# Power System Dynamics, Control and Monitoring

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## Lecture – 05
## Power System stability (Contd.)

So, we are back again; whatever we finished in a previous lecture, so we are starting only from that point, right.

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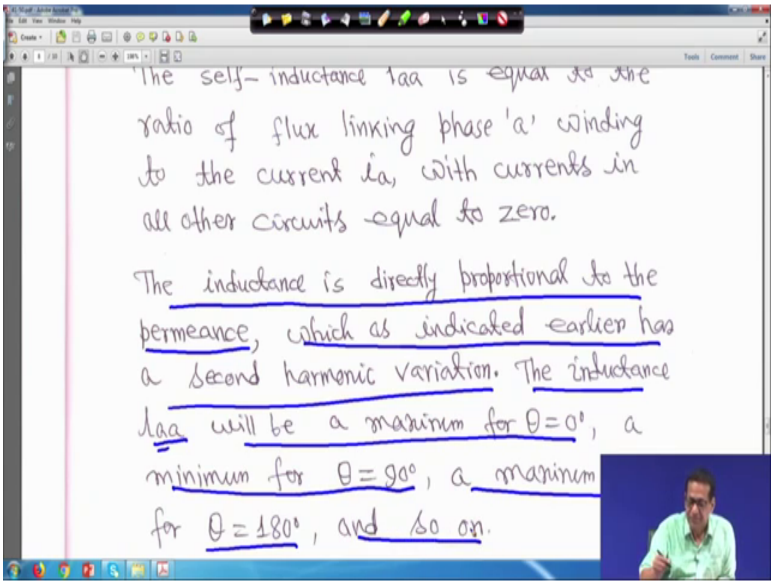


So, earlier we have seen that inductance is directly proportional to the permeance, which as indicated earlier has a second harmonic variation right. The inductance l aa right, actually l aa, l bb, l cc we will see their these are the you know stator yourself inductance right of a a means phase a, b means phase b, and c means a phase c, right will be a maximum at theta is equal to 0 degree and a minimum of theta is equal to 90 degree, right. And again, a maximum at theta is equal to 180 degree and so on, right.

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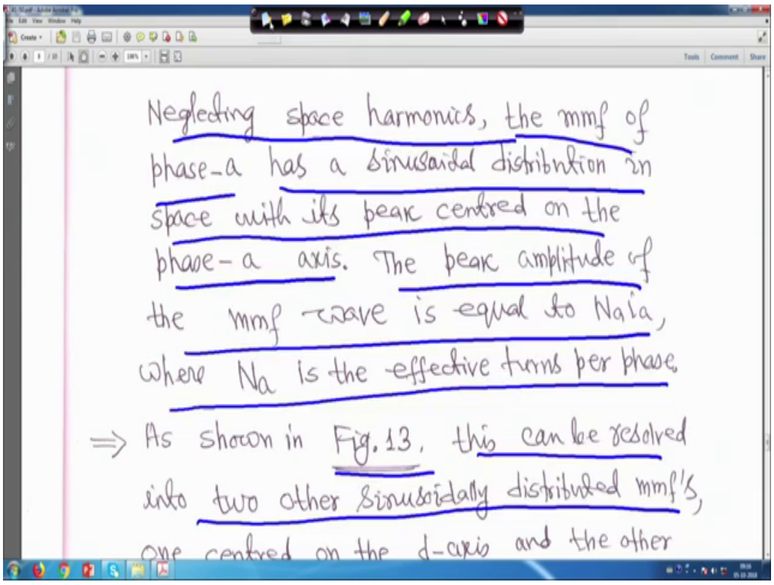


So, that means, just hold on. So, if we neglect the space harmonics right. If you neglect the space harmonics right, the MMF of phase a has a sinusoidal distribution in space with it is peak centered on the phase a axis, right. The peak amplitude of the MMF wave is equal to the Na ia that was I mean it is your number of your ampere turns right Na ia, where Na is the effective turns per phase, right. So, as shown in figure 13, I will show you the earlier you know in the previous lecture also you have seen that, this can be resolved into two other sinusoidally distributed MMF's, right.

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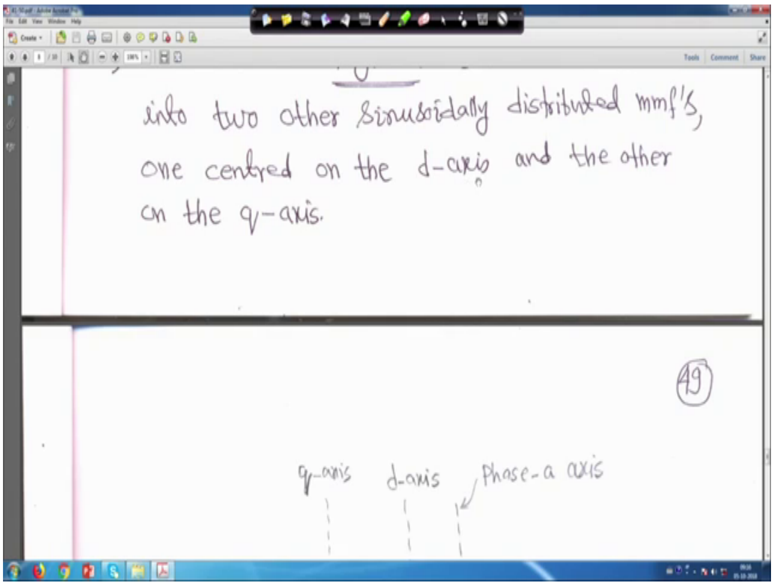

So, if we come to this, right. So an centered on the d axis and the other on the q axis.

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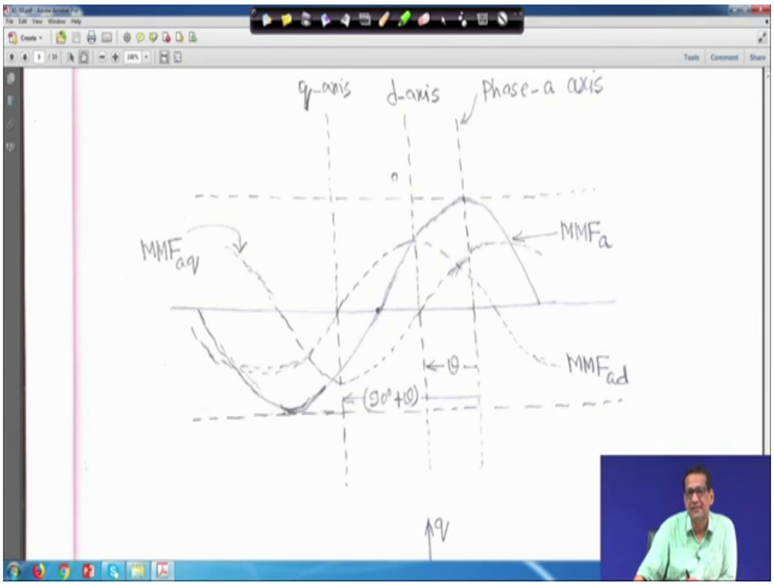


So, this is the diagram we have seen earlier. So, this is your MMF a and this is your MMF ad right, d x is 1 and this is another one that MMF aq. This is MMF aq right and from this point that phase a axis point theta is measured. So, this is theta and angle between q axis and d axis is 90 degree. So, from here if you measure it will be theta plus 90 degree, right.

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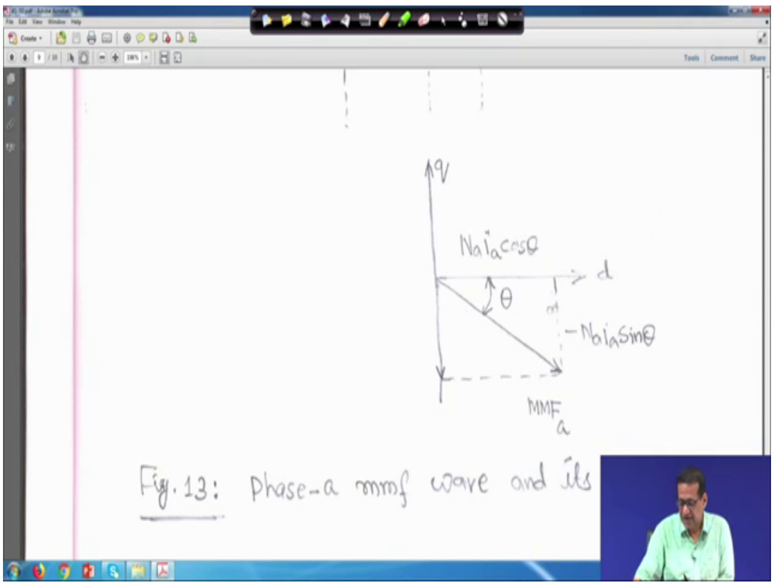

Now, if you dissolve this one in the q axis and d axis component, though this side on d axis. It will be Na ia cos theta and on d axis will be minus Na ia sin theta, because it will be Na ia cos 90 degree plus theta, right. So, that that will become minus Na ia sin theta.

So, these d axis and this is q axis and this is the resultant one MMF a, right.

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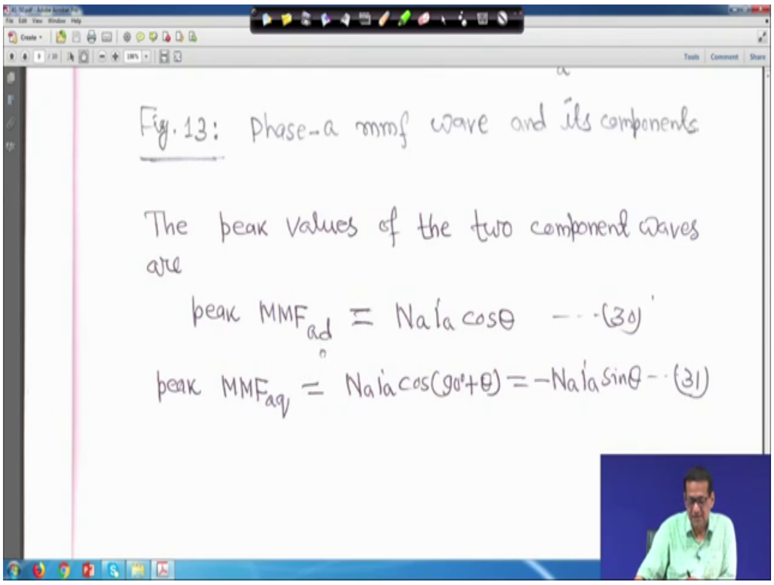


So, this is actually figure 13, phase a MMF wave and it is components. Now the peak values of the two component waves. Now, here it is given that MMF ad will be Na ia cos theta right and MMF aq will be Na ia cos 90 degree plus theta I told you, that will be minus Na ia sin theta. This is equation 31.

