Power Systems Dynamics Control and Monitoring
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Lecture - 55
State estimation in power system (Contd.)

Ok, so in the previous lecture right, we have seen that your, what you call that power flow from line 1 to 2 ultimately it is coming your 61.4 megawatt, and P 1 37.1 megawatt and P 3 2 37.7 megawatt right. And P g 1, P g 2, P g 3 and the total load it is coming like this and this is your delta 1 estimated and this one is your delta 2 estimated right.

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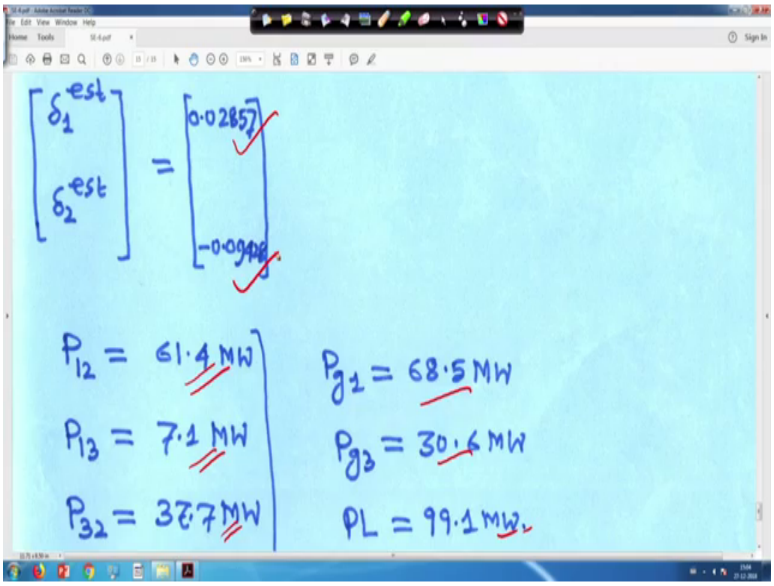

So, next we will go to the next slide tight. So, hold on this says because of scanning this first this two pages will come later right. So, now we will come continuation of this one right.

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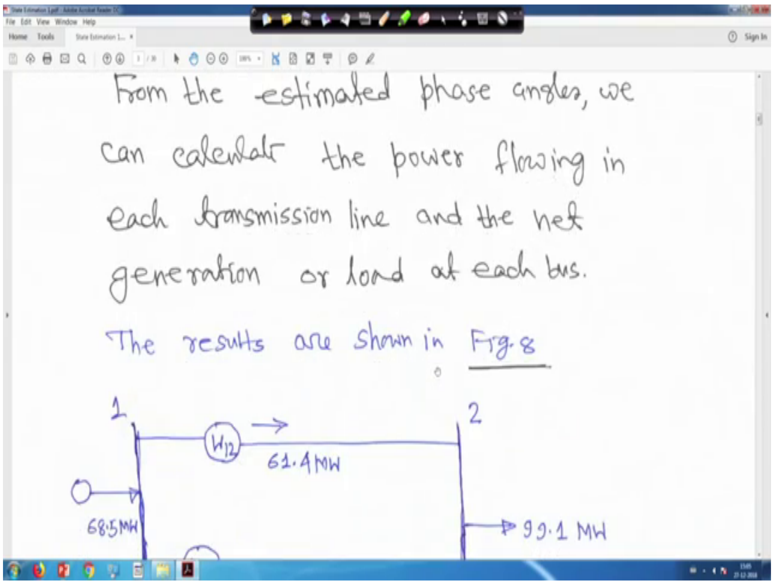

So, from the estimated phase angle I mean it is a continuation of the previous problem then we will go back to the previous two pages when I was scanning actually that that two paper pages have been scan before this one right. So, from the estimated phase angle so right. So, this will be for that was 60 plus 65, you come to 66 first right. So, from the estimated phase angles, we can calculate the power flowing in each transmission line and the net generation or load at each bus right. So, results are given like this. So, this is that your line flows and these are the metre W 12, W 13 and W 32.

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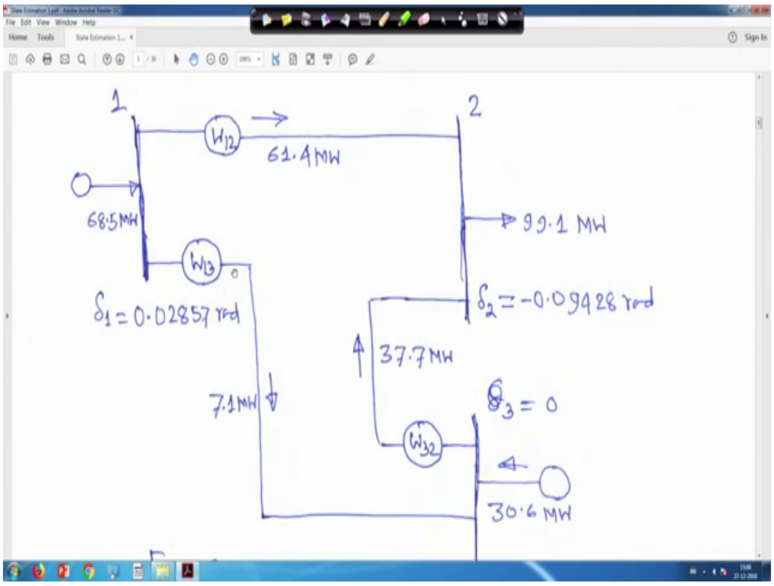

So, P 12 is 61.4 megawatt that is f 12, that is I told you f 1, f 2, f 3 three lines right. And this is the angle delta 1 estimated on 0.02857 radian and this is the power flow 7.1 megawatt, line 1 to 3 and from 2 to 3 it is 37.7 megawatt right. And power injection here it is this is 30.6 megawatt right. 30.6 and 7.1 so total will be 37.7 megawatt. So, 61.4 and 37.7 so, it will be 99.1 megawatt and delta 2 after estimation we got minus 0.09428 radian right. And this is a slag mass so delta 3 is equal to 0 that we have assume right.

