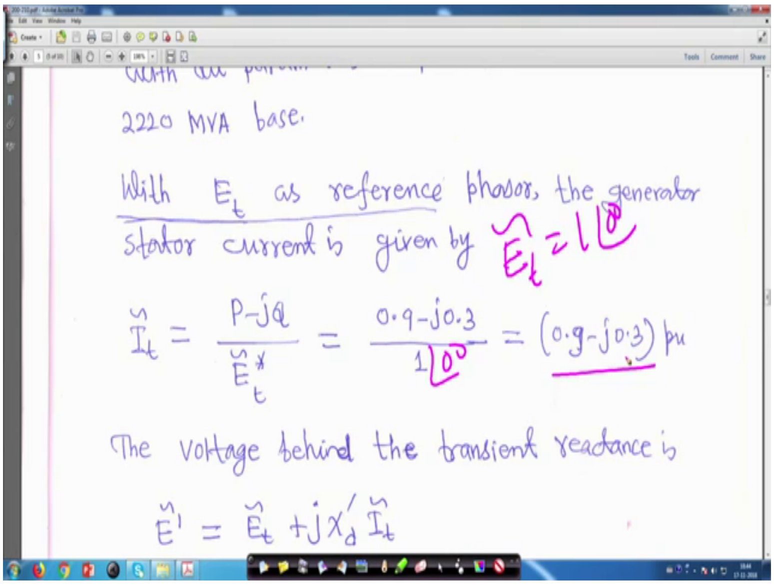


That means E t tilde is equal to 1 angle 0 degree right; that means, this is actually 1 angle 0 degree. So, this is nothing, but 0.9 minus j 0.3 per unit right. Next just let me clear it, next if you come back to this your what you call to this figure.

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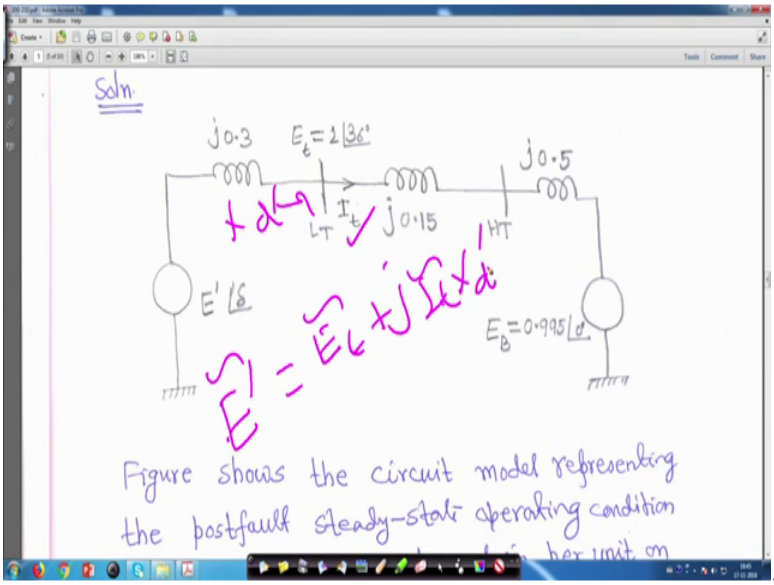


That means, from there also you can write that my in general my E dash tilde is equal to say E t tilde in phasor form plus j I t then x d dash because this is my x d dash and current flowing through this is I t.

Therefore, E dash tilde will be E t tilde plus j I t your x d dash right. So, that is why we are writing that.

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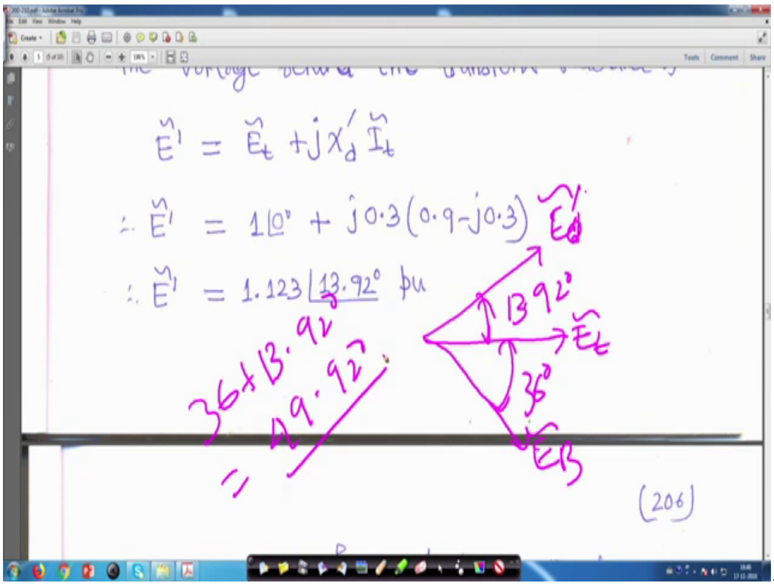