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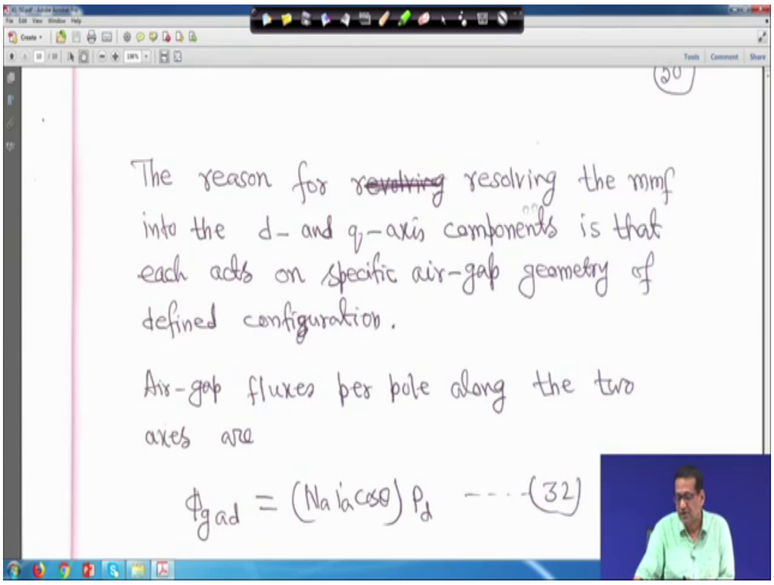


Now, next is the reason for resolving the MMF into the d axis and q axis component is that each acts on specific air gap geometry of defined configuration, right. So, air gap fluxes per pole along the two axes are, right. So, it is actually synchronous machine when we will go through that, we will see that every time that this little bit what you call I mean some certain assumption. We will make and another thing is that we will try to see that as simple as possible to the present these kind of thing. Therefore, air gap fluxes per pole along your two axes are given.

So, air gap has phi g a d with d axis, it will be Na ia cos theta into pd, right. So, that we will see that pd and pq are the permeance coefficient of the d axis and q axis respectively. And, this is Na ia cos theta into pd because we know that in general flux is equal to ampere trans into permeance.

This, we have seen when you are brushing up our memories for your couple circuit, right. Similarly, for phi g a q this we have seen the component minus Na ia sin theta into P q right. So, this is equation 32 and this is equation 33.

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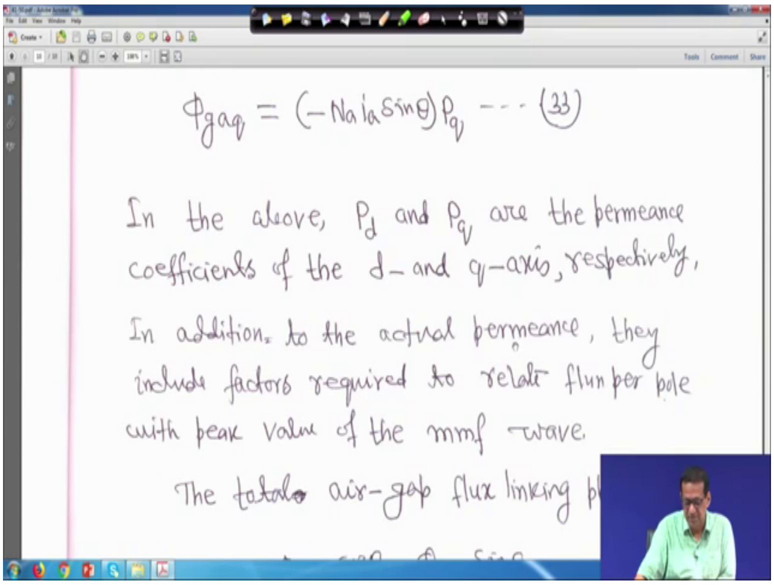


Now, in the above pd and pq are the permeance coefficient of the d and q axis respectively. Now, in addition to the actual permeance, they actually include factors required to relate flux per pole with peak value of the MMF wave, right. Therefore, the total air gap flux linking phase a is actually it will be phi g a.

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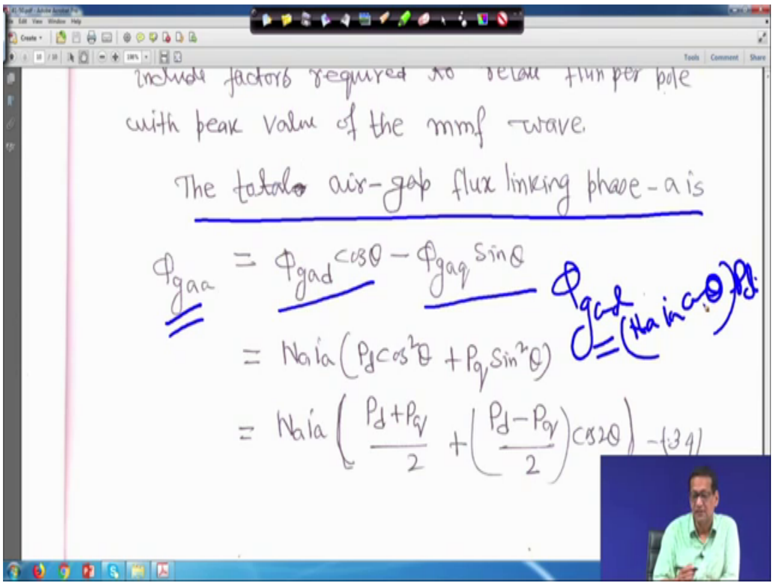


This is that your this is your, this is your that total air gap flux linking per phase a will be that is phi g a right is equal to phi g a d cos theta and minus phi g a q sin theta.

Now, p from previous expression, from previous expression, you will substitute here that your phi g a d is equal to Na ia pd cos theta. From the previous equation, that is your 32 and 33 and similarly, phi g a q you put it as minus Na ia that from previous expression sin theta into P d. These two if you substitute; that means, from previous expression that phi g a d is equal to Na ia cos theta right, that you substitute here then it, and your into your what you call into your P d, right. So, if you substitute here it will be Na ia pd then cos square theta because cos theta into this cos theta, cos square theta.

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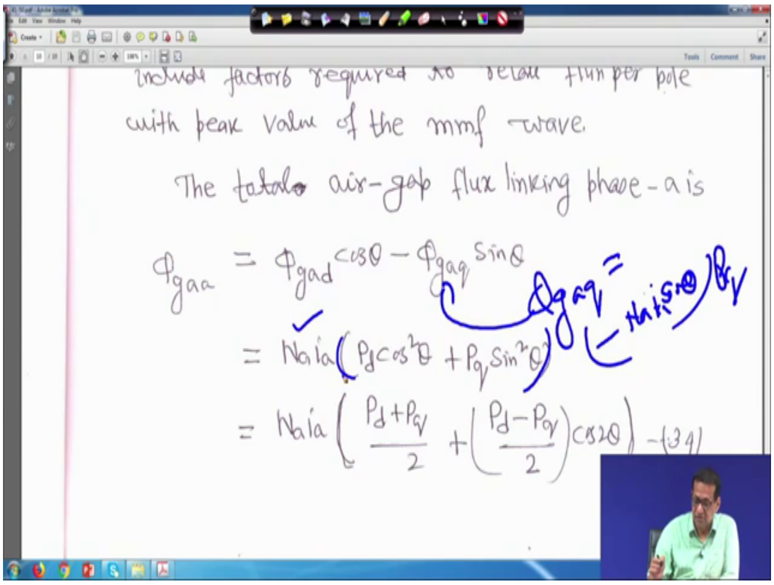


Similarly, for phi g a q from the equation your 33 right, phi g a q is equal to minus Na ia your sin theta into your P q. So, you substitute, you substitute here, this expression you substitute here. So, it will plus then your Na ia will be common because this is in bracket Na ia is common. Basically, it will be become Na ia pq sin square theta right. So now, let me clear this.

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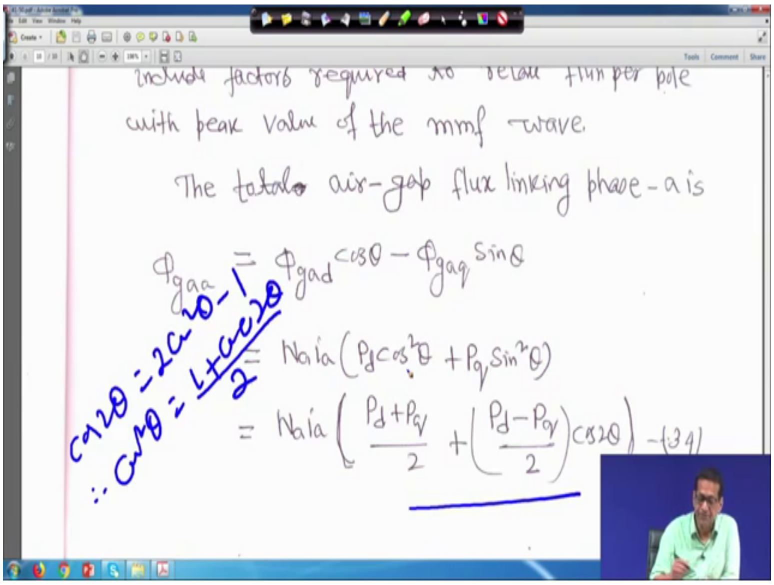


Now, after that you simplify this one, because we know that your cos 2 theta is equal to 2 cos square theta minus 1; that means, your cos square theta is equal to 1 plus cos 2 theta divided by 2. So, cos square theta, you substitute here, 1 plus cos 2 theta by 2.

