Power System Dynamics, Control and Monitoring

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Lecture - 54
State estimation in power system (Contd.)

We are back again. So, again we have come back to equation 13 right.

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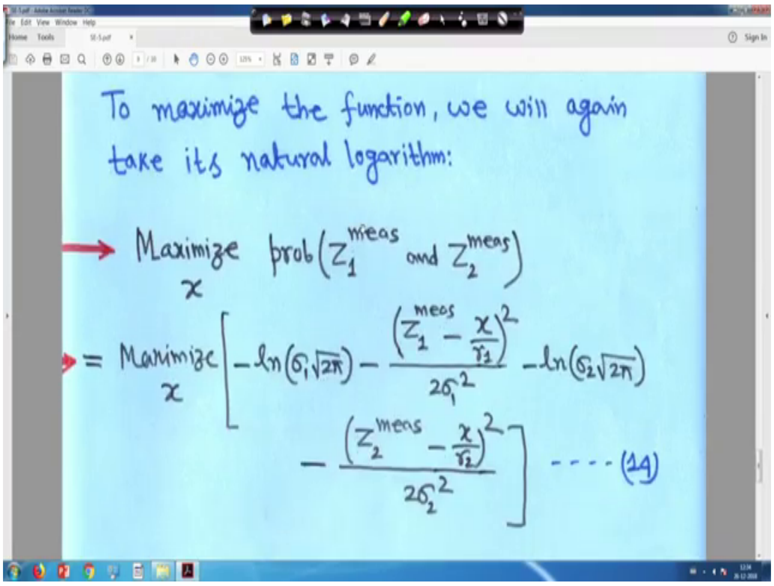

Now, next that to maximize the function, we will again take its natural logarithm I told you right therefore, maximize x that is probability of Z 1 measure and Z 2 measure is nothing, but maximize x that will be minus ln sigma over root 2 pi minus Z 1 measure minus x x upon r 1 whole square then minus ln sigma 2 root 2 pi minus Z 2 measure minus x upon r 2 whole square divided by 2 sigma 2 square that means that you are what you will take that these two PDF functions you will take this one, this one and this one and then you take its natural logarithm product actually.

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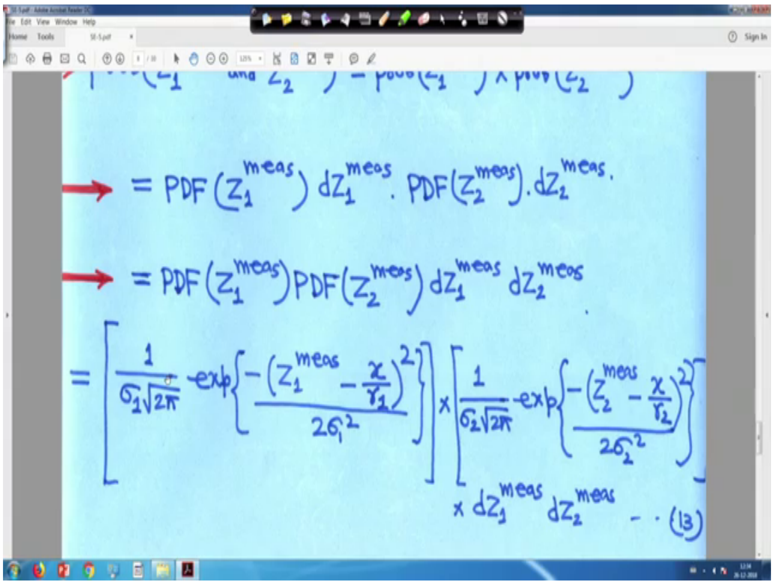

So, ultimately all will be this one will be minus of sigma root 2 pi, this one will be minus of sigma 2 root 2 pi ln ln that minus of ln sigma root 2 pi then minus of ln here what you call Z 1 measure minus x upon r 1 whole square upon 2 sigma square, then minus of ln sigma 2 root 2 pi, then minus your what you call it will be Z 2 measure minus x upon r 2 whole square upon 2 sigma 2 square because you take just product and you take natural logarithm right.

So, it will come like this. So, this term and this term is constant and same as before right we want to maximize x. So, 1 minus sign is before this term, minus sign is before this term and this term will be added I will minimize this term such that we will get the x estimated value right.

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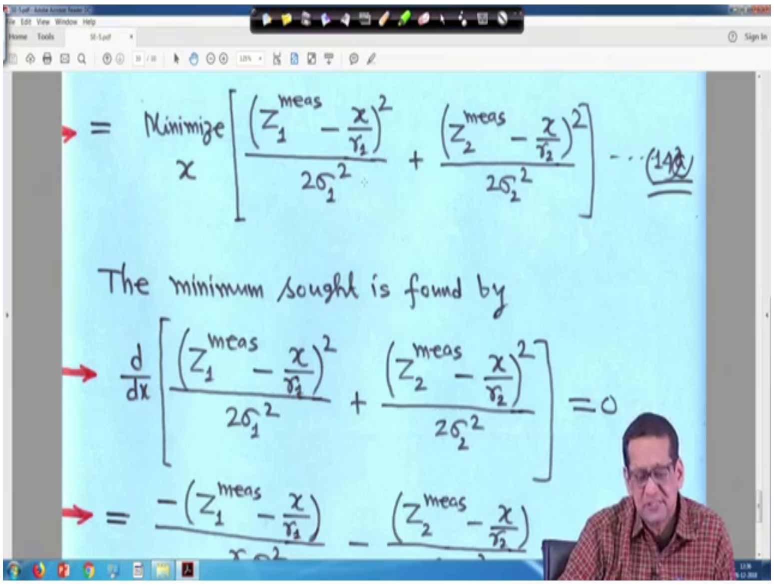

Therefore this, the minimum sought is found right; that means, you minimize x; that means, this is j this two term I told you before also for single ammeter case in this case you minimize x that is minimize this function for which you get the estimated value of the x. So, Z 1 measure minus x upon r 1 whole square upon 2 sigma 1 square plus Z 2 measure minus x upon r 2 whole square upon 2 sigma 2 square right.

So, now the minimum you will get if you put d dx is equal to 0, d dx of this term is equal to 0.

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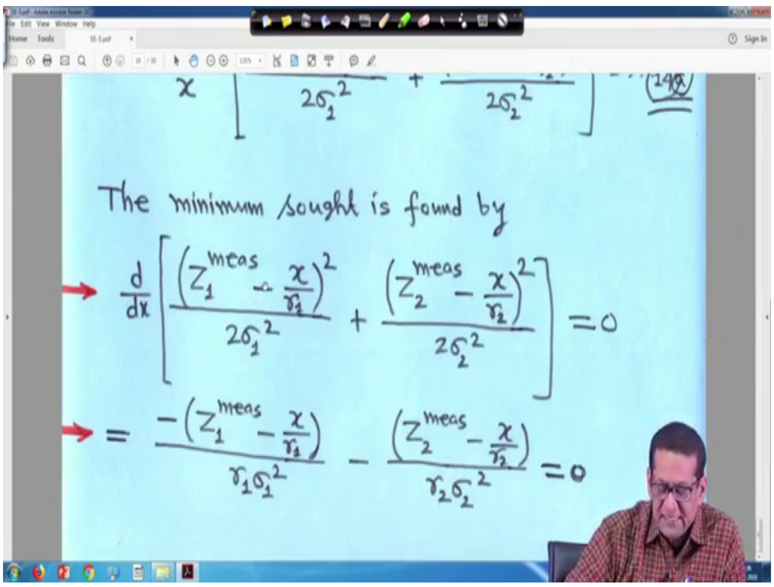

If you do so and take the derivative that two term anywhere right hand side is 0 you will not be there so minus of Z 1 measure minus x upon r 1 divided by r 1 sigma 1 square minus Z 2 measure minus x upon r 2 divided by r 2 sigma square is equal to 0 right and then from where you solve x is equal to x estimated.