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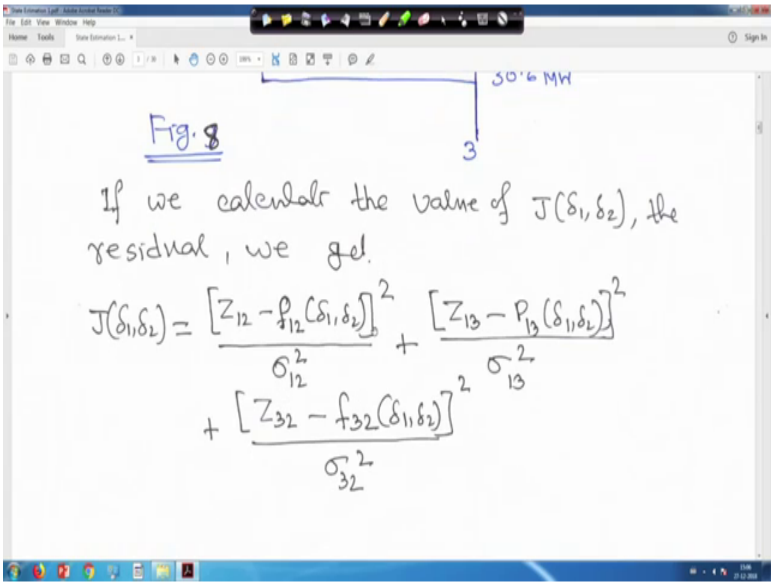

So, now if we calculate the value of J function of delta 1 and delta 2 the residual will get actually J delta 1 delta 2 is equal to your Z 12 minus your p 12 delta 1 delta 2 find P 1 2 function of delta 1 delta 2 then Z 13 minus P 13 and Z 32 minus f 32 and these are all square then, divided by their respective meters your standard deviation square that is variance right. So, when we compute this that actually the Z 12 measure was given 0.62 and your here one thing, one thing it is there that is you just hold on.

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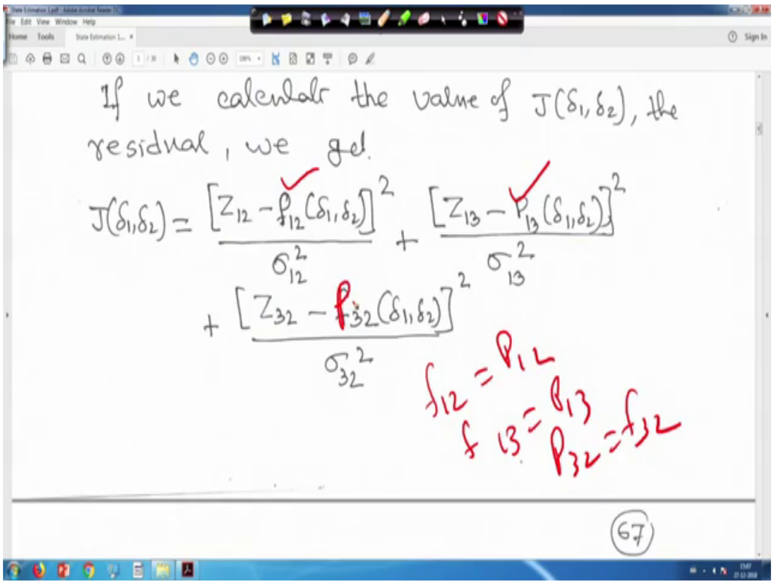

This is P 13 this is P 1, just hold on. This is your P 13, this is your P 12 and this should be actually P 32 although all are same although we have seen f 12 is equal to P 12 f 13 is equal to P 13 and P 32 is equal to nothing but, f 32 so this is actually P 32 or f 32 same right.

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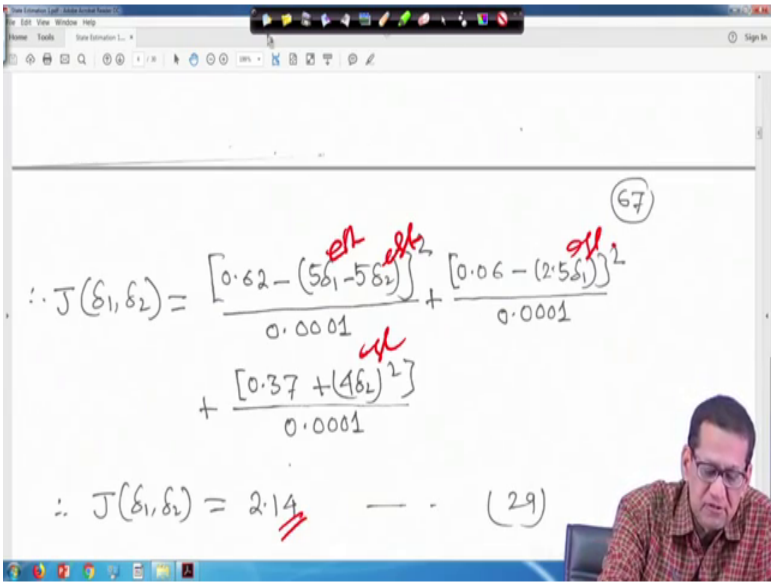

So, now all these things that you are measure your what you call? This was your measured value that is your W 12, that is Z 12 the measure value so, it is 0.62 and your a P 12 is equal to f 12 that was nothing but, minus 5 delta 1 minus 5 delta 2. This we have seen in we are constant we are trying to find out the power flow from 1 to 2 right and also the each matrix. So, this is whole square then your this one Z 13 that is W 13 the metre reading 0.06 minus your it will be 2.5 delta 1 because P 13 is 2.5 delta 1, that is whole square plus your it is your this your Z 32 right. So, 0.37 and this one is minus your f 32 that is your minus 4 delta your this thing what you call this P your a P 32 that is I told you that is f 32 and P 32 are same. So, that they are also you will get.

That you are it is actually minus four delta 2. So, it is minus minus plus. So, 4 delta 2 square divided by their respective your here it is variance sigma 12 square, sigma 13, square sigma 32 square. So, all this things are divided. If you do so and compute this one delta 1 and delta 2 means these are all estimated value. These are all estimated value right; these are all estimated value right; these are all estimated value. So, all estimated value. So, you whatever you have got you substitute there and ultimately you will get that your J delta 1 delta 2 will be 2.14 right; so that is the residue.