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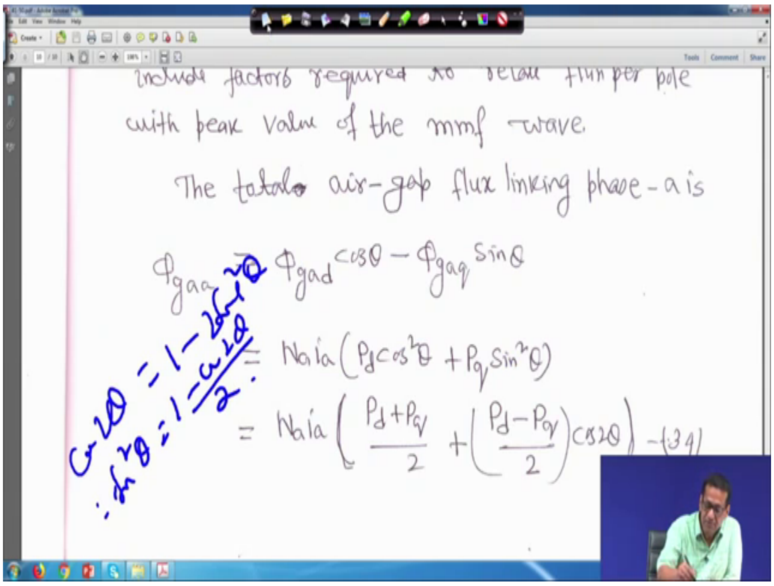


Similarly for sin square theta, similarly for sin square theta again that cos 2 theta is equal to 1 minus 2 sin square theta; that means, sin square theta is equal to 1 minus cos 2 theta by 2, right. Here also, you substitute and then simplify. If you do so, you will get Na ia, then P d plus P q by 2 plus P d minus P q by 2 into cos 2 theta.

So, this is equation 34 right, I hope this part is understandable. Now, once you have once you have done, right. Just hold on, we will go to your next one, right.

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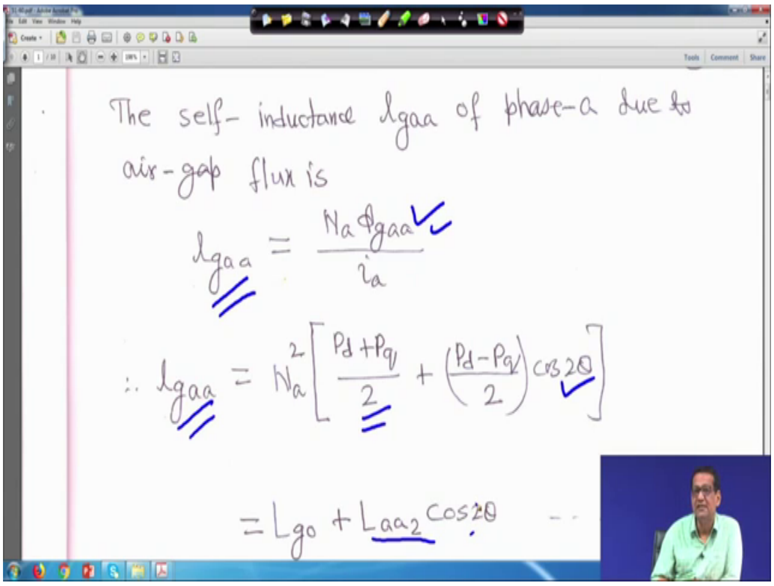


Now therefore, we know this the self inductance L gaa of phase a due to the air gap flux is, it will be n if general formula n phi upon I right. Therefore, your these thing, this Lgaa right the self inductance Lgaa of phase a due to air gap, flux will be Na phi g a upon i a, right. So, that phi g a expression we have seen, you substitute here in that case what will happen ia, ia will be cancel and Na Na will be multiplied. So, it will N a square sorry plus P d plus into P d plus P q upon 2 plus P d minus P q upon 2 into cos 2 theta, right.

So, these equation, this one is equal to you can right Lg 0 plus laa 2 cos 2 theta; that means, one this is that that Lgaa is not a constant it is function of theta, right.

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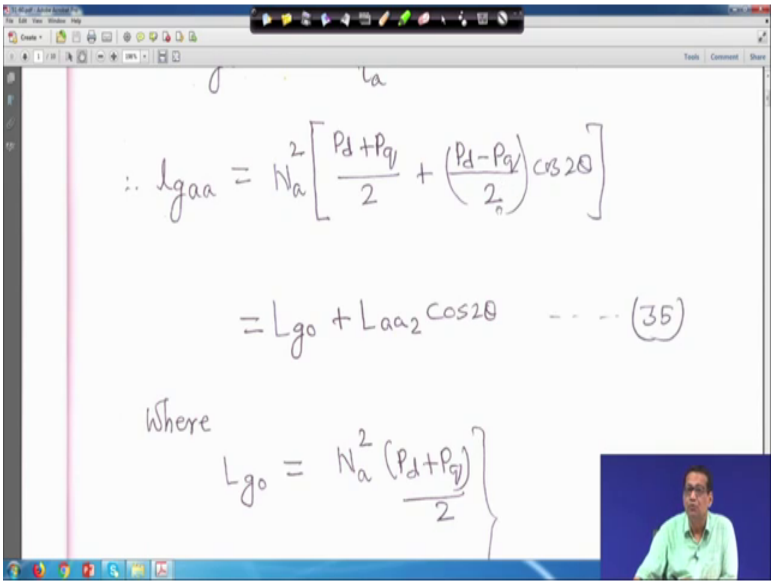


So, where Lg 0 is equal to your na square into p d plus p q upon 2. So, it is given N a square into p d plus p q by 2 and laa 2 will be P d minus P q by 2. So, this is into N a square, right.

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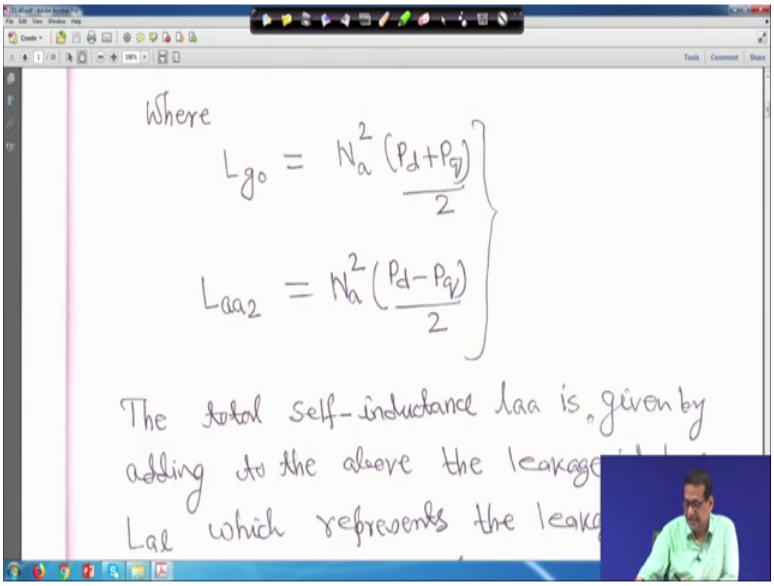


So, the total self inductance L aa is given by adding to the above the leakage inductance L als, I mean we are considering say some leakage inductance is also there, right.

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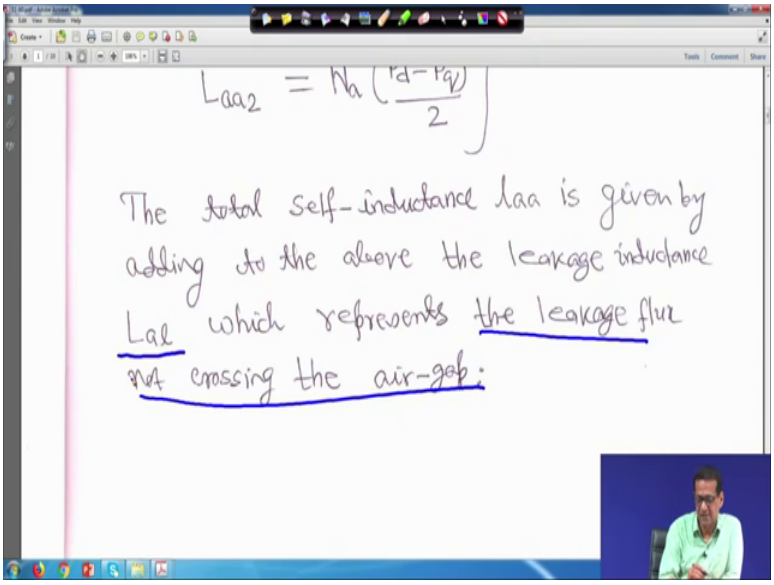


Just your some leakage inductance L al is there capital a suffix is small al there which represent the leakage flux not crossing the air gap. So, very small amount, but I mean we are we are adding to this. If you do so, then it will be L aa will be L capital L al plus L gaa right which is actually now, this L g 0 your la L al is there then L g 0 lgaa is that equal to Lg 0 plus L aa 2 cos 2 theta, right.