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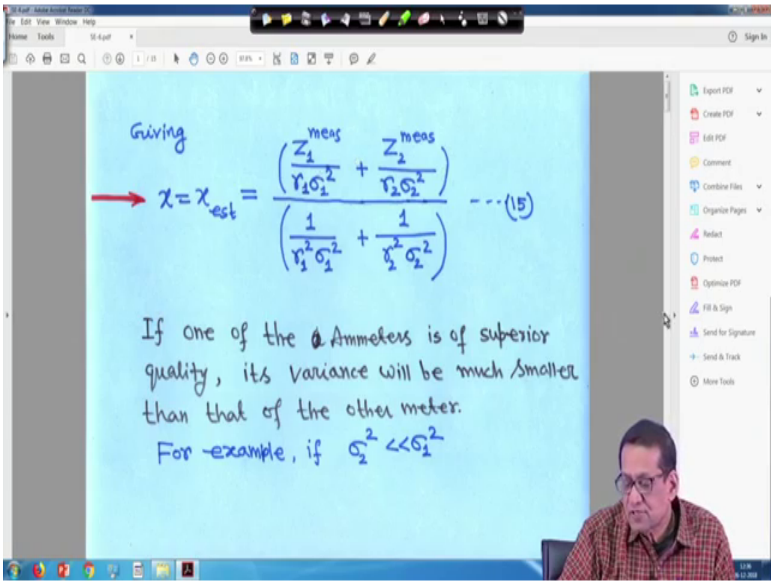

So, if you solve this 1 hold on so, if you just hold on. Now if you solve this, then you will get x is equal to x estimated is equal to your Z 1 measure upon r 1 sigma 1 square plus Z 2 measure upon r 2 sigma 2 this way we have written plus divided by your 1 upon r 1 square sigma 1 square plus 1 upon r 2 square sigma 2 square this is equation 15, you can easily write it from that equation just simplify I did not write intermediate state, but this is the expression. If one of the ammeters is of superior quality its variance will be much smaller than that of the other meter. For example, if 1 meter I mean superior instrument means its sigma the standard deviation will be very low; that means, various variance also will be low right.

So, then that for then that of the other meter for example, if sigma 2 square is your what you call much much less than sigma 1 square right therefore, therefore, we can ah; that means, that ammeter 2 is superior quality, then sigma your ammeter 1 because its standard deviation is much less than your what you call, this standard deviation of your what you call that you are ammeter one so where we have taken the variance also variance sigma 2 square variance also will be much smaller compare to your what you call the other meter variance sigma 1 say this is the this is something like this, we assume right.

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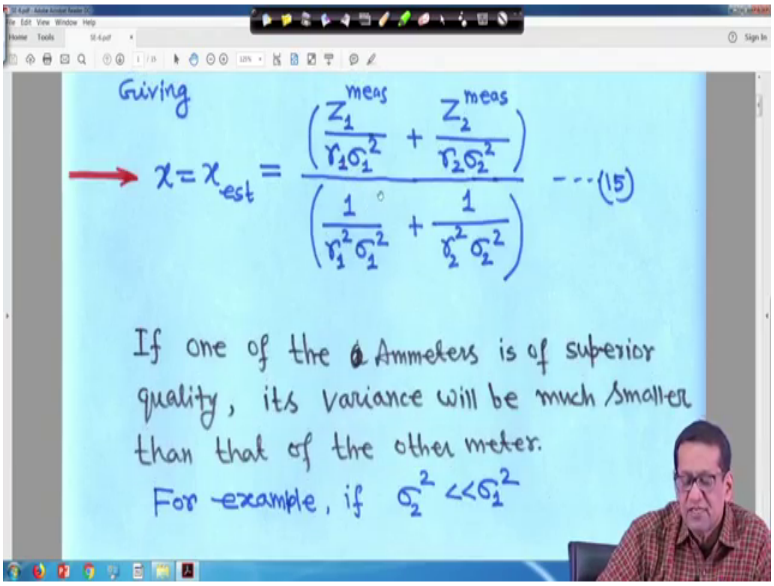

Then what will happen that x is equal to x estimated you can write from this equation only from equation 15 only, just simplify from this equation 15 only right. You can write x is equal to x estimated is equal to r 2 sigma 2 square Z 1 measure plus r 1 sigma 1 square Z 2 measure divided by r 2 square sigma 2 square plus r 1 square sigma square sigma 1 square into r 1 r 2 right.

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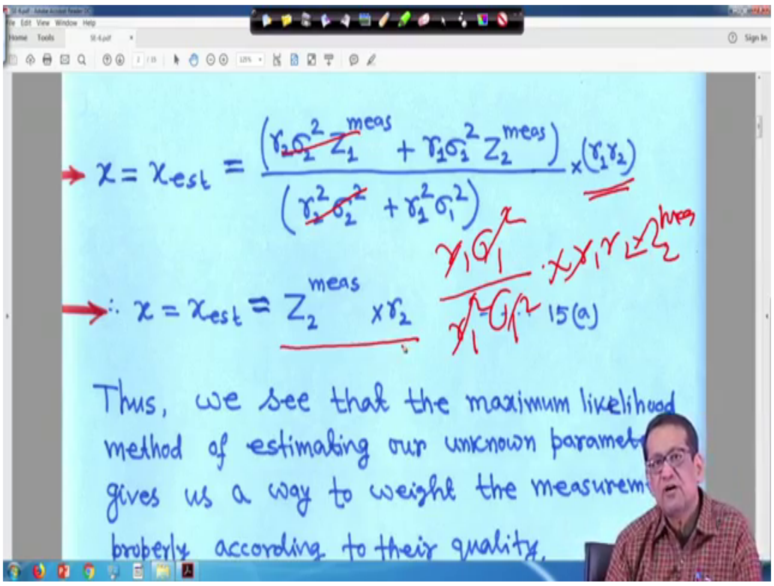

Now, if you are now if you look into that if sigma 1 square is much much you are greater than your what to call sigma 2 square.

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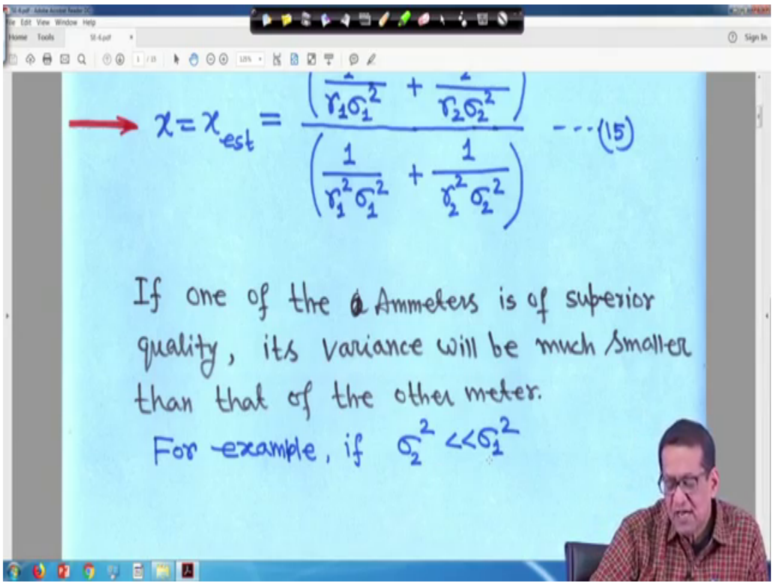

In other way that sigma 2 square is much much lower than sigma 1 square right then you can only you can just what you call that you what you will get that in that case you neglect the neglect this your what you call that sigma 2, because sigma 2 is your what you call sigma 2 square is very small compared to sigma 1 square.

So, if you neglect this term, that you are if you neglect this term for example, (Refer Time: 05:28) for just for our say your analysis so, this term should not be there, this term should not be there then this term should be there so it will be r 1 sigma 1 square by r 1 square sigma 1 square into this term r 1 r 2 right. So, in that case so what will happen? Sigma 1 square into Z measure into Z 2 measure right and r 1 r 1 and r 1 square so, this one will be cancel ultimately it will remain as Z 2 measure into r 2 right.

Therefore, x is equal to x estimated you will get Z 2 measure into r 2 it is a assumption. Otherwise, that you when you have two meters and you have different measurement from that you can see that x estimated value it is not like a first case where we to call 1 meter and 1 whole meter so everything is related to other.

So, you will get the best estimation of the volt voltage right x estimated x is nothing, but your voltage right. So, thus we see that maximum likelihood method of estimating our unknown parameter gives us a way to weight the measurements properly according to their quality; that means, it depends on sigma 1 and sigma 2 the standard deviation, in other way it is variance in this case expression is coming all sigma 1 square, sigma 2 square it is variance right.

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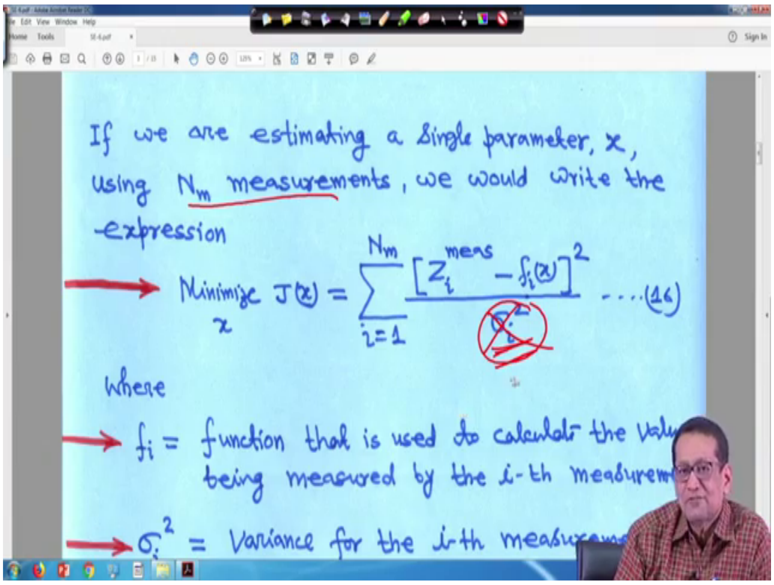

Now, that is that is actually whatever we see for 2 ammeters. Now come to little bit much deeper. So, now, in this case what we will do, that if we are estimating a single parameter x using any measurement we would write the expression I mean you have any number of measurement right I mean m stands for measurement. So, in that case you have to say minimize x function is given J x right. So, say i is equal to 1 to N m Z i measure that is the ith state variable minus fi x right and sigma i square and you write.