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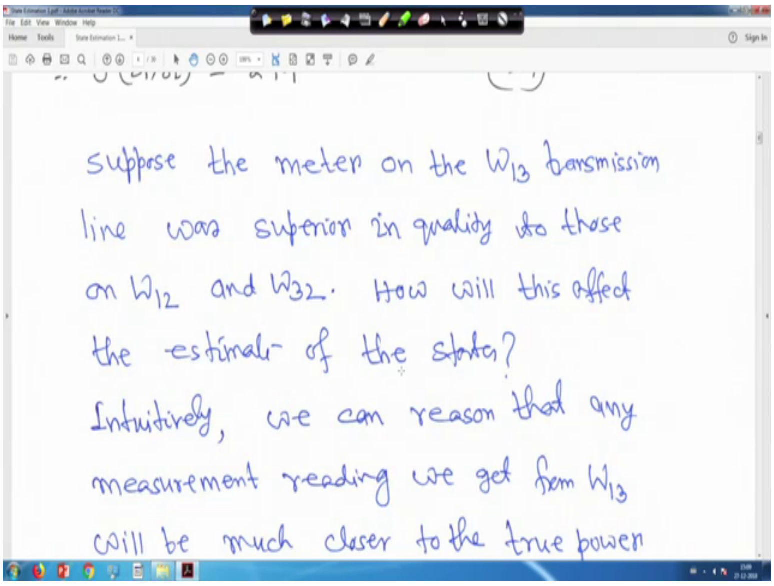

So, with this us now, question is that, that suppose the metre on W 13 transmission line was superior in quality to those on W 12 and W 32 right. So, how will this affect the a your estimate of the states right. Intuitively we can reason that any measurement reading we get from W 13 that is right, will be much closer to the true power, flowing through true power flowing through line 1-3 than can be expected when comparing W 12 and W 32 to the closer lines 1-2 and 3-2 respectively right.

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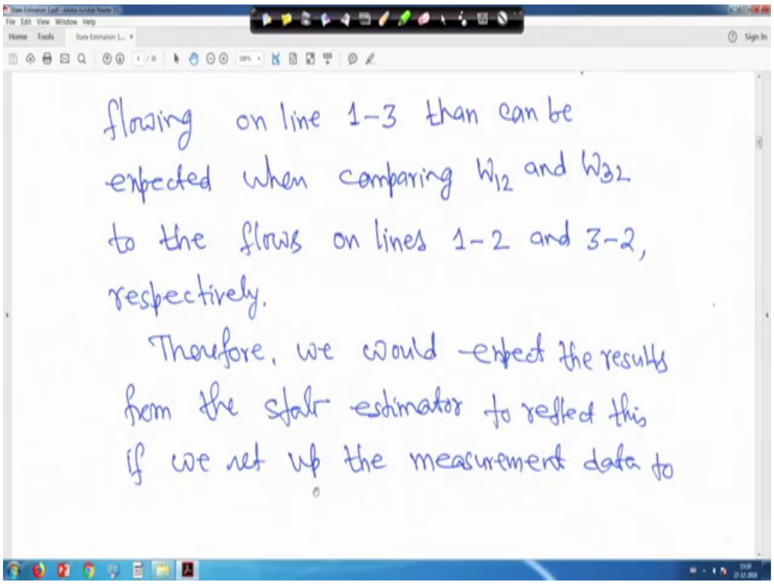

Therefore, we would expect the results from the state estimator to reflect this if we set up the measurement data to your re, your reflect the fact that W 13 is a superior measurement right. To show this we use the your what you call following your metering data.

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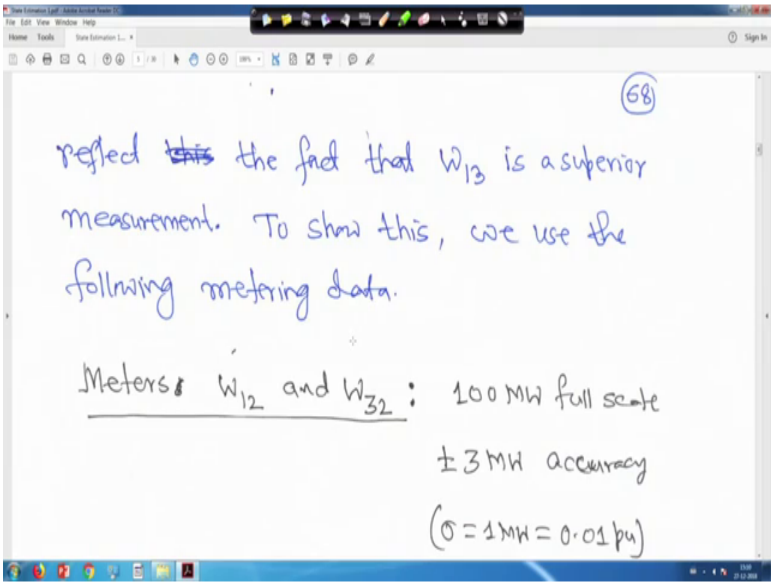

For example; I will come to that previous two pages first say W 12 and W 32, suppose it is 100 megawatt full scale reading. And plus minus 3 sigma is the megawatt accuracy, that means it is 3 megawatt means plus minus it is 3 sigma. So, sigma is equal to 1 megawatt that divided by 100 that will be 0.01 right. That is the standard deviation. Now meter W 13 is 100 megawatt, but is stand it accuracy is plus minus 0.3 megawatt right. Therefore, sigma is equal to 0.1 megawatt then it 0.001 per units. So, sigma actually you are here it is very small, that means meter 13 is more accurate. Than meter your 12 and 32 right.

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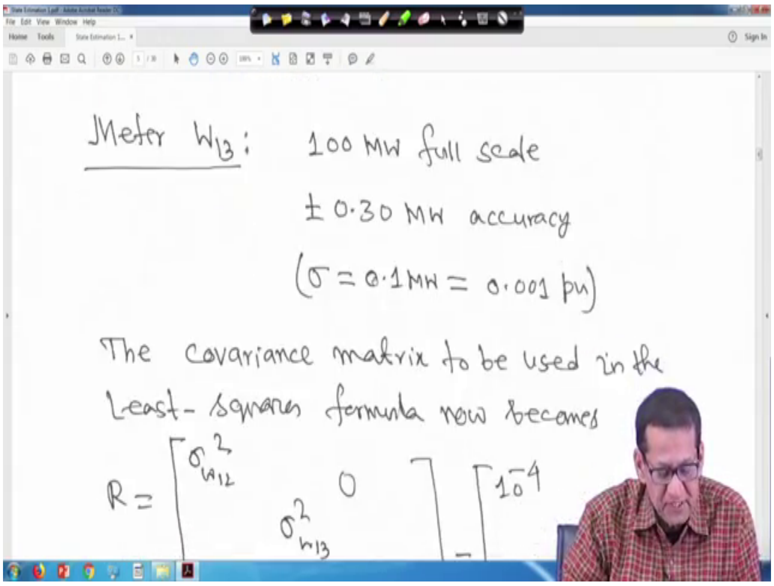