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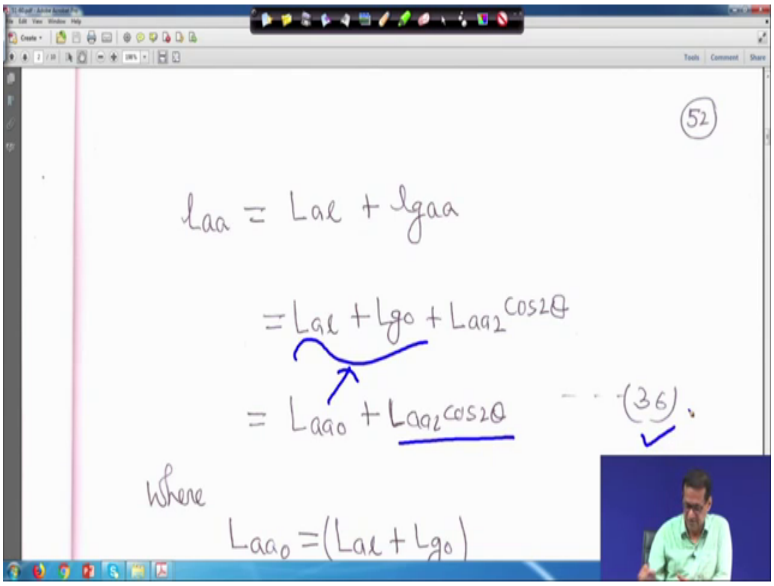


And this can be written as L aa 0 plus L aa 2 cos 2 theta. Actually, this part actually this part, this little bit of a leakage flux will be there which is not crossing the air gap. So, along with this, you have to add this right. So, if it is so, and L ga is equal to Lg 0 plus Laa 2 cos 2 theta all Lg 0 and Laa 2 we have defined just in the previous page, right.

So, this can be totally it can be written as your totally it can be written as your L aa 0. So, L aa 0 is your this term, L al plus Lg 0 and this is L aa 2 cos theta this is equation 36, right.

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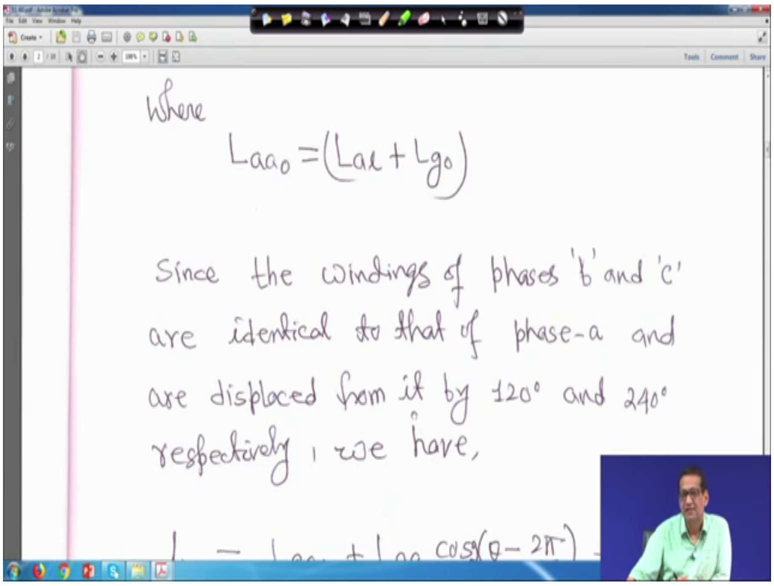


So, therefore, L aa 0 is equal to L al plus lg 0. Since, the windings of phase b and c are identical right, because it is a symmetrical winding and 120 degree apart, right. So, that the phase a and are displaced from it by 120 degree and 240 degree respectively, right.

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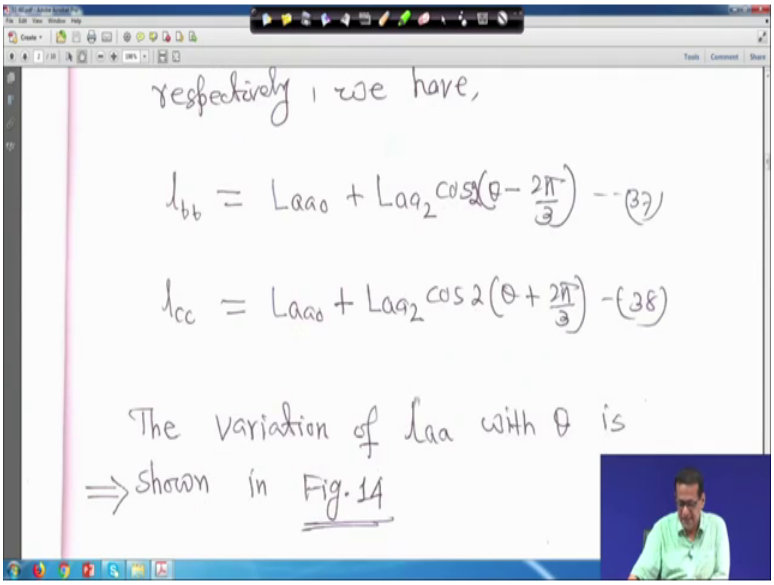


So, similarly therefore, l bb and l cc will get because we write L aa 0. We will not write l bb 0 or l cc 0, because they all are same from the symmetry right. So, l bb is equal to instead of writing l bb 0 l cc is equal to l cc 0, because all are same L aa 0 is equal to l bb 0 is equal to l cc 0. So, we are representing only this term L aa 0 again and again right. Similarly, this term also from symmetry we will not write bb 2 or cc 2 will same thing because only 120 degree apart, but this magnitude will remain same right. Therefore, L aa 0 will be L aa 2 cos 2 into theta minus 2 pi by 3 right because 120 degree apart and second one l cc will be L aa 0 plus L aa 2 cos 2 into theta plus 2 pi by 3. This is equation 37 and this is equation 38.

So, only we are finding out that L aa. After that from the symmetry, we are just replacing the theta by theta minus 2 pi by 3 for the phase b and for phase c, that is it is theta your what you call replacing theta by theta plus 2 pi by 2 pi by 3 right. So, the variation of L aa with theta is shown in figure 14.

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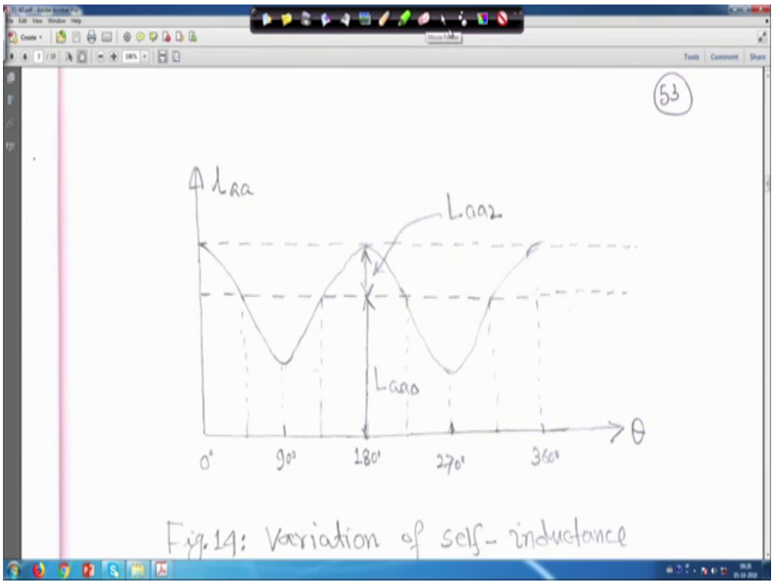


So, this is the variation of L aa. So, if you come to this, that your expression for your L aa, right. So, this is what you call this is your L aa. So, when theta is equal to 0. Therefore, your maximum value will be L aa 0 plus L aa 2 right and when theta will be is equal to phi by 2 right.That means, theta pi by 2 means 2 theta is equal to phi, so it will be L aa 0 minus L aa 2.

So, maximum plus and minimum minus, so if you plot this the plotting will be like this right when theta is equal to 0, this is the for your what you call that for maximum that positive value when it is theta your pi by 2, 90 degree. It is the minimum one right. And this is part is L aa 2 and this L aa 0 is constant because L aa 0 plus L aa 2 cos 2 theta. So, this is your this is your L aa 0 line I mean this is your, this is your L aa 0 right because this part is L aa 0. And this part, your what you call from this part is L aa 0 plus L aa 2 and this part is L aa 0 minus L aa 2, right.

So, this is your l plot plot of L aa versus theta, variation of self inductance. So, it is not a constant it is, it is varying with theta, right.

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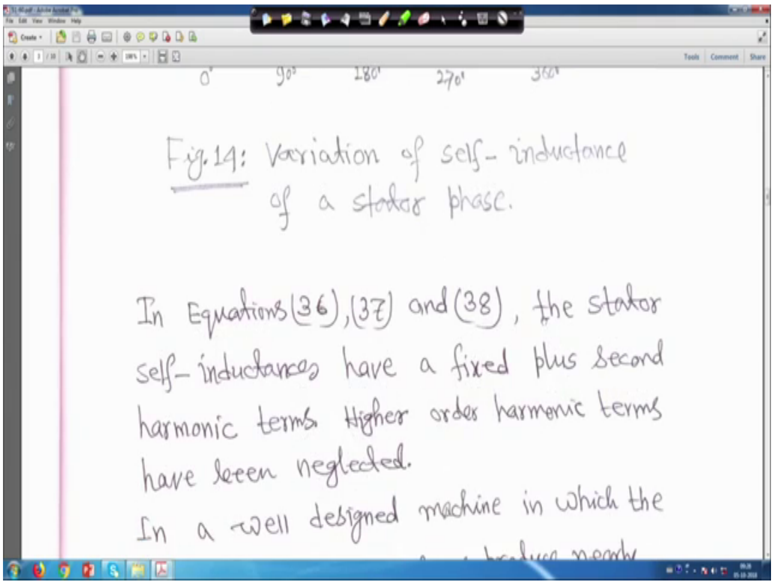


Now, in equation 36 and 37 and 38, the stator self inductances have a fixed plus second harmonic terms right, that is fixed term is a L aa 0 plus you have the term L aa 2 into cos 2 theta right. Higher order harmonics terms have been neglected for this study otherwise things will become complicated.