So, this thing actually we have taken Z i measure minus pi x square the whole square the Z i measure minus fi x whole square divided by sigma i square right. So, in that case this function we have taken and we are esti single parameter any measurement, suppose you have different measurement devices and measuring, but you know the standard deviation.

So, a see divided by your sigma i 1 square is given right. So, this is this is this is a question to you that I should have taken also Z i measure minus fi x whole square instead of divided by sigma i square right that variance so, but we have divided it by sigma i square why this is question to you. Why this function is divided by the answer is very simple, but why this function is divided by your what you call that you are just a you are what you call that sigma i square I should have; I should have not given that one also, I should have taken simply squad error right, but we have divided it by sigma i square, but these a question to you what is the reason right.

Now, where you will find just hold on where you will find that fi actually is function that is used to calculate the value being measured by the ith measurement right we will see that what is fi and sigma i square variance for the ith measurement.

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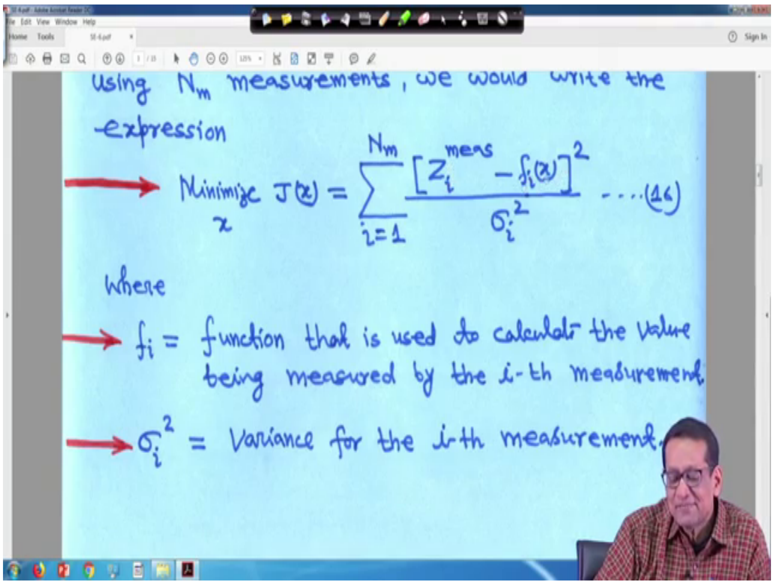

So, these function this function you have to here what your call you have to minimize. So, this way we have taken and you have N m number of measurements.

Now, J x is equal to measurement residual, because we are taking measured value my actually this is measured value and this one you are calculating actually fi x the way in the load for studies you have seen know that for convergence pi minus pi schedule, then qi minus qi schedule the schedule value is there, but pi and qi your computing thorough your algorithm that is your through your mathematical expression right so and you are; and you are trying to miss trying to minimize your what you call minimize that error such that it should be equal and then you are just trying to your what you call checking the convergence. So, here also Z i measure minus fi x basically nothing, but the residual.

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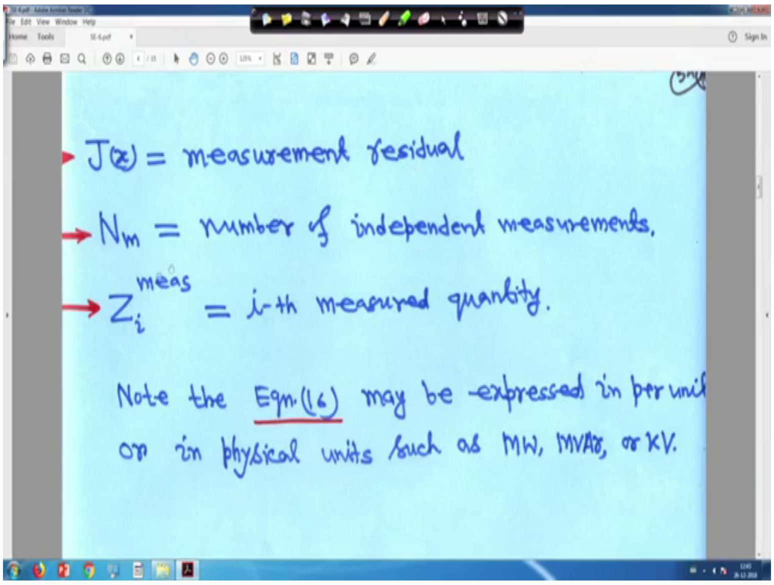

So, that is measurement residual and N m is number of independent measurement right. So and Z i measure is equal to ith measure quantity right so this is the nomenclature. Now note that, equation 16 may be expressing per unit or in physical unit such as megawatt, mega wire or kV. Then in these unit Z i measure or fi x sigma you can take it per unit or in real quantities megawatt, mega wire like this or voltage right kV.

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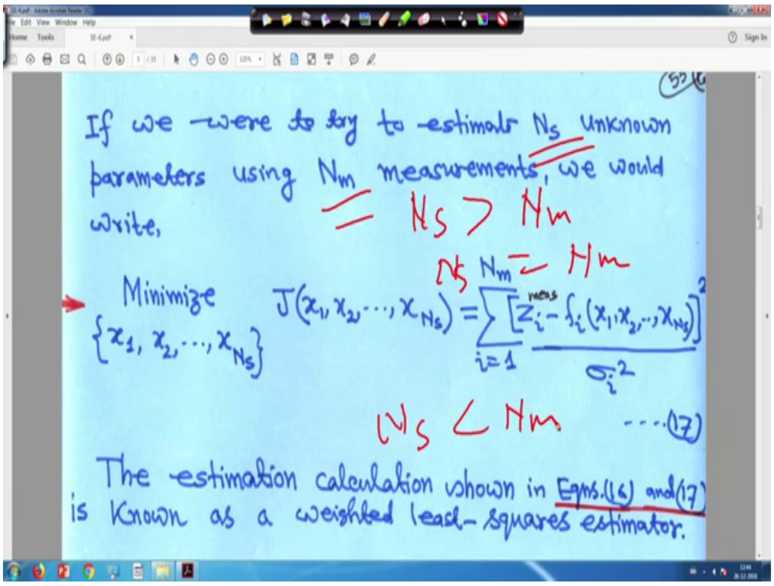

So, now suppose if you have to try to estimate N s unknown parameters using N m measurement, now suppose we have ns state number of state variable right. Suppose we have N s number of state variables right and we have N m number of measurement. So generally, we can find three things, one is when N s greater than N m right when N s number of state variable is equal to number of this thing another is N s less than N m right, all the three cases are there so we will see that.

So, here we have suppose we have N s number of parameters that is state variables using N m measurement right we would write that you are minimize your x 1, x 2 up to N s; that is x N s means it is number of state variable right that is your J is a function of x 1, x 2 up to x N s and number of measurement i is equal to 1 to N m Z i measure minus fi also function of your x 1, x 2 up to N s and this is whole square right divided by your sigma i square.

So, you have to minimize this function when f is a function of your in all the state variable that is N s number of state variables and you have N m number of measurement right. So, the estimation calculation of equation 16 and 17 right. So equation 16 we have seen. So, this is equation 16 and then equation 17 right is known as a weighted least square estimator right. So, actually it is called that you are weighted least square estimators right.

So, again there is a question why it is divided by sigma i square.

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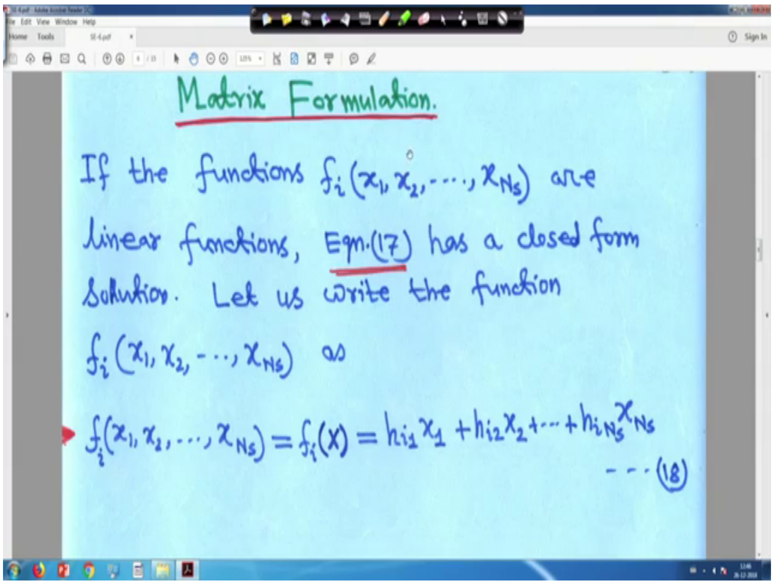

So, now matrix formulation that how we will do this. If the function say fi it is function x 1, x 2 up to N s that is number of state variables linear function generally whatever we are trying to do is first you assume that fi is a linear function of all the state variables in general AC circuit also we will see later, when you come to the AC circuit that your power function, power injection and other things are not linear it is a non-linear right.