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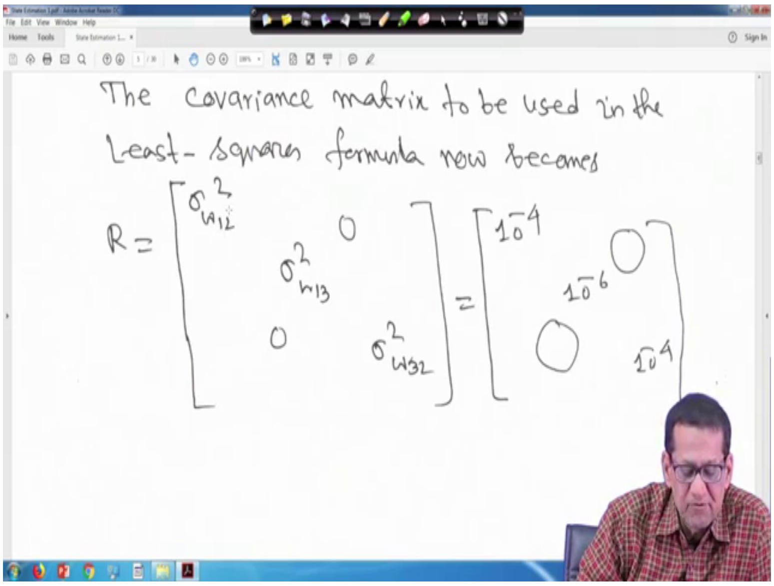

Now, the covariance matrix to be used in this least square formula now it becomes actually that R is equal to sigma, directly I am putting W 12 square, sigma W 13, square sigma W 32 square; so basically in this case it will be your what you call 10 to the power minus 4, 10 to the power minus 6 and 10 to the power minus 4 right. That will be my R right.

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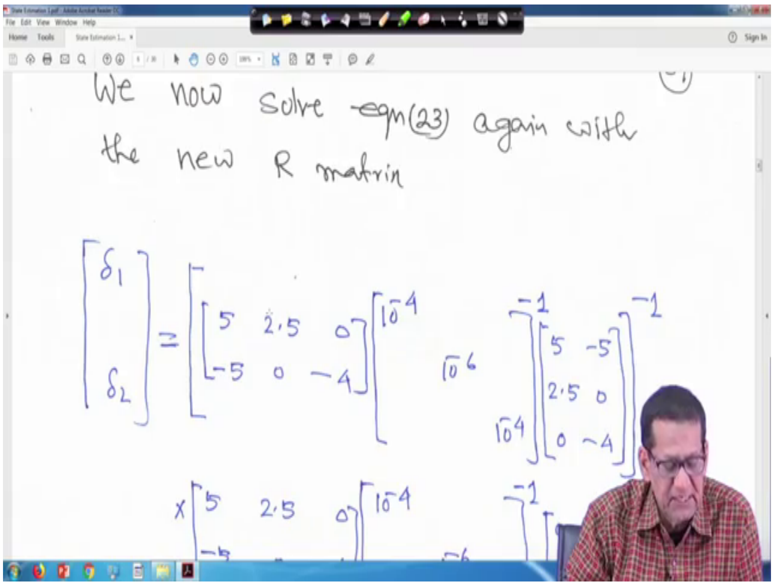

And then we now solve equation 23 again with the new R metrics. So, that R is change rest are remain same for the problem right. So, this is your h transfers, this is your r inverse, this is h right into this is your h again and this is r invar and this is completely whole to the by inverse right and this one and multiplied by this major value 0.62, 0.06, 0.37.

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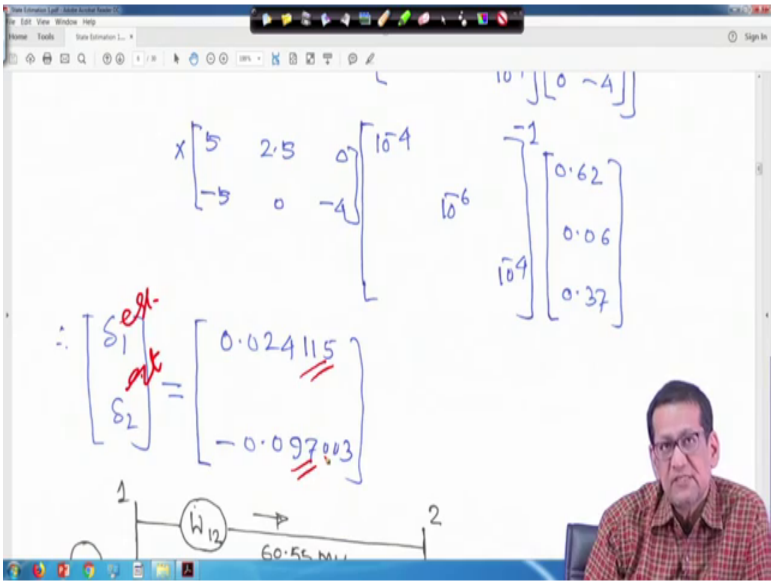

Now if you compute this one, you will get this is your delta, this is your delta 1 actually estimated. This is delta 2 estimated right. You will get 0.024115 radian and delta to estimated you will get minus 0.0957003 radian right. Therefore, if you re calculate the power flow using the same thing. Then you will get that W 12 power flowing from 1 to 2 actually, that whatever meter reading was given that 60.55. Simply you can calculator this one? Right directly you can calculate delta 1 minus delta 2 upon x 12; only thing is that that all these bus voltage magnitude we have assume 1 right; 1 per unit.