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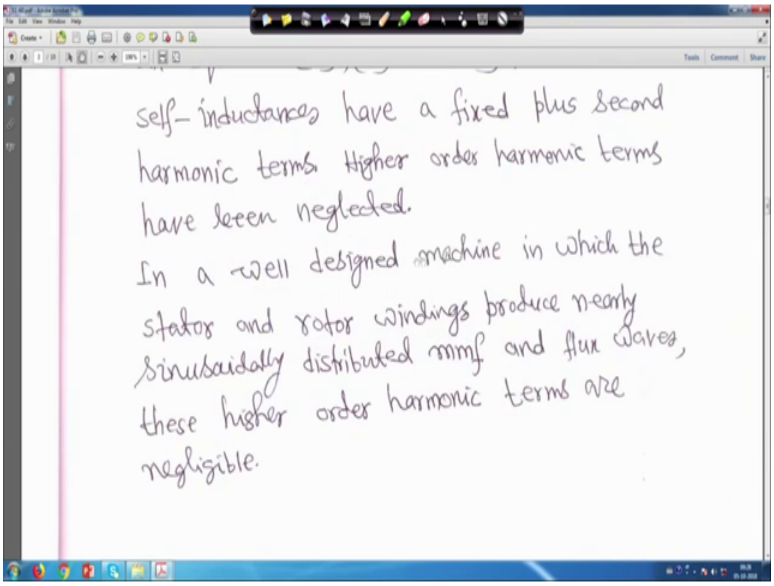


So, in a well designed machine in which the stator and rotor winding produce nearly sinusoidally distributed MMF and flux wave, these higher order harmonics terms are negligible, very small. That is why it is neglected.

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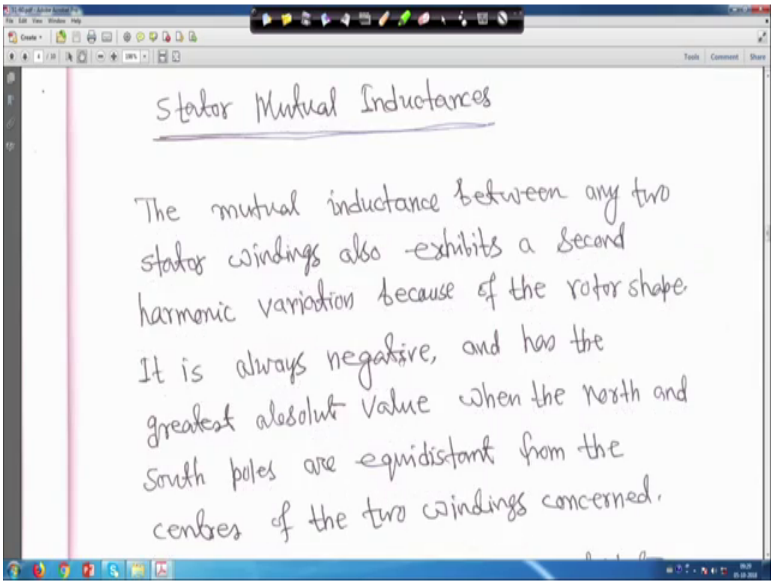


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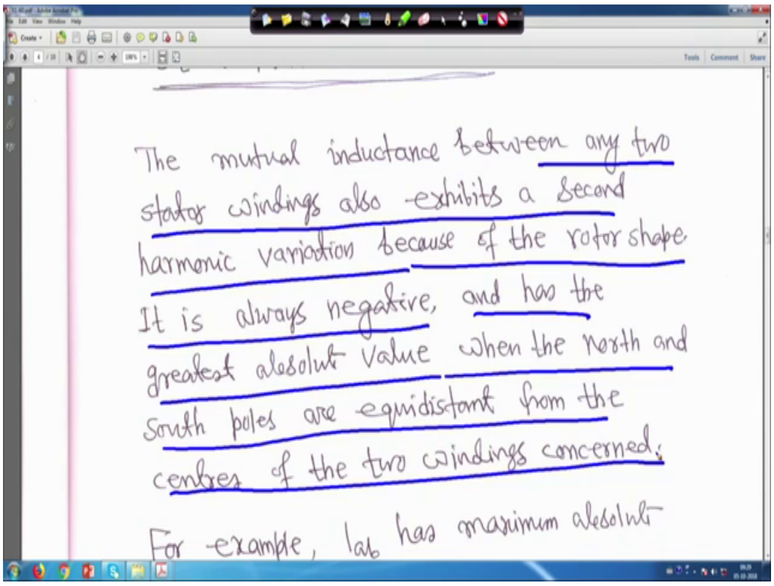


Now, stator mutual inductances, the mutual inductance between any two stator windings also exhibits a second harmonic variation, because of the rotor shape right. It is always negative and has the greatest absolute value when the north and south poles are equidistance from the centres of the two windings concerned, right. So, the mutual inductance between any two, so what you call stator windings that is in between phase a or b b or c or c or a right exhibits a second harmonic variation, because of the rotor shape right. It is always negative and has the greatest value when the north and south poles are equidistance from the centres of the two windings concerned, right.

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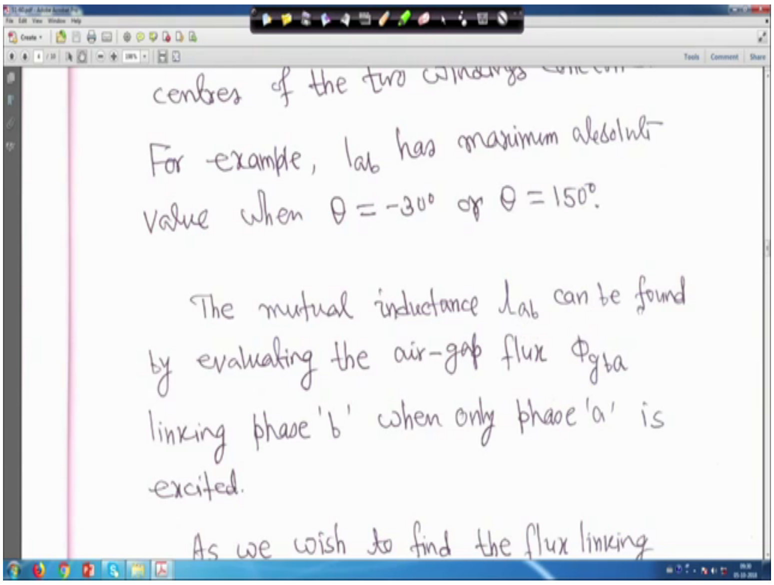


For example, suppose l ab, l ab, l ab has a it is between your phase a and b of stator winding right had maximum absolute value when theta is equal to minus 30 say or theta is equal to 150 degree. The mutual inductance l ab can be found by evaluating the air gap flux phi g b a linking phase a when only phase a is excited the way. When you are revising that your couple circuit, right. So, same philosophy; so the mutual inductance lab can be your found by evaluating the air gap flux phi g b a linking phase b when only phase a is excited, right.

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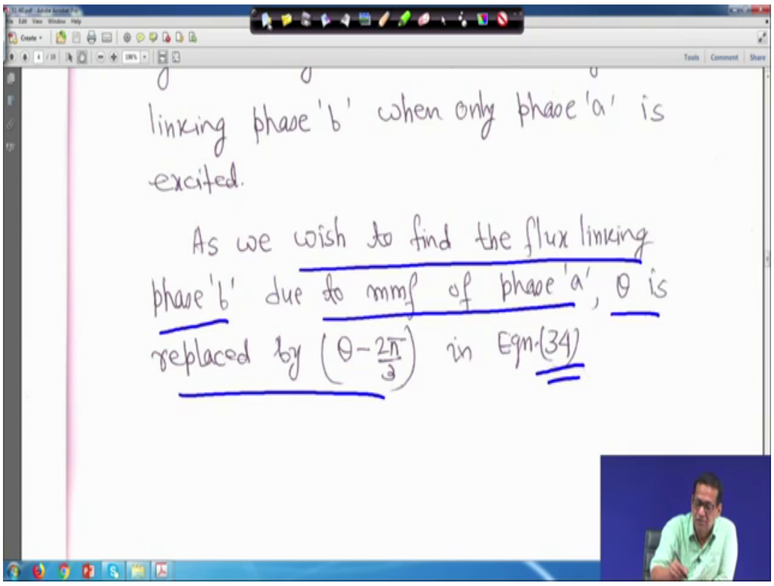

So, as we wish to find the flux linking phase b, due of MMF of phase a right. So, theta is replaced by theta minus 2 by 3 in equation. This is very important. You need not you need not derive many things look as we wish to find the flux linking phase b. Due of MMF of phase a, so, then theta is replaced by theta minus 2 by 3 in equation 34 right.

So, some our intuition, we can make this one.

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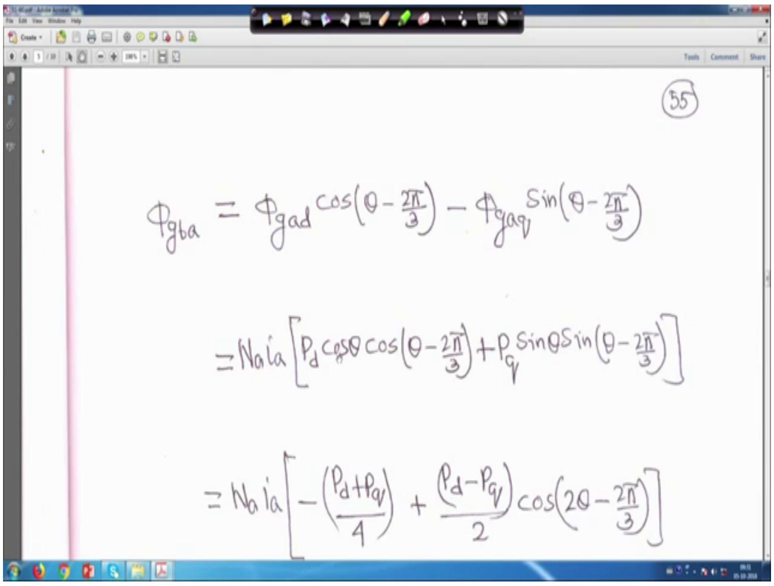


That means, that phi g b a right is equal to phi g a d; just same thing we are writing replacing theta by theta minus 2 pi by 3 cos theta minus 2 pi by 3 minus phi g a q sin theta minus 2 pi by, right. Now, if you substitute the expression of phi g a d that was Na ia pd cos theta and here also, phi g a q if you substitute minus Na i a your what you call sin theta into P q, if you substitute, then it Na ia you will take common then it will be P d cos theta into cos theta minus 2 pi by 3 plus P q sin theta into sin theta minus 2 pi by 3. Only same expression, earlier whatever we did phi g a a right, just we replacing theta by theta minus 2 pi by 3 that was in equation 34, right.