So, that we will be seen later first let us see that fi function of x 1, x 2, N s are linear function that is equation steps 17 it has a closed form solution let us write the function that fi x 1, x 2 up to x N s number of state variable. Therefore, that fi x 1, x 2 up to x N s is equal to you can write fi x right is equal to you can write hi 1 x 1 plus hi 2 x 2 plus plus up to hi N s x N s right.

So, this is actually equation of xi that is a linearly related right say it is a linear function, in AC circuit it is not right because sine cosine terms are involved.

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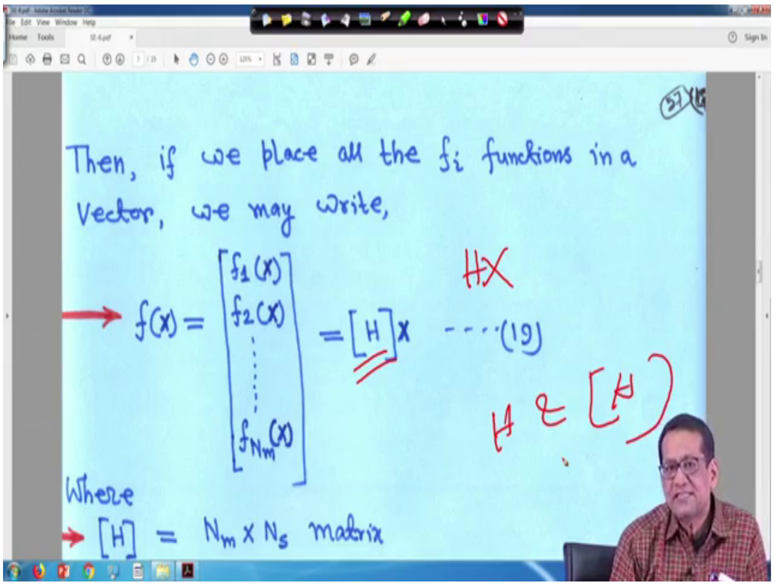

So, therefore, then if we place all the fi function in a vector, then we can write this one f x is equal to f 1 x, f 2 x you have up to N m number of measurement that f N m x that is equal to H into x because you have this fi whatever you write that i y i is equal to 1, 2 up to your N m number of measurement so it is h i1 x 1, h i2 x 2 up to h i N s.

So, these are into x N s number of state variable and this i actually is equal to 1, 2, N m that is your number of measurement so right. That is why this f x can be written as f 1 x, f 2 x because x is nothing, but your it is a vector x 1, x 2 up to x N s right up to f N m the number of measurement x is equal to H into x so this is a matrix actually, H is a matrix here I put in like this in bracket in your next video lectures instead of avoiding all the time writing bracket simply I might have written H x right so meaning is same this H and your bracket H both are same right similarly other matrices are also same .

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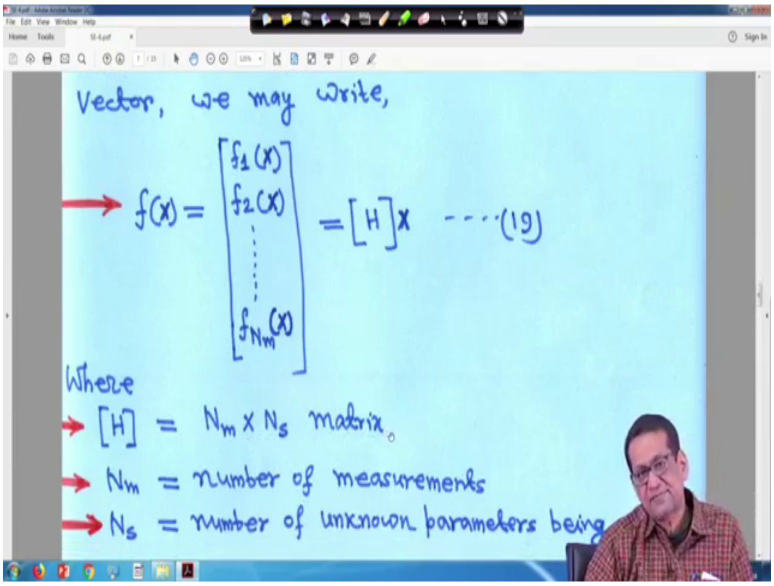

So that means, this H matrix right, this H matrix actually it will be N m into N s because you have N m number of measurement and N s number of state variables right. So, it is N m is equal to number of measurement and N s is number of unknown parameter being estimated that is actually nothing, but the state variables right.

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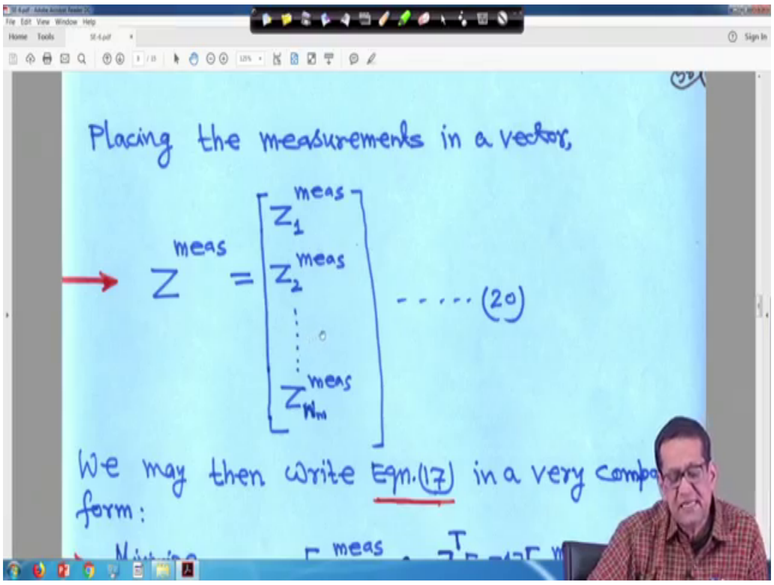

Therefore, placing the measurement in a vector so, Z o Z measure is nothing, but Z 1 measure, Z 2 measure up to Z N m measure, this is Z N m measure that is equation 20 right.

So that means, this is also putting it in vector form H x form and this Z also we are putting like this Z measure right and we may then write equation 17 in a very compact form, that is your minimum because it is a if you go to equation 17 right; if you go to equation 17 it is a quadratic function z i measure minus fi of x 1, x 2, x N s whole square divided by sigma i square it is basically your what you call is a quadratic function and you have N s number of state variable and N m number of measurement; that means, mathematically sorry mathematically we can write this equation that your minimize x right.

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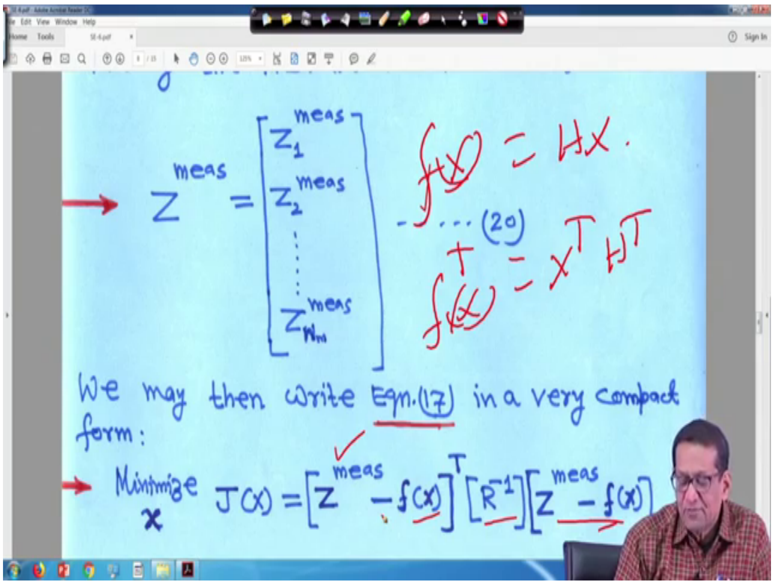

You have find out x value zx equal to minimize x that is Z measure minus f x transpose right then into R inverse your Z measure minus f x. So, this quadratic function can be written like this because this matrix R we are writing in R matrix, the matrix R actually what is matrix R if you come to come here this is actually your putting right that sigma i square it is sigma i square.