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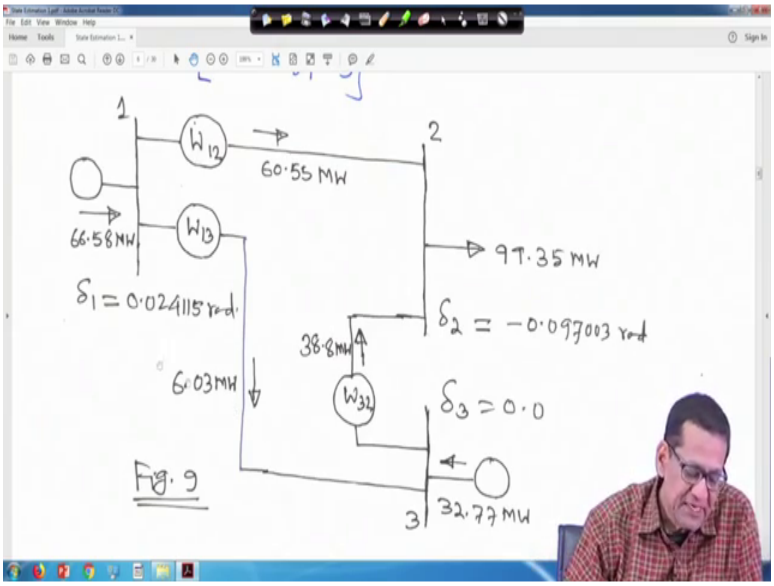

So, similarly here also your delta so, that power flow from line 1 to 3 sorry it is actually 6.03 megawatt. But look it is metre accuracy is very I mean pa plus minus 0.3 megawatt. So, sigma is equal to 0.1 megawatt, so accuracy is I mean quite good that is why it is very close to the ammeter reading. Because meter reading was given for this meter W 13 was 6 megawatt so, it is coming 6.03 and similarly W 32 according to this calculations 38.8 megawatt.

So, similarly you can find out that what will be your total? 60.55 and this side 38.8 so, it will become 99.35 megawatt right. And delta 2 is equal to this much and delta 1 estimated is this much, that you we have done it. And similarly here 6.03 coming and 32.77 is coming so, it is 38.8 megawatt right. So, with this your what you call that with this is this shows that how one can your what you call, estimate that angle delta 1 and delta 2.

Now, I will go back to this that you are this is equation 29 is. But we will go back to that you are other two things. That is your this two right. So, so there is also a closed form solution so, first we will go to this, then will come your 65 A or 65 B that 2 pages right. Because I stand in before, so there is also a closed form solution to the problem, when N s greater than N m, that is your un underdetermined right.

Although in this case we are not estimating X to maximize a likelihood functions since N s greater than N m right. So, usually implies that many different values of X estimated can be found right. And that cause actually estimated to equal to Z i measure for all i is equal to 1, 2 up to N m exactly right.

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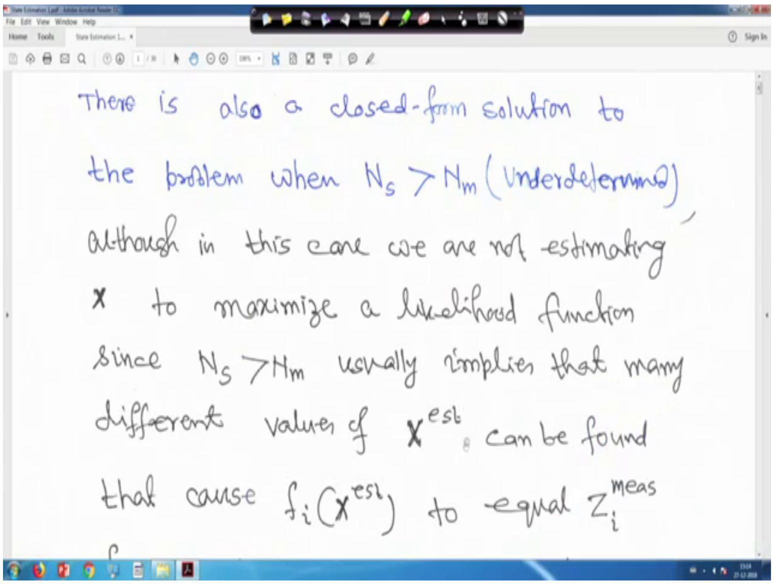

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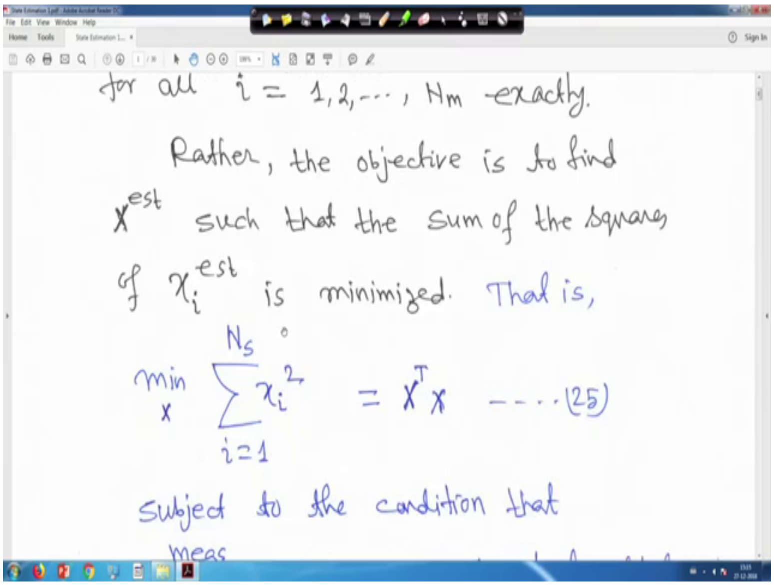

Rather the objective is to find X estimated, such that the sum of the squares of x i estimated is minimised right. That is, you minimise your i is equal to 1 2 n is x i x i square and that can be written as X transpose X. So, this is equation actually 25 right.

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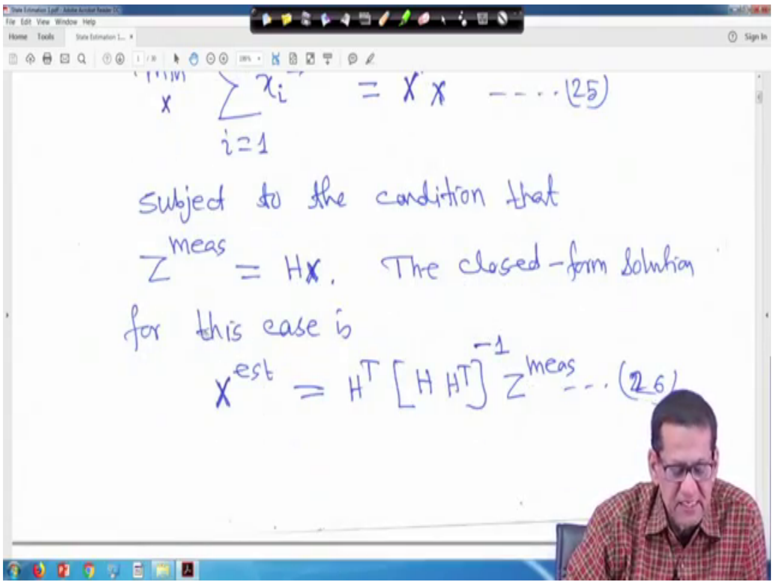

Subject to the condition that Z measures should be is equal to H into X. This we have seen early earlier right. The closed form solution for this case will be X estimated will be H transpose H, H transpose inverse into Z measures. So, this will be the solution but question is that, if time permits I will derive it I will derive it for you right. But let us see, right. So, if from that equation only slight change will be there and it can be derived. So, this is equation actually 26. So, easy to remember actually H transpose then into H, H transpose inverse into Z measure right.