So, if you simplify this one is an small exercise for you because if I try to do all these things here, then it will consume more time right. So, if you just simplify this one, then it will become your what you call minus P d plus P q upon 4 plus P d minus P q upon 2 cos 2 theta minus 2 pi by 3, right. So, this way what you call this has been simplified. So, this is actually cos a cos b and this is your sin a sin b and just on that just giving you some hint. So, try to derive this one right.

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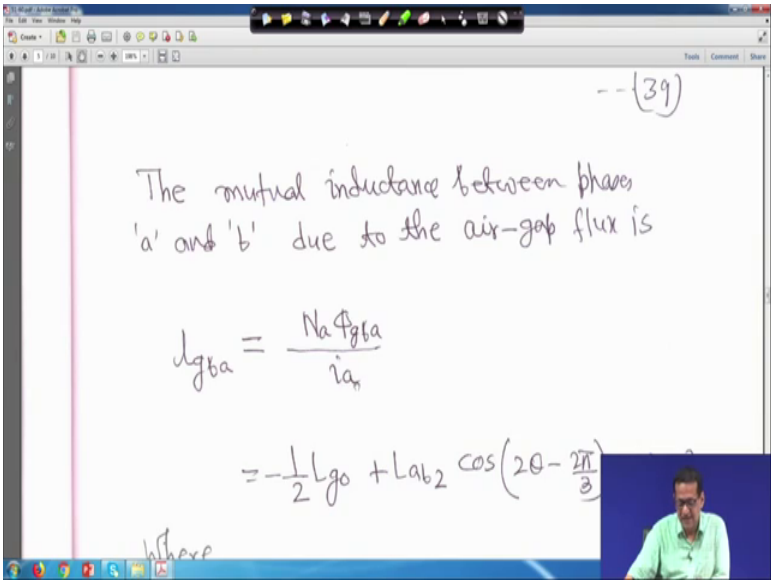


So, if you make it,it will be coming like this. So, this is equation 39 after simplification.

Now, the mutual inductance between phases a and b due to the air gap flux is, now we have to find out the mutual inductance between phases a and b due to the air gap flux that mean l gba right, this is the mutual inductance between your what you call phases b and a it will be Na into this your phi g b a divided by the current because x phase is exited. So, divided the current in phase a. So, it will be Na phi gba upon ia. Now, if you substitute the expression from here phi g, this is your phi g b a this is your phi g b a, right.

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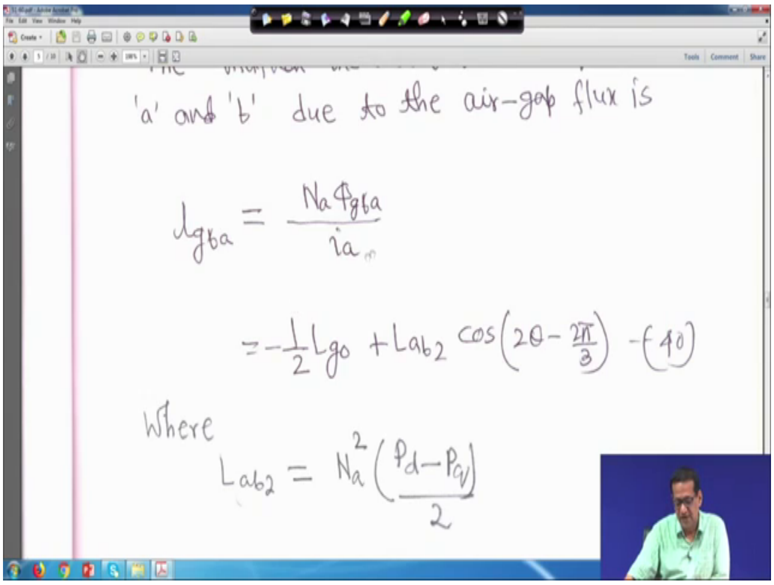


You substitute your ia, ia will be cancel. And it will finally, after simplification we can write that l gba will be minus half lg 0 capital lg 0 plus capital l ab 2 cos 2 theta minus 2 pi by 3. This is equation 40 where L a b 2 will be N a square into P d minus pq upon 2 is substitute and simplify write, then you put in this form and lg 0, right.

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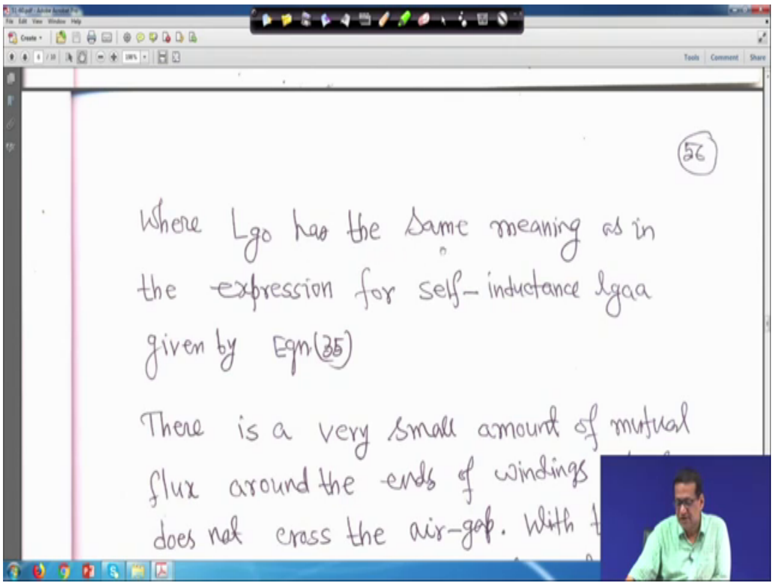


And where lg 0 has the same meaning as in the expression for self inductance L gaa given by equation 35.

So, whatever lg 0 is there in equation 35 same meaning, so just you put it just you put it and simplify and get in this form right. So, it will be la b 2 will be Na square into P d minus P q by 2.

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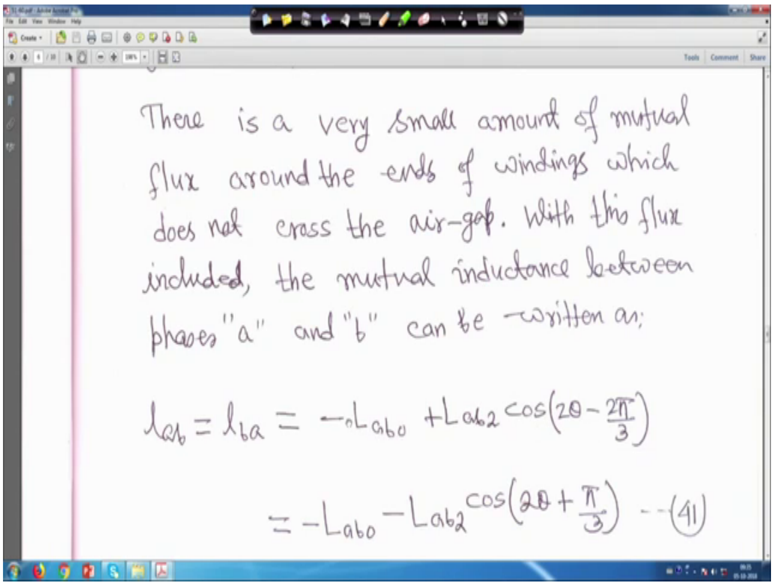


So, there is a very small amount of mutual flux around the ends of windings which does not cross the air gap. That which discussed earlier also with this flux included the mutual inductance between phases a and b can be written as we can write l a b is equal to l b a from the symmetry right, is equal to minus say lab 0 plus lab 2 cos 2 theta minus 2 pi by 3 or we can write this is minus L ab 0 minus lab 2 this expression. This expression, after simplification can be written as cos 2 theta plus pi by 3. This is equation 41. This little bit you do from your side right; directly, I am writing.

So, this equation can be written like this, but here it is plus. Now, it is minus and it is 2 theta plus pi by 3 right similarly this L ab is equal to L ba.

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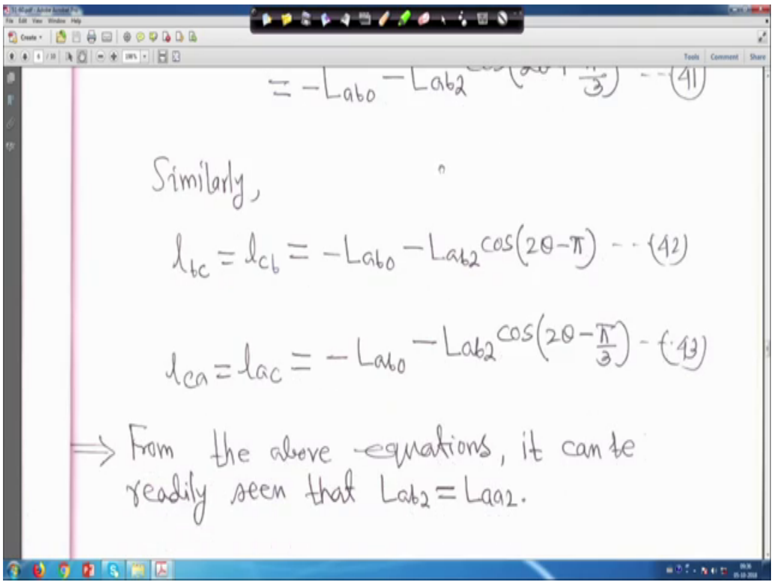


Similarly, because from your what you call that windings are 120 degree apart, similarly, L bc is equal L cb will be it will remain as it is minus lab 0, it will remain as it is minus lab 2 because they are magnitude for all these things are same. So, you are not making it. You are bc means not making it bc 0 or bc 2 right, just we because magnitude remain same. I know this way I mean lab 2 is equal to L bc 2 is equal to L ca 2 like this. So, all are same only thing is that that their things are 120 degree apart. So, this one will be cos 2 theta minus pi this is pi by 3. This will be cos 2 theta minus pi. And similarly, L ca will be lac will be minus L ab 0 then minus L ab 2 cos 2 theta minus pi by 3 because 120 degree plus 240 degree this is a part.