Your E dash t tilde is equal to E t tilde plus j X d dash I t. Now, E t tilde we have taken as a reference 1 angle 0 and this one is j and x d dash is given know x d dash is given here that is your 0.3 right. So, it is 0.3 into that I t tilde we have got 0.9 minus j 0.3 if you do so, then E dash tilde is actually 1.123 13.9 degree right.

Now, if you draw the phasor diagram; if you draw the phasor diagram I am making it here say this is we have taken reference this is my E t right, this is E t. Now, E dash actually it is positive angle leading. So, it is actually this angle say 13.92 degree right. This is my E dash your what you call sorry just hold on, it is E dash tilde right and this is my E t and E B as E t as taken reference. So, this is my E b right and this angle is 36 degree right.

Therefore, this E dash tilde leading E B by how much that is 36 plus 13.92 degree right so; that means, 49.92 degree. I hope it is understandable to you right.

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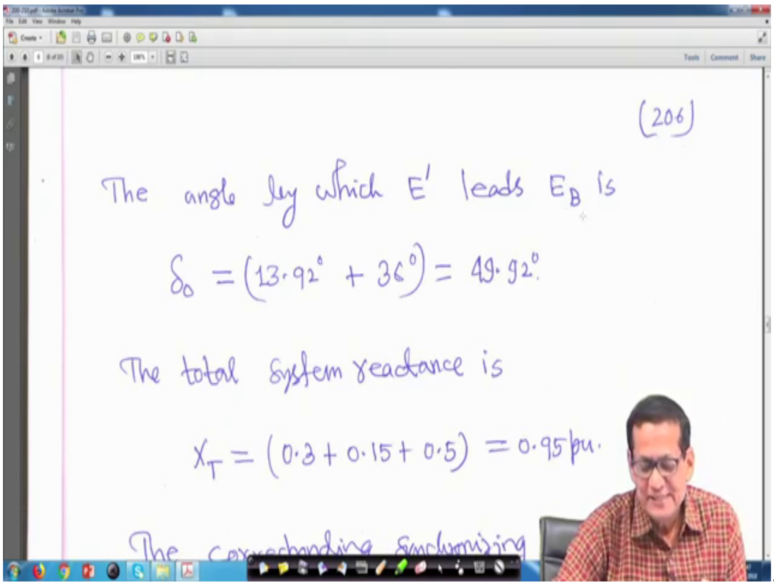


So, therefore, the angle by which E dash leads E B, I told you it will be 13.9 plus 36 of 49.92 degree right. Now, the total system reactance is that is X d dash plus your transformer reactance 0.15 plus line reactance 0.5. So, total is 0.95 per unit right.

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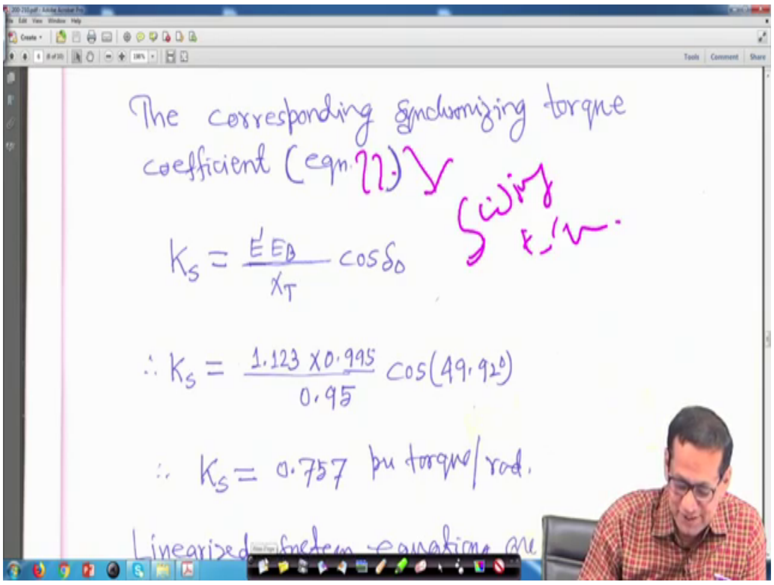


The corresponding synchronizing torque coefficient equation, actually we know the synchronizing torque coefficient this equation number I have forgotten, because you just go to those swing equations right. You go to that topic where I covered, I mean maybe few lectures back swing equation, there you know that is that your synchronizing torque coefficient right. And, I put a question also why it is called synchronizing torque coefficient you will put this answer into the forum when we will go through it right.