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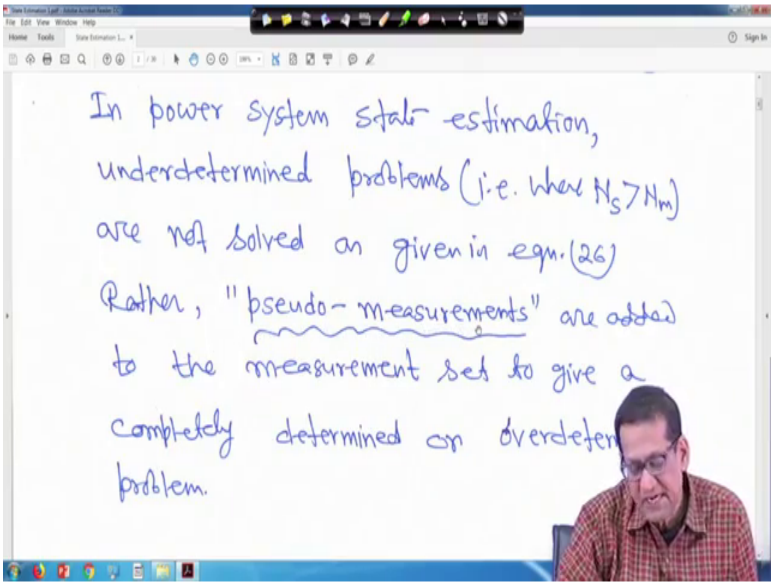

And in power system state estimation you underdetermine problem that is when number of state variable greater than number of measurement right are not solve as given equation 26. So, we do not solve it like this right. So, rather "pseudo measurement" are added to the measurement set to give a completely determined or over determined problem right. So, these are the things if time permits will see that, but question is that that time will not permit to solve on this problem, because it takes time right; so that is all there right.

So, again will go back to the your next thing. So, whatever little bit we have seen that is particularly suitable for your DC in a DC network or dc circuit right. So, and all this examples that two different data I showed you that how it works.

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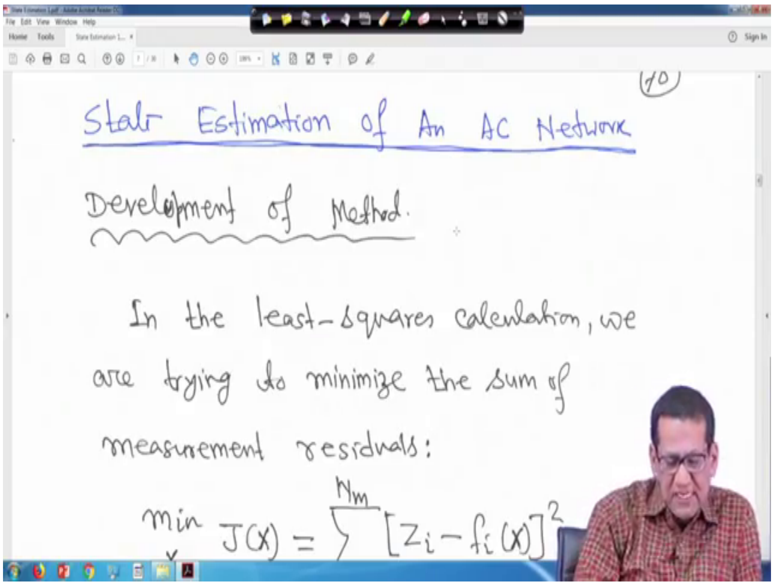

Now, we will come to state estimation of an often AC Network right. In the AC network sorry that power flow in any line this a it is actually a non-linear function right because it at cosine and sine terms are involved. And similarly, that power injection also it is non-linear because sine and cosine transfer involved right. So, quest question is that, how one can develop this method for state estimation of an AC network?

So, in this case a philosophy will remain more or less same, but mathematical development will be different right. So, now in the least square calculation right; we are trying to minimise the sum of the measurements residuals. So, we know that minimise your J, J X that sigma i is equal to 12 N m, N m is the number of measurement right.

That is into their J your bracket Z i minus f i of X because, X is vector right. So, it is function of so many state variables right.

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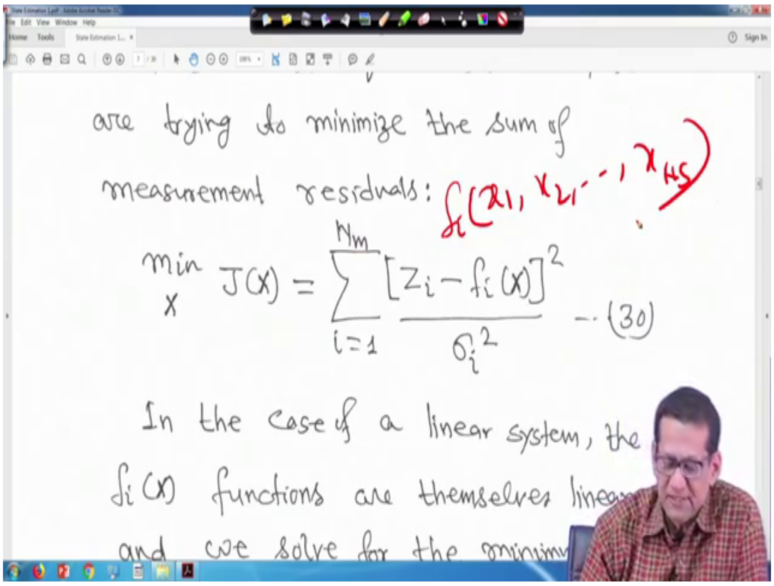

Square divided by sigma is square actually this is actually this f i for i right it is function of 7, it is actually I am written in a thick X right. So that means, that is actually f i it is function of x 1 x 2 right your x Ns right. So, that is why it is actually given like this.