So, this way this way you can write. If you, if you, if you just try to your see this one then, then, you will see that difference will be 180 sorry 120 degree, right. So, this is equation 41 this is 42 and this is your 43. Now, from the above equations it can be readily seen that L ab 2 will be is equal to your L aa 2, right. So, all these expression if you see that whatever has been done, whatever has been simplify right that l ab 2 is equal to L aa 2 because earlier we have seen know L aa 2 is equal to Na square P d minus P q by 2 and this is nothing but also is equal to l ab 2, right.

So, that is why it is written lab 2 is equal to L aa 2, right.

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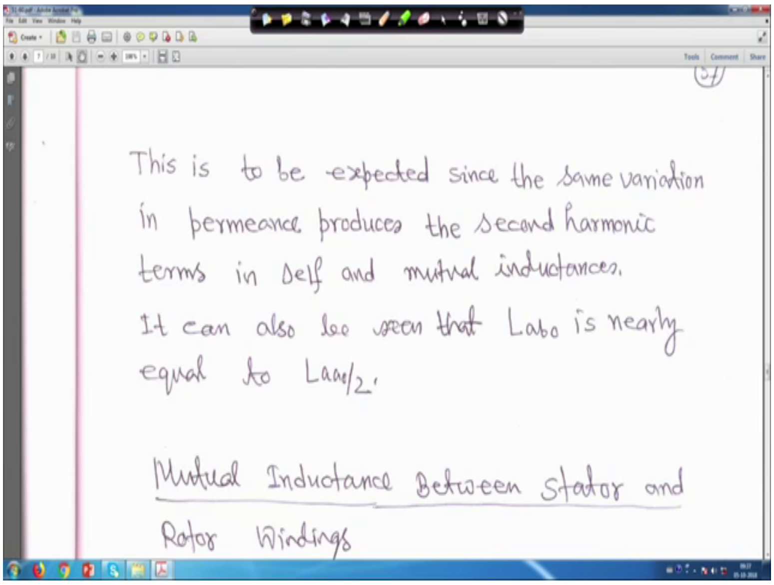


Now, this is to be expected since the same variation in permeance produces the second harmonic terms in self and mutual inductances. It can also be seen that L ab 0 is nearly equal to half of L aa 0 right. If you neglect that your leakage one; so, this is what you call that is your mutual inductance between the, these are all these expression this is b b your mutual inductances between the two phases of stator winding, right.

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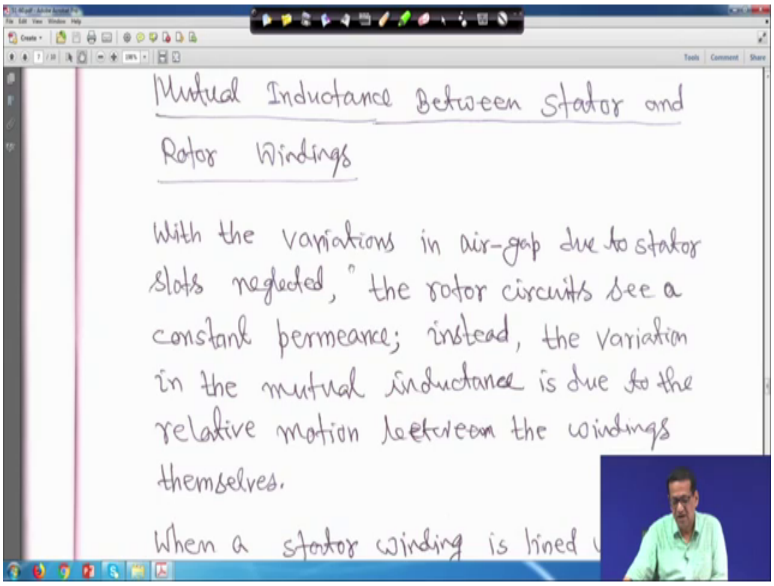


Next is the Mutual Inductance between Stator and Rotor Windings.

Now, with the variation in air gap due to stator slots neglected, the rotor circuits see a constant permeance because we have to make some assumption to simply our analysis right. So, this is now for rotor windings we have to obtain. So, with the variation in air gap due to stator slot neglected, the rotor circuits see a constant permeance right instead the variation in the mutual inductance is due to the relative motion between the windings themselves, right.

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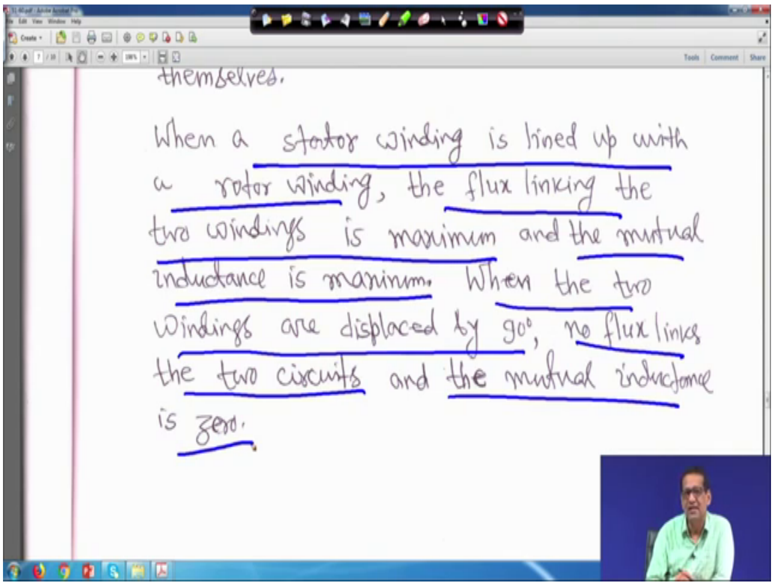


That means, when a stator winding is lined up with a rotor windings, when a stator winding is lined up with a rotor winding, the flux linking the two windings is maximum and the mutual inductance is also maximum, right. I mean when a stator winding is lined up with a rotor winding. The flux linking the two windings what you call is maximum and the mutual inductance is also maximum right. And when the two windings are displaced by ninety degree right no flux links the two circuits and the mutual inductance is 0, right.

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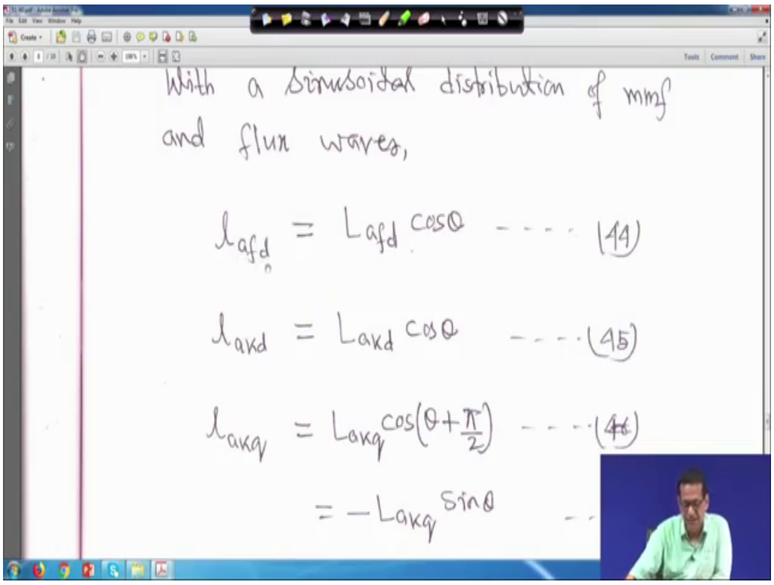


So, therefore, it is sinusoidal distribution of MMF and flux wave. We can write this is what you call we are a this is fd means field circuit and this side your it is. Basically, it is what you call this is mutual inductance between stator and rotor windings, right. Therefore, it is between your field is on the field is on the rotor and this is L afd. We can write that L afd is equal to capital l a f d this is the peak value say cos theta right. And on the direct axis apart from this field winding, you have the amortisseur winding right therefore, small l akd is equal to capital L akd cos theta this is equation 45, right.

Similarly, on the quadrature axis right, you have only that amortisseur winding between these small aq akq will be capital L akq cos theta plus pi by 2 because direct axis and quadrature axis the angle between these two axis is 90 degree. That is why it is cos theta plus pi by 2.

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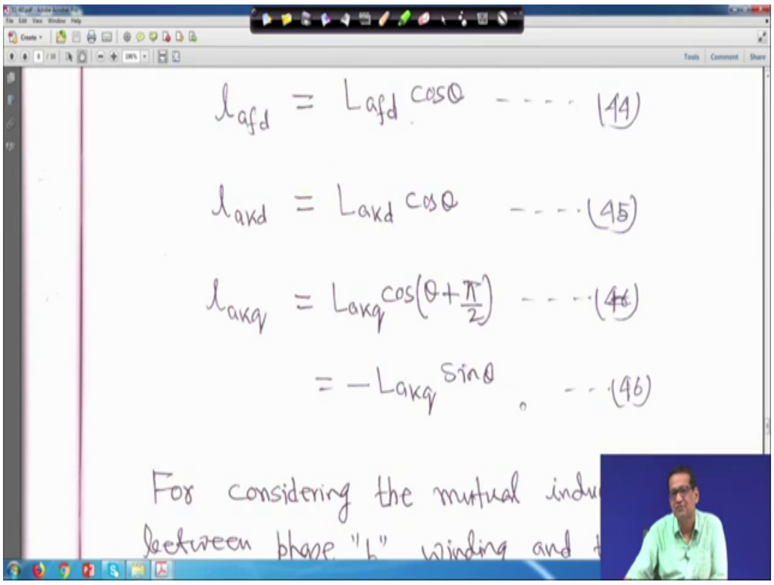


That is actually is equal to minus capital L a k q sin theta, this is equation 46. So, this is actually between stator and rotor that your mutual inductance. This is the expression.