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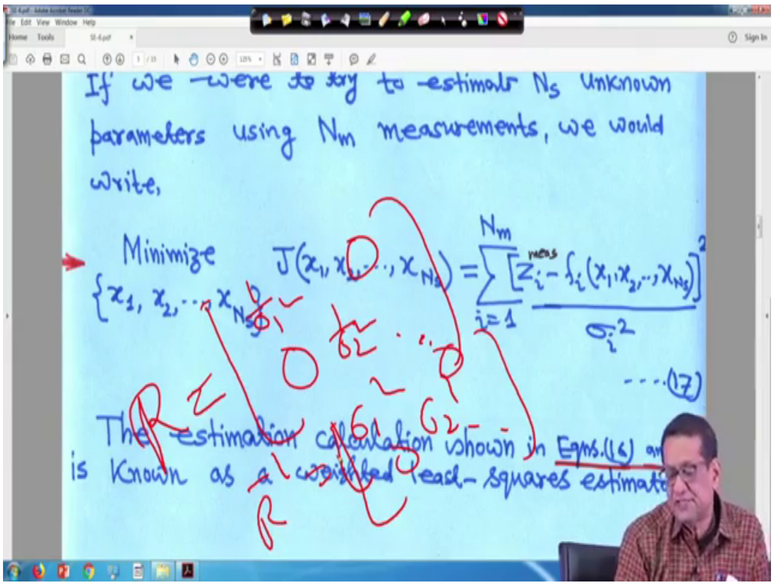

So, basically making it here later it is given what is 1 upon sigma i square. So, R actually it will be diagonal matrix 1 upon sigma 1 square, 1 upon sigma 2 square right up to like this and this is 0, this is 0 right. Now if you take R inverse, the inverse of this matrix it will be nothing, but you are sigma 1 square, sigma 2 square like this it will come this is 0, this is 0 right. So, that is why that actually so many state variables are there, that is why you have made it in the matrix form diagonal matrix right.

So, that is why it Z measure minus f x transpose R inverse right Z measure minus R inverse into Z measure minus f x and another thing is that your f x is equal to you are here I am putting it look later I have written directly that f x is equal to say H x right then when you will take the transpose Z measure transpose minus f x transpose so f x transpose actually it will be x transpose H transpose right so put all these things and then multiply the way you multiply, but note that this all this is vector right, these are matrix right.

So, when you multiply that your Z measure f x is equal to H into x right all this even multiply, multiply carefully do not write the way you make a minus b into a plus 2b not like that right. So, just go for this multiplication so I will not show this one, but directly I will write, I think it is understandable for you right.

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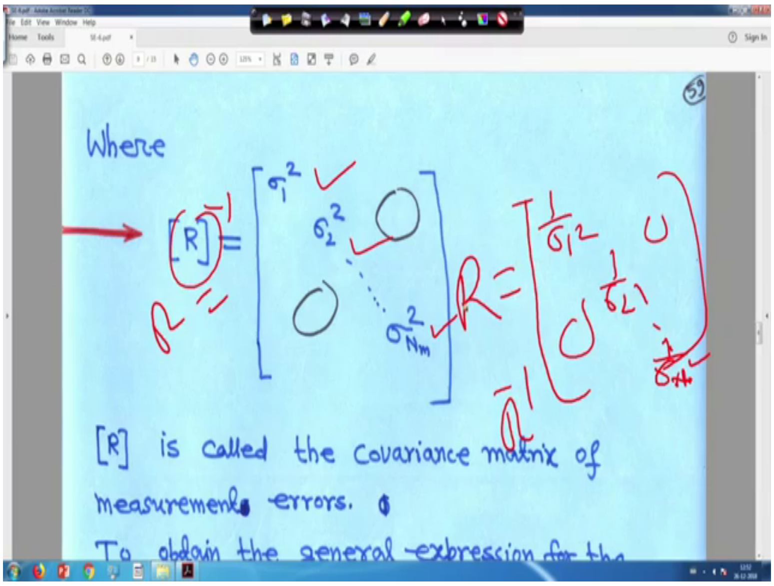

So, when you multiply, R I told you that R actually I told you that this is I let us tell I did not put in bracket simply I have written R. So, R is equal to actually I told you that your this is actually this one actually it is R inverse right. So, R actually 1 upon sigma 1 square, 1 upon sigma 2 square upto 1 upon sigma N m square right and your R is called the covariance matrix of the this thing and it is 1 upon c and this is 0, this is 0 if you take R inverse then only it will be sigma 1 square, sigma 2 square up to sigma N m square right.

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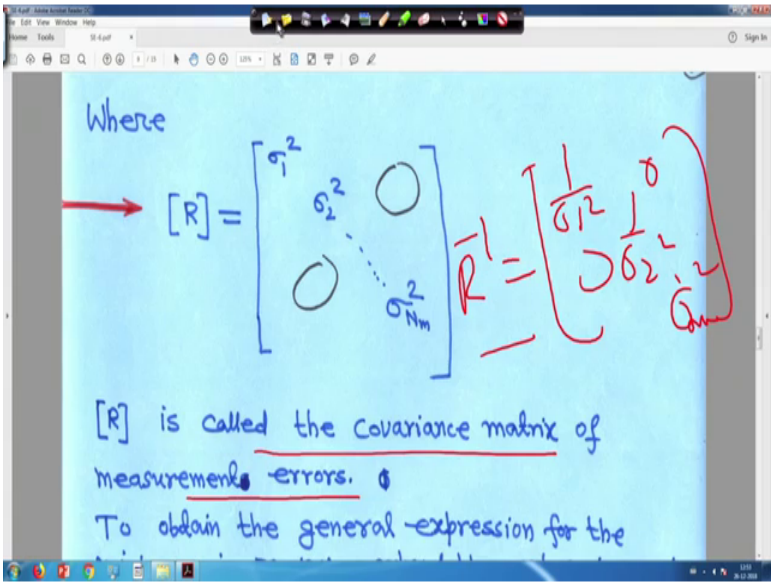

So, that is why here it is actually it is R inverse right. So, this is corrected.

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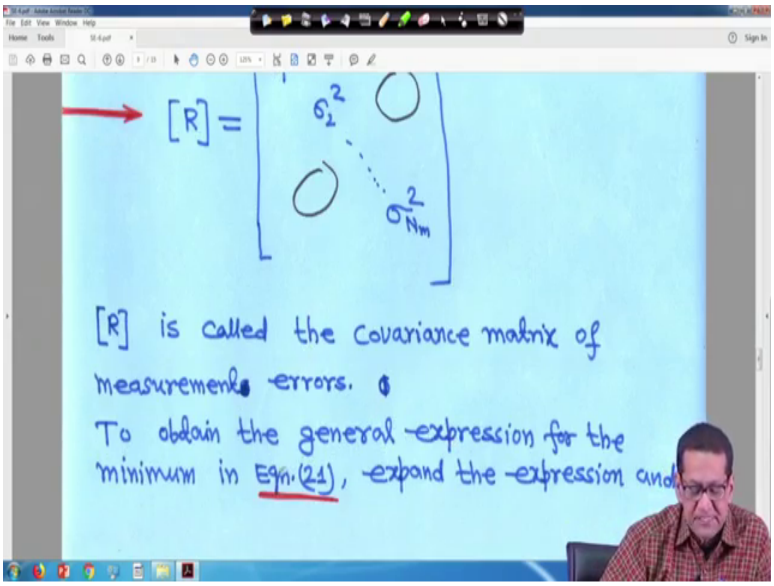

Now R is called the covariance matrix of the measurement error right to obtain the general expression for the minimum that we can expand the expression and if you expand the expression, you will get your what you call just 1 minute this is R inverse;

just 1 minute this is R inverse is equal to sigma no it is correct it is correct just hold on R is equal to we have taken say sigma 1, sigma 2, sigma 2 square so, we are writing R inverse.