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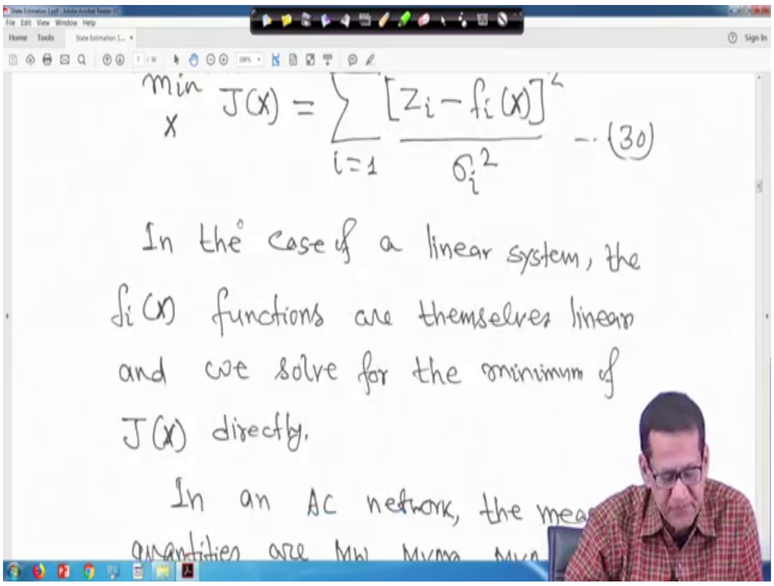

So, in the your case of a linear system the f i X we have taken a f i X is equal to h x. So, we have seen know h i 1 x 1, h I 2 x 2 like this right. Are your them self linear and we solve further minimum J X directly that we have seen right when you told that function f x is equal to a h x.

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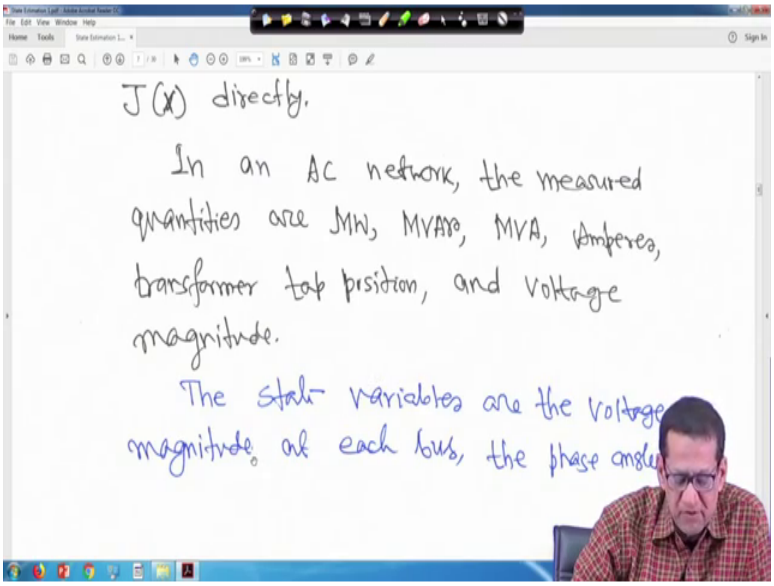

Now, in an AC network the measured quantities actually are megawatt megavar then mega volt ampere, amperes, transformer tap position and voltage magnitude. So, so many you are your what you call the measurement quantities are there and depends on which one you are trying to estimate right. So, I mean the line flows or power injections or the other voltage magnitudes or its angle right.

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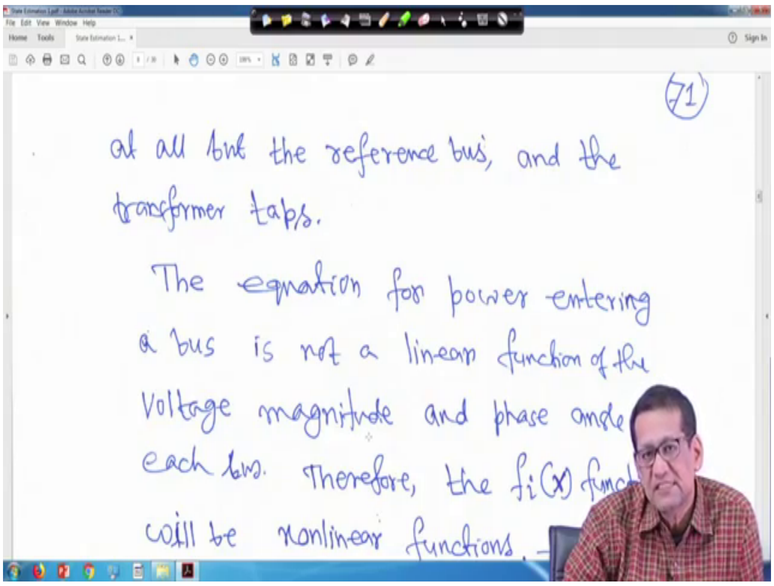

So, the state variables are the voltage magnitude at each bus right. The phase angles at all, but the reference bus and the transformer taps. Because at reference bus means that is a slag bus that we need not estimate and the transformer tap position right; that also can be considered as a state variable. So, the equation for power entering a bus is not a linear function of the voltage magnitude and phase angle at each bus; that you know for AC network.

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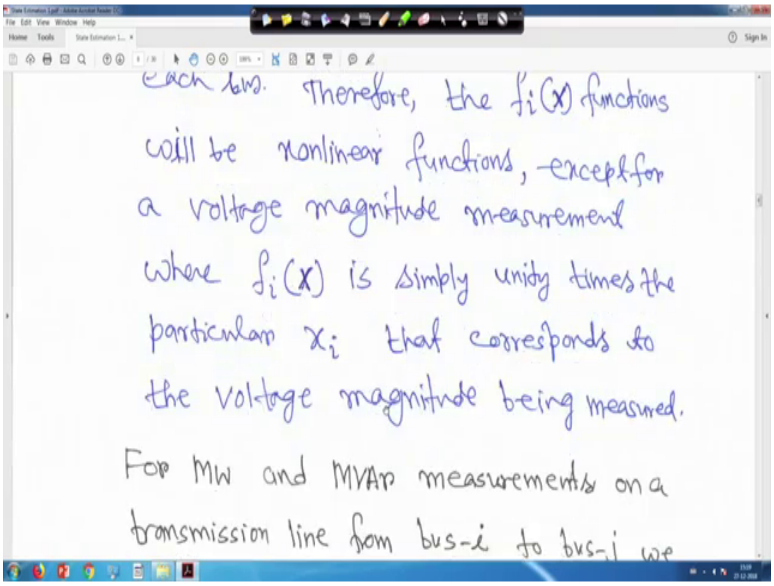

Therefore the f i X function will be non-linear function except for a voltage magnitude measurement, where f i X is simply unity times the particular x i right; that corresponds to a voltage magnitude being measured. So, for megawatt and megavar measurement right on a transmission line from bus-i to bus-j we would have the following terms in J X matric.