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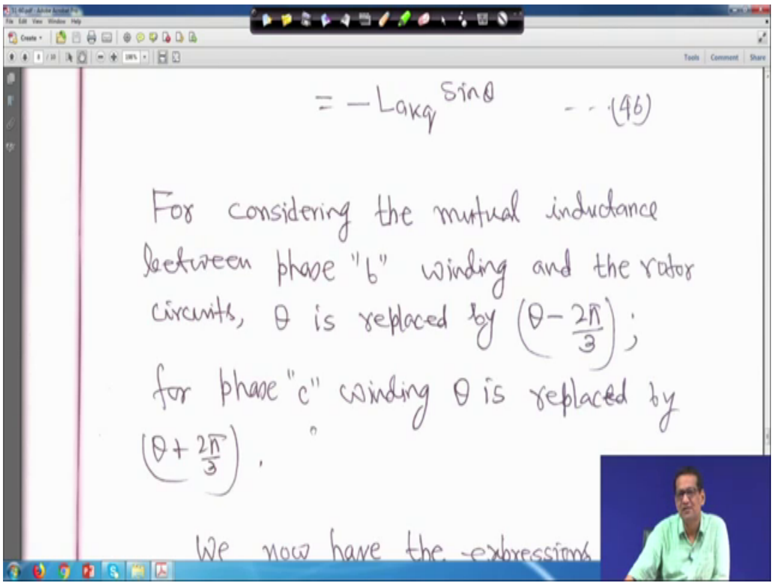


Now, for considering the mutual inductance between phase b winding and the rotor circuit theta is replaced by theta minus 2 pi by 3. And for phase c winding, theta is replaced by theta plus 2 pi by 3 because same philosophy from the symmetry, it is same philosophy.

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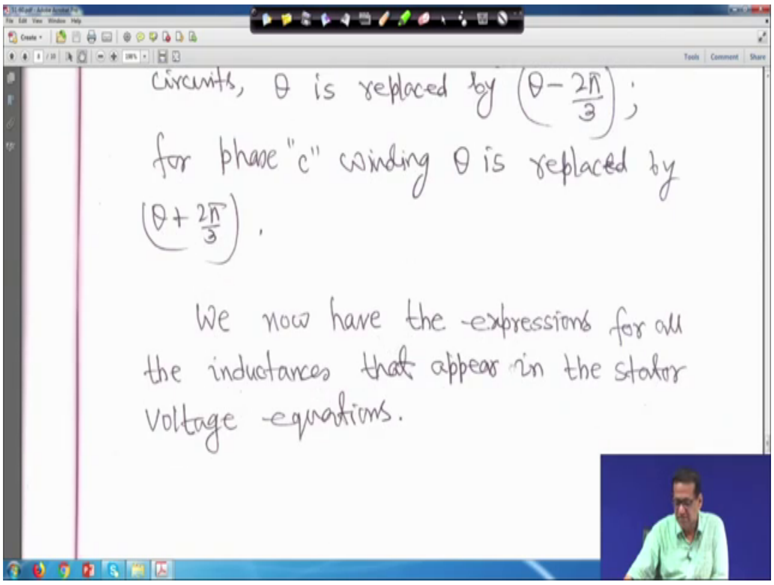


Therefore, we now have the expressions for all the inductances that appear in the stator voltage equation, right.

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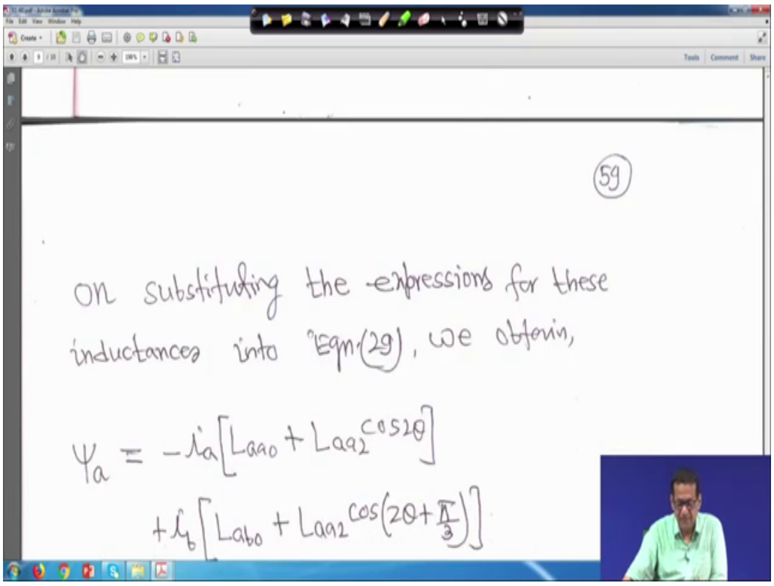


On substituting the expression for these in equation 29, we are not going back to equation 29.

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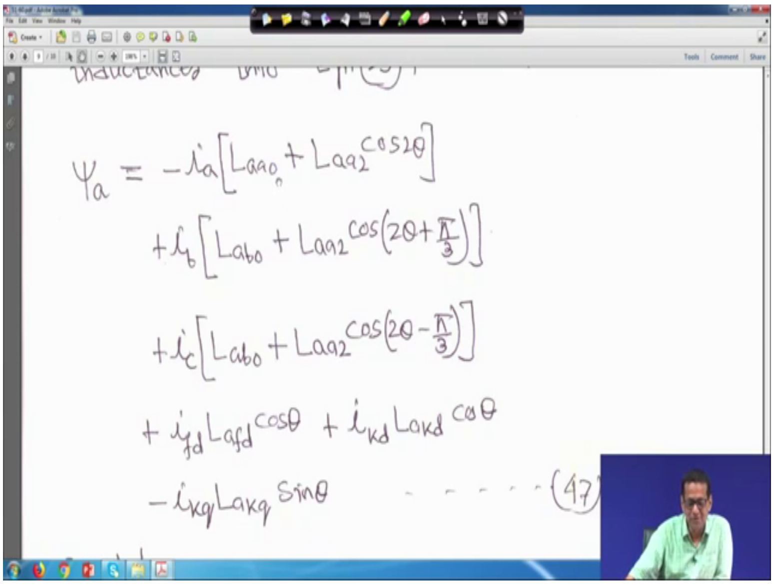


But just whatever flux linkage equation is written for phase a, what you do, you substitute all. Then you will get psi will be minus I a into L aa 0 plus L aa 2 cos 2 theta plus ib into L ab 0 plus L aa 2 cos 2 theta plus pi by 3 plus ic into L ab 0 plus L aa 2 cos 2 theta minus pi by 3 right plus you will see ifd L afd capital L afd cos theta plus ikd capital L akd cos theta.

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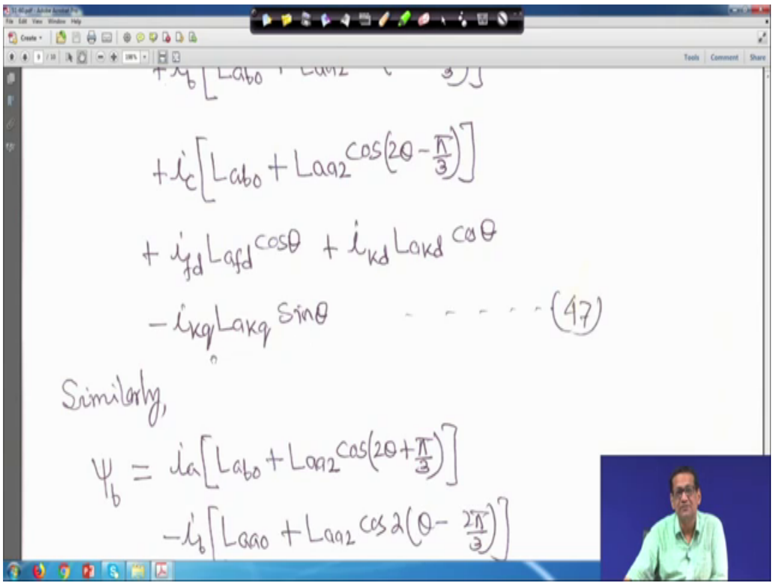


Then, this one minus your ikq l capital laq sin theta this is equation 47, right. So, when you that all the flux linkages, I am not going to just hold on if your equation 29, right. I have to find out which phase it is. So, equation 29 probably here, right. So, this is your just hold on, just hold on equation 29, I will see where it has gone, just hold on, not this one.

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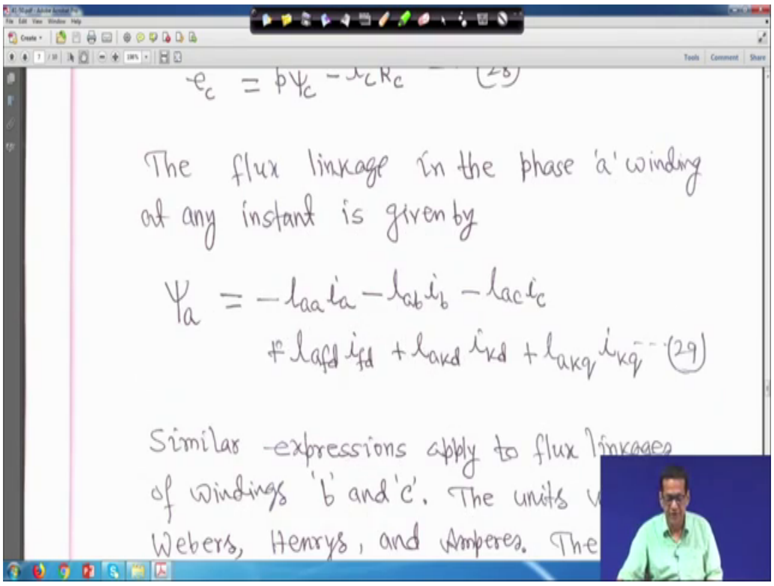


So, here only just have a look that equation this is equation 29 right. And here we actually this minus L aa ia minus L abib minus L acic plus all these things, right. So, just you have to substitute la expression of L aa L ab lac and here also you have to substitute your L afd L akd and L akq, you substitute right and then only, you will get this your what you call these expressions ia right, it will be minus ia whatever it is. So, this is equation actually 47.

Thank you very much. We will be back again.