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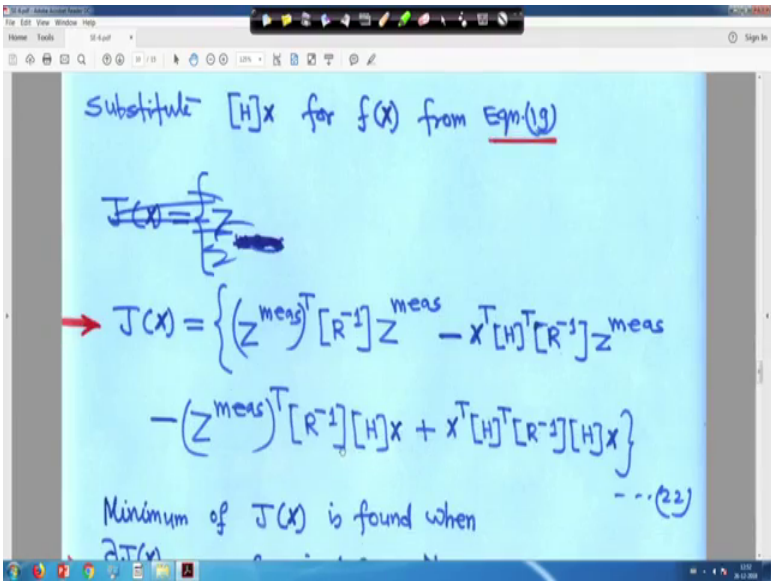

So, we are writing R inverse say it is actually 1 upon sigma 1 square, 1 upon sigma 2 squares up to sigma N m square this is 0, this is 0 you. So, this is actually R is equal we have taken this is called the co covariance matrix of measurement error right. So, here we have taken like this. So, therefore, this when you multiply all this terms, when you multiply all this things so it is Z measure terms R inverse Z measure minus your Z measure transpose R inverse Z measure minus x transpose H transpose R inverse Z measure minus Z measure transpose R inverse H into x plus x transpose H transpose R inverse into H into x. So, everywhere I have put it in bracket, later I have removed the brackets remove the brackets such that it will save the space, but understandable that is a matrix right.

So, so in this case the minimum of J x is found right. So, whatever we will do that minimum of J x that you have to take that your what you call the del J x upon del x is equal to 0, this is equation 22. So, you have to you substitute that f x is equal to H x you have substituted and then we got this thing, now minimum will come when your del J x upon del x i is equal to 0 for I is equal 2 N s number of state variable right.

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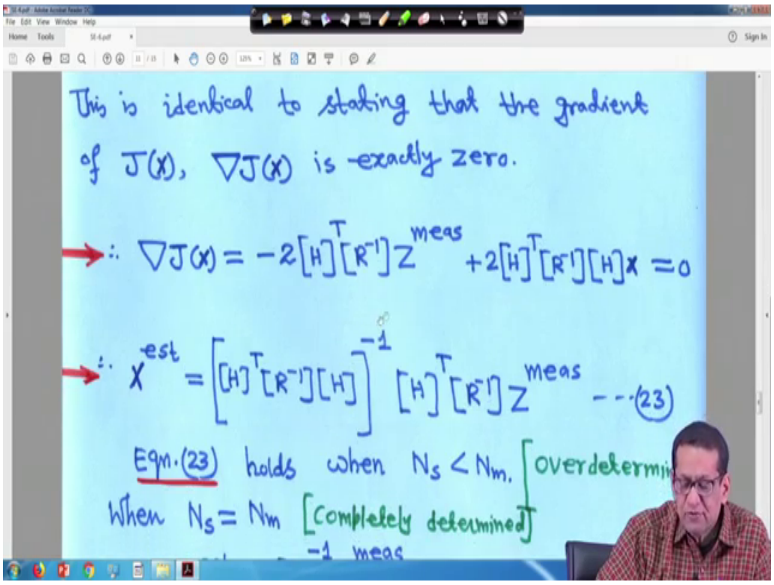

Therefore if you take the derivative this is nothing but, call that gradient right and we have to make it exactly delta J x gradient of J x is equal exactly 0. If you take the derivative with respect to x, it will come like this that gradient of J x will be minus 2 into H transpose R inverse Z measure plus 2 H transpose R inverse H into x is equal to 0 right and if you solve it 2 2 will be canceled, you solve it you will get x estimated is equal to H transpose R inverse H whole inverse then H transpose R inverse Z measure this is equation 23 right. So, equation 23 holds when N s less than N m that is over determined case.

So, in number of state variables are less than the number of measurement, when number of measurement is more than number of state variable it is called over determined case and when N s is equal to N m completely determined it will become x estimated will be H inverse if time permits this completely determine case I derived at the end if time permits and another thing is that, the underdetermined case in this case N m less than N s or N s is greater than N m that also derivation if time permits will see that, but for the further for this till now we will only consider the over determined case.

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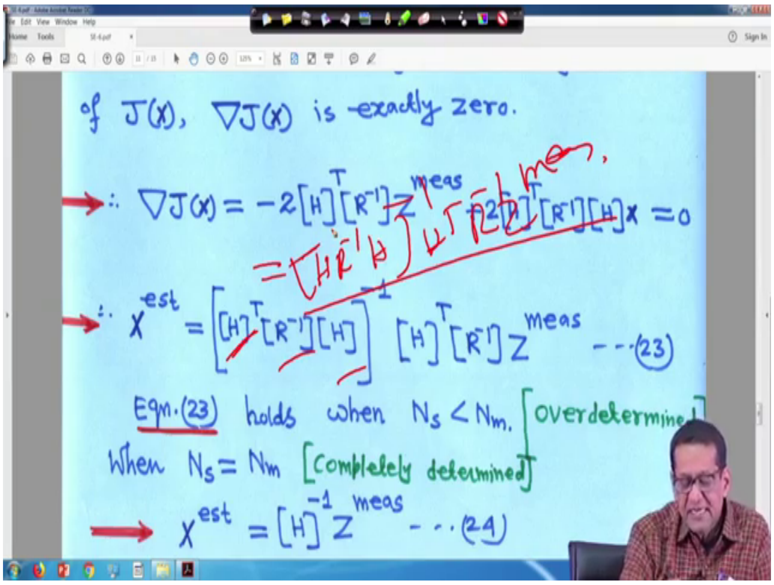

So, what happened that later stage all this matrix whatever is having H, R and this thing I did not put in that your bracket brackets, simply what I write for example, this example that x estimated simply we will write that H R inverse H whole inverse right then your H transpose then R inverse then J over writing on it Z measure this way we will write right.

So, again and again we will not put it in bracket. So, if you just take the derivative and it is a quadratic function; so definitely, we will get the solution of this one. So, this is actually possible if f x is equal to H x I mean it is linearly dependent right. So, and this one your what you call this H your whatever we will start you know over determined case they are completely determine and under determine case if time permits we will derived right.

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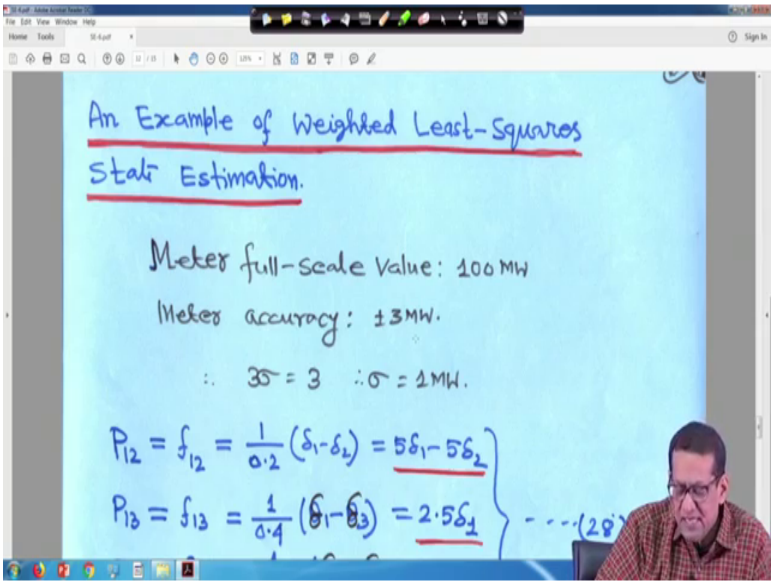

Now, consider that your what you call example of weighted least squares state estimation. Now suppose meter school full scale value is 100 megawatt. So, meter accuracy is plus minus your 3 megawatt that is 3 sigma is equal to plus minus 3. So, 3 sigma is equal to 3 so sigma is equal to 1 megawatt and you take 100 megawatt full scale value so in per unit it will be 1 upon 100 so 0.01, sigma will be point 0.01 right.