So, question is that this one E dash E B. So, this equation number I have forgotten I have written what I forgot to write it, but you know that K S is equal to E dash E B upon x t cos delta 0. So, E dash is equal to we got 1.123 E B is 0.995 and xt is 0.95 and delta 0 we got 49.92 degree; that means, case is 0.757 per unit torque per radian, which means this is a dimensionless quantity right.

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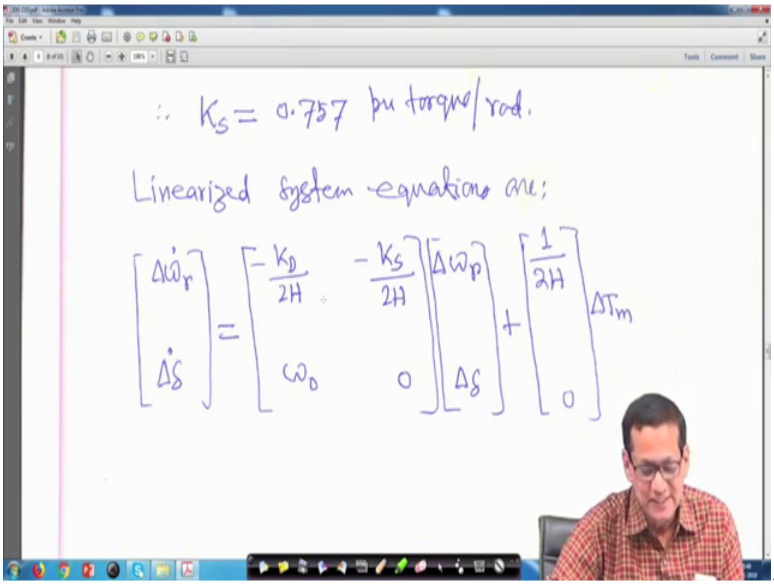


So, linearized system equation also we have seen in that swing equation that delta omega dot delta delta dot is equal to minus K D upon 2 H minus KS upon 2 H omega 0 0 delta omega upon delta delta plus 1 upon 2 H 0 T m this equation we have seen before right. I did not write the equation number, but same in that swing equation where we derived this same equation is there. I mean same equation we are just that linearized E Q system equation that we are using here go back to that swing equation thing and your what you call and your this equation right.

So, now just hold on so, this is actually earlier we have derive this equation right. So, or that K D we are using K D is equal to 10 value of H is also given right, value of H is given K S we have just determined right. So, all these values you substitute here.

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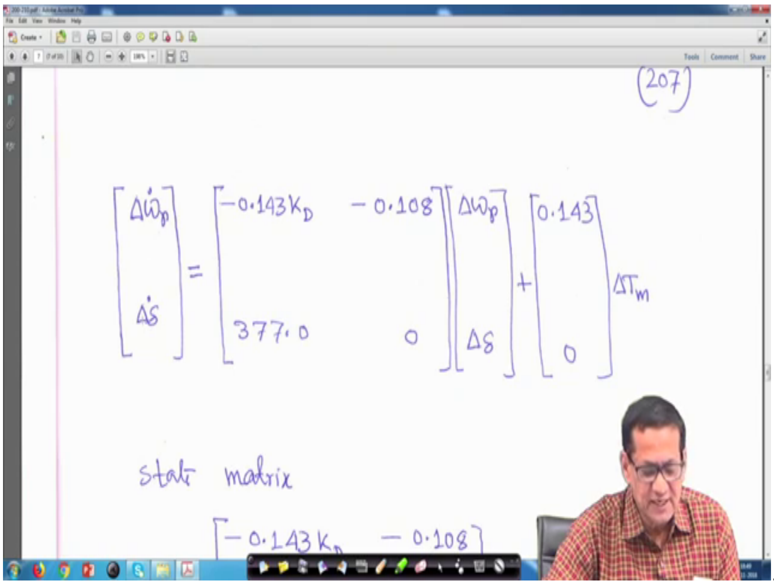


And if you do so that delta omega dot delta omega dot delta delta dot will be minus 0.143 K D minus 0.10 8 this is 377 2 pi f f is 60 hertz you have taken right. And, this is 0 delta omega delta delta plus this is this is 0.143 is coming this is 0 and delta T m right. So, this is actually state matrix it is 2 into 2 the simplest system.

Thank you very much we will be back again.