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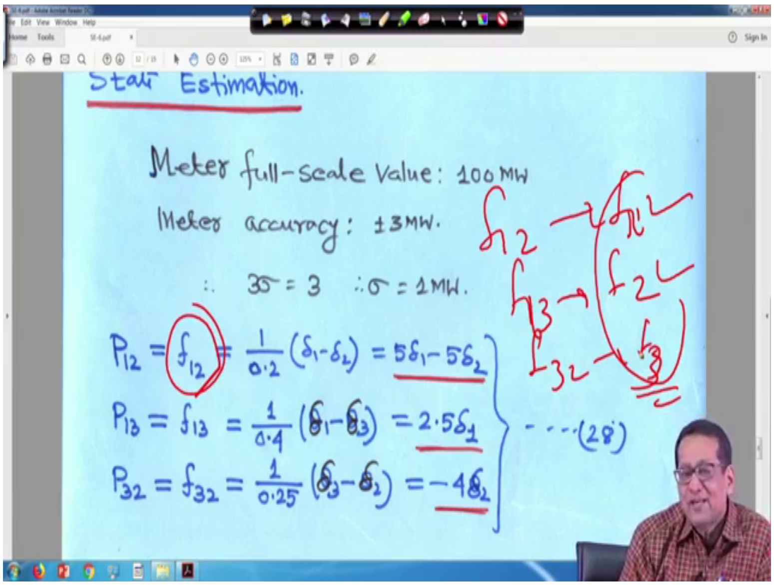

Now, that same example, same data that power flow that is 1 to 2 that is f 12 that x 12 value earlier we have taken we are not going back to the diagrams. So, already 2 3 times we have seen the diagram right. So, it is 1 upon x 1 to delta 1 minus delta 2 this is actually 5 delta 1 minus 5 delta 2. Similarly, power flowing in line 13, that is P 13 this is nothing, but your f 13 there in the mathematics it was f i f i your what you call f 1, f 2 up to f N m right.

But when your solving the problem actually it is f 12, f 13 that means, if you if you define like this that my f 12 actually it is line 1 that is f 1, if my f 13 define line 2 that is my f 2 if f 3 it will define line 3 f 3. So, it is 1, 2, 3 right this is basically 1, 2, 3 so that in the mathematics it was f i, but here it is when you coming to the problem it is line 1 to 2, 1 to 3 and 3 to two so it will make a f 12 is f 1, f 2 f 3 here, what you call f 13 is f 2 and f 32 is f 3.

So, your understanding actually there will be no confusion actually; no confusion only thing is that that regarding that R initially I saw; I saw it is R actually, but it was R inverse right [Laughter] so that is why so everything all the nodes everything is correct to the based of my knowledge right everything is correct.

So, question is that your similarly f 13 is 1 upon 0.4 delta 1 minus delta 3, but delta 3 is a slack bus so it is so delta 3 is 0. So, it is 2.5 delta 1 and f 32 that is nothing, but f 3, that is P 32 it is in 1 upon 0.25 because x 32 0.25, x 1 3.14 initially all this parameter so it (Refer Time: 25:22) and it is actually delta 3 0 so it will be minus 4 delta 2.

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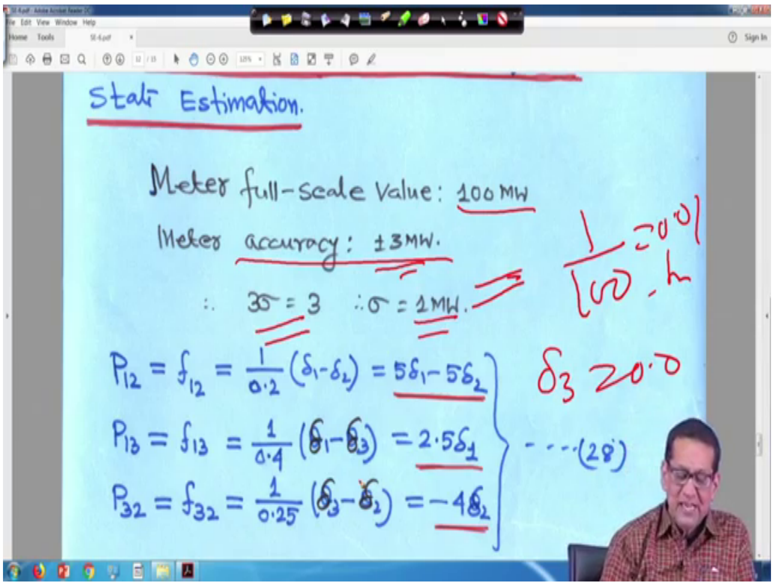

So, this sigma actually this sigma actually 1 upon 100 because a meter accuracy if you look into the meter ammeter (Refer Time: 25:34) another thing this meter accuracy will be given plus minus 3 percent something it is given it is given in percent; that means, it is in per unit, but if the meter full scale value is 100 megawatt and if it is given plus minus 3 megawatt; that means, basically 3 sigma is equal to 3 plus minus, but we will take only what we call that your sigma, sigma is equal to 1 because our case that sigma square will come right.

So, basically it is 0.01 in per unit and bus 3 was the slag bus so delta 3 was 0.0 in this equation delta 3 is 0.0. So, we get that matrix each matrix you have to get. So, actually; that means, if you write like this your x estimated actually nothing, but you are delta estimated right.

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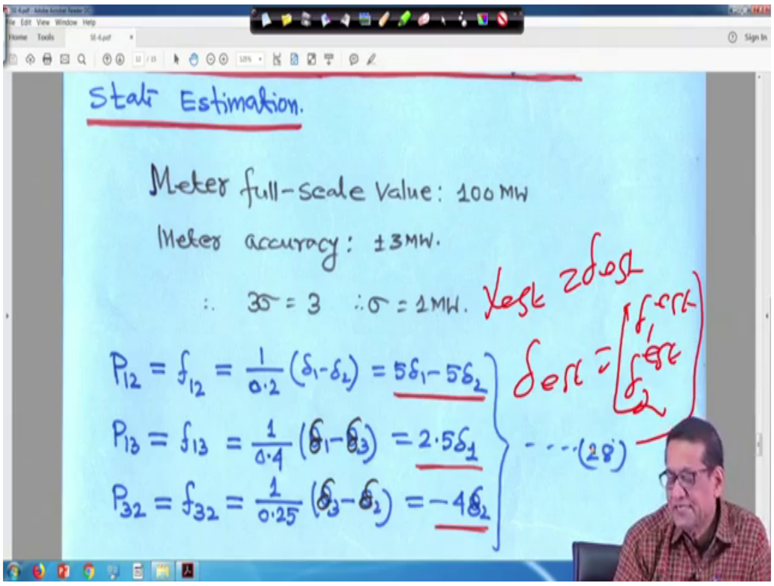

So, delta estimated is equal to actually delta estimated 1 is delta 1 estimated and delta 2 estimated right. So, that is why if you form this equation that you will this thing then your H actually will be come this matrix this is H matrix right.

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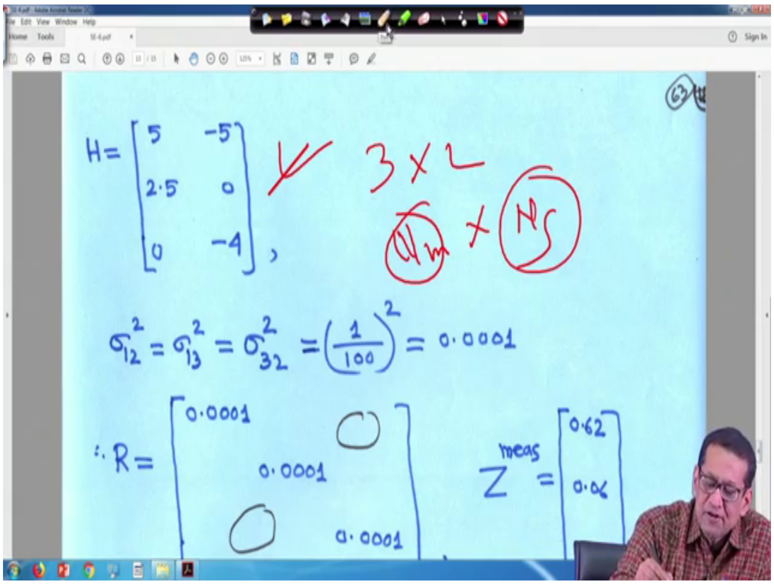

So, if you see that it is 3 into 2 matrix that is nothing what your N m into N s because your number of measurement is 3 because all the 3 lines measured value was given, but number of state variable actually it is 2 right. So, that means, you are what you call that is why this matrix H matrix is 3 into 2 and another thing I have missed 1 equation number in this note that x estimated is nothing, but your delta 1 estimated and delta 2 estimated right it is somewhere this equation number I think it will 27 somewhere right.

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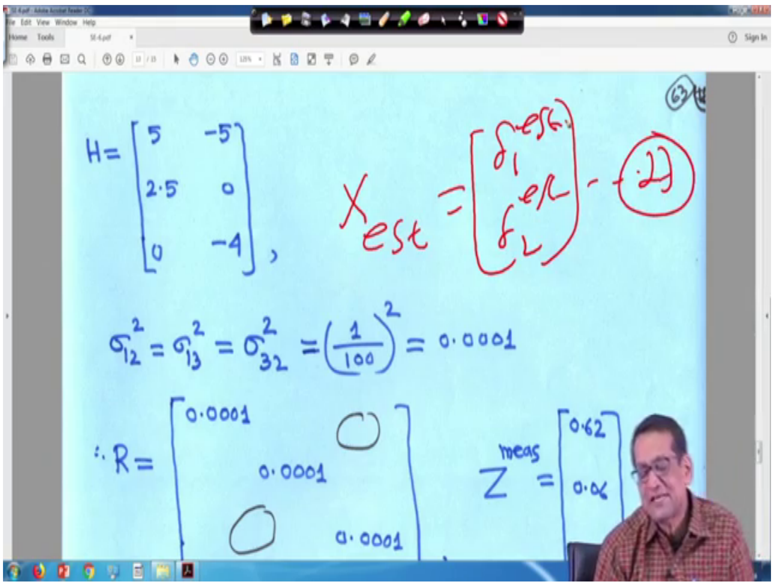

So, that is why this sigma 12, sigma 13 all the meters are same sigma, same standard your deviation so their variance actually sigma 12 square, sigma 13 square, sigma 32 square is nothing, but your pi 1 upon 100, I told you 0.0001 right it is 0.0001 1.

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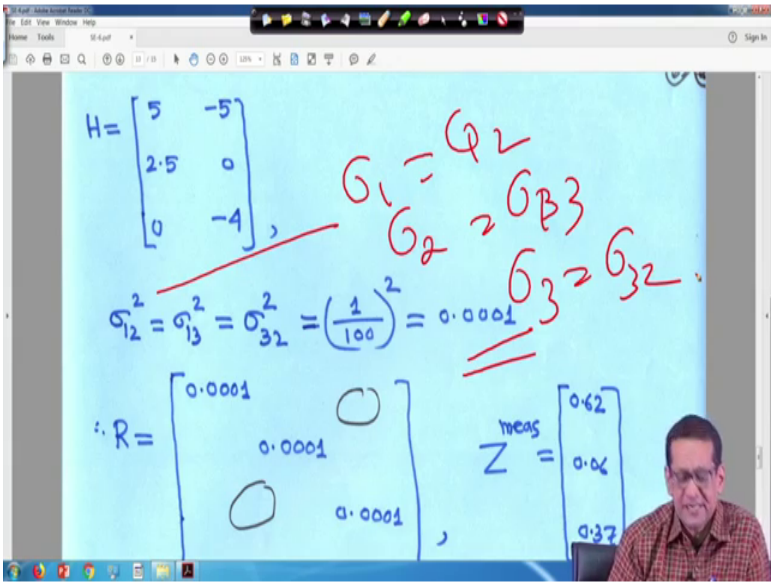

So, basically it is a mathematics sort of a derive. Basically sigma 1 is equal to sigma 12, sigma 2 is equal to sigma your 13 and sigma 3 is equal to sigma 32 right.

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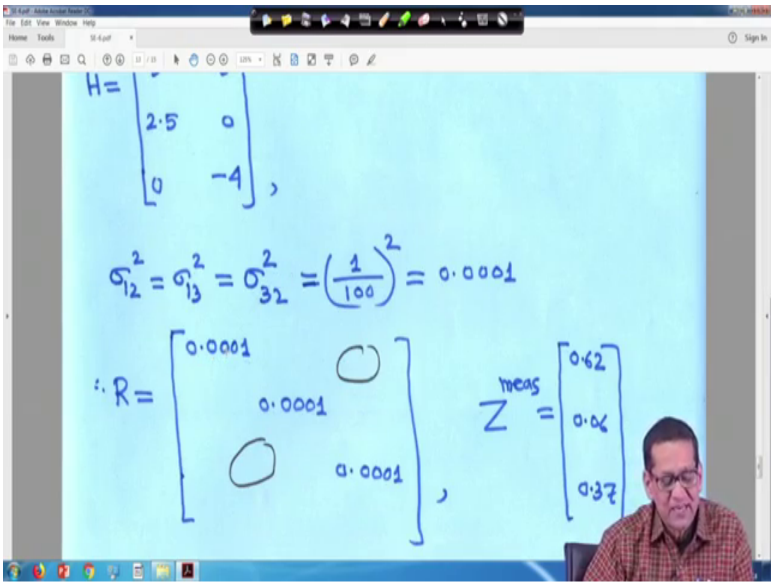

Therefore, therefore, your this is my R is equal to this one sigma 12 square, sigma 32 square or other elements are 0 and Z measure already earlier it was given same data. So, it is actually 0.62, 0.06 and 0.37 right.

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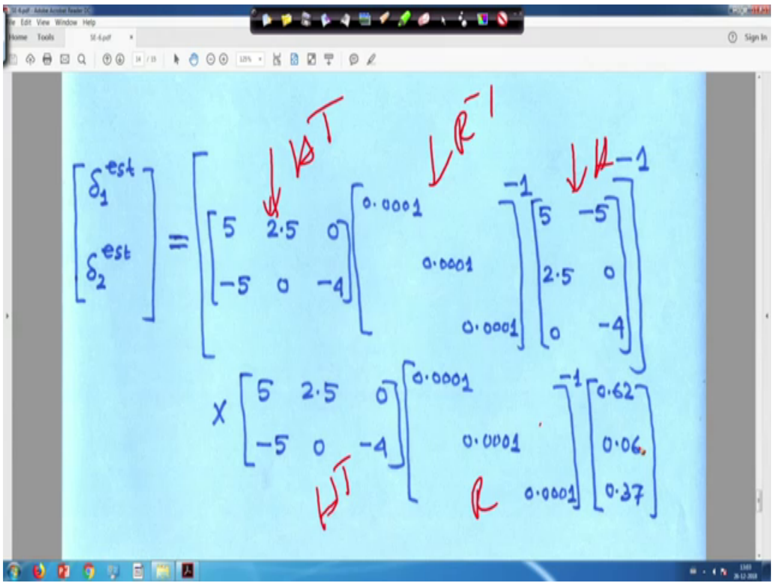

And next that is delta 1 estimated and delta 2 estimated right I told you that will be equation 27. So, this is if you look into that this is my H transpose right, this is my H transpose this is my R inverse this is inverse and this is my H right and this is again if you look into that this is my H transpose this is R inverse is given this is Z measure right so, everything is multiplied.

So, just put every value, just you put every value in that expression that is your in this expression I will go back to that just hold on ; just hold on here you go back to this expression, this equation your that equation 23 you go to this expression right equation 23 right.

So, what you will see, that this is equation your 28 I told you after 27, 28 has come and I told you that 27 that x a delta 1 estimated, delta 2 estimated and if you put all these values here in this expression so if you just put it is H transpose R inverse H whole inverse, then it is again your H transpose R inverse, then it you are whatever is coming Z measure is given just put everything value and you just simplify then you will what will get this inside this matrix, it will be 2 into 2 only so it is manageable because in a diagonal matrix, it is diagonal matrix all other elements so all other elements will be 0 right. So, it is easy to multiply and get the result right and in calculator 2 into 2, 3 into 3 matrix inverse easily you will get it and if you do, so.

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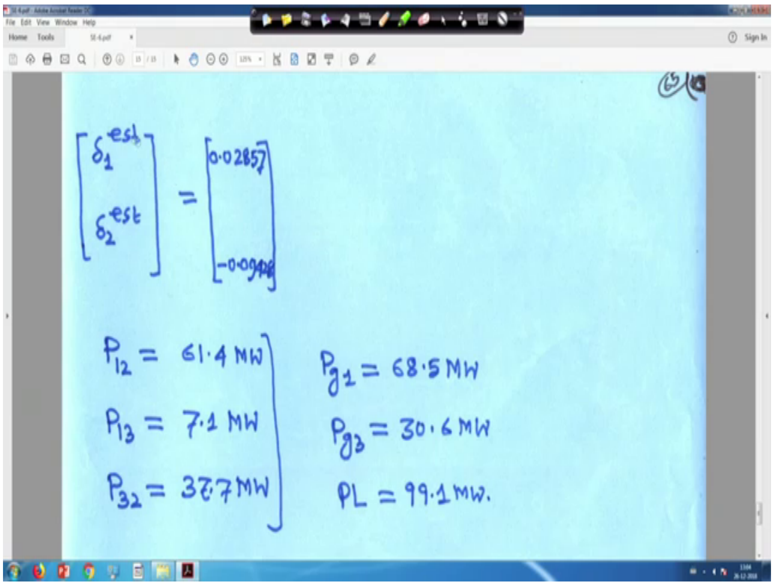

You will get delta 1 estimated and delta 2 estimated delta, delta 1 estimated is 0.02857 radian and delta 2 estimated will get minus 0.09428 radian right and based on this estimated value you will get P 12 is equal to 61.4 megawatt, P 13 you will get 7.5 mega 1 megawatt and P 32 37.7 megawatt and P g1 if you calculate 68.5 megawatt, P g3 30.6 megawatt, P L is equal to 99.1 megawatt.

So, from this example you can see that we have made we have used all the 3 state variables right, we have used all the 3 state variables and based on that 3 state variables we have compute a delta 1 and delta 2 estimated values, based on that this result we have got P 12 power flows line 12, 13 and 32 and earlier you we have given example that only 2 reading and third reading redundant, but in this case the we have used all 3 readings and this is the best values right.

So, thank you very much, we will be back again.