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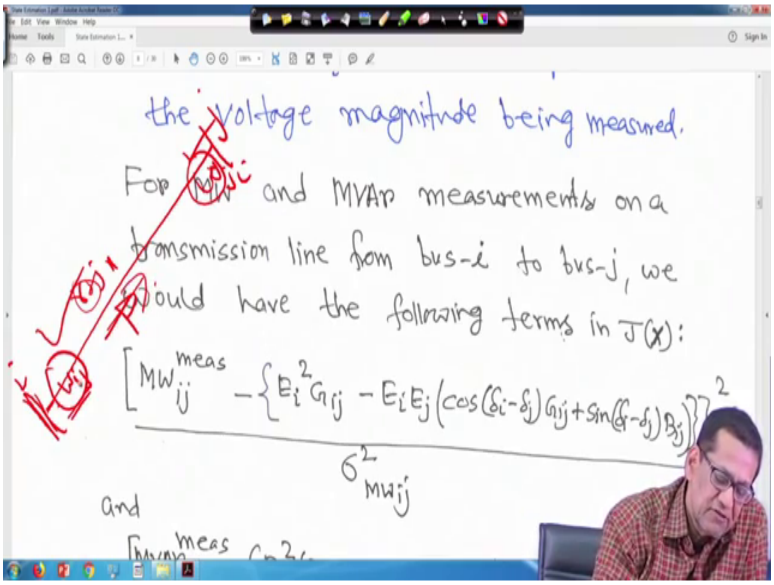

So, that actually this that power flow from bus-i that is from bus-i to bus-j right; this I am use this expression right, it depends on your which convention you are taking regarding G i j negative or positive it does not matter final result will remain same right.

So, in that case what one can do is, that this is my say power flow that your power is flowing from i to j you can measure right. And another thing is that, that you have to also find out that what is your E i what you call that your mathematically whatever is expression that is P i j the power flowing from line i to j. So, one small thing that suppose this is my line suppose forget about this. Suppose this is my bus 1 bus-i and this is my bus-j, say this is j this is i right. I have a metre say it is called W i j if you put a metre and also say W j i right.

So, if you put in just near the bus this meter to measure the power flow. So, line has resistance r plus j x say direction of the power flow is like this right. So, in this case whatever power your flowing will measure near this bus, I mean in the line near this bus i right. So, if you try to measure here that it will not be same because line is r so, it has the loss so, whatever it will measure here it is here that lost will be excluded.

That means, if you assume that that lossless line that is r is not there then, this reading and this reading you can take same. But line and loss so, what whatever reading it will be there whatever reading of this what watt meter it isdirection is this way right. So, it will be this one minus your what you call that power loss in the line that is i square r will be subtracted that will be the meter reading right.

So, similarly when you measure power right; megawatt i j you have to see that we have your what you call place the meter near in the line near bus i right. And that is your megawatt i j measure right. Similarly minus this power flowing equation power is flowing from bus i to j so, I have used this your what you call this equation. So, that is E i square G i j minus E i E j in bracket G i j into cosine of delta i minus delta j plus B i j sine of delta i minus delta j right, whole square this is whole square right.

So, because we have to take the residue right divided by sigma square megawatt i j, that means, the metre which you have placed near the your near bus in the line i j nearby near bus-i right. That that accuracy is also is given. So, that is your standard deviation;. so sigma square megawatt i j, so this is actually equation 31 right.

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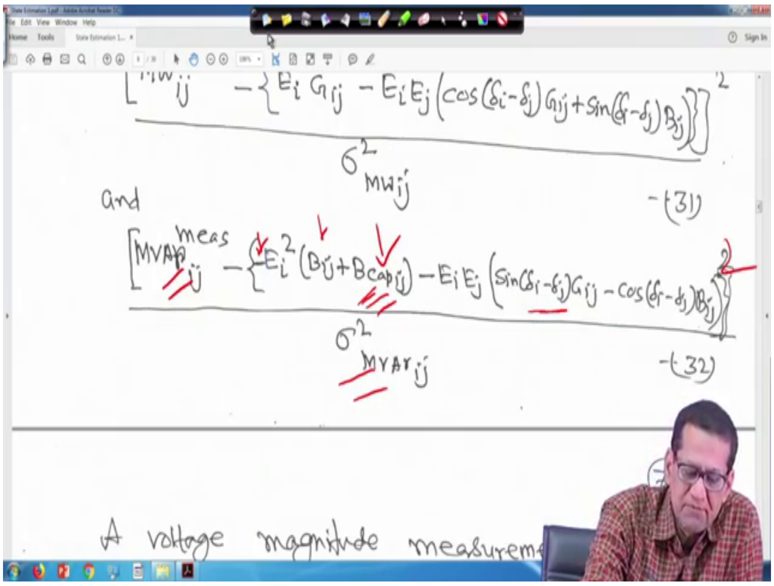

So, next is and similarly if you are if it is a megavar if it is a megavar. Then it is megavar i j that is your measured value minus E i square minus of this is actually minus another minus is here right. So, minus in bracket minus E i squared B i j plus B cap i j. Because line i and j you have charging your capacitance that means, you have the charging susceptance right, that also need to be taken care of. So, that is why B i j is there plus B cap i j this notation I have you used right. So, if you if you pi network. So, in accordingly you have your what you call you have to take that values right. So, that is why this B i j plus B cap i j is taken minus E i E j in bracket G i j sine delta i minus delta j then minus B i j cosine delta i minus delta j right.

So, your what you call is another thing is if in the line also you will have that, your if it is a series compensated line say transmission lines series series capacitor are there in line. Then also that your what you call the reactance also you have to consider right. So, all these things you have to you, to consider right when you are measuring that your what you call that your megavar, megavar power that is you reactive power right. So, all these things you have to consider that is why this B cap j is also added here right.

So, this is actually your what you call that that reactive power measurement right. So, then whole square because, residue you have to take divided by that metre which you are using for measuring the megavar; so sigma square megavar i j. So, in the case of real power no need to consider any your what you call that shunt or series capacitors or anything right. But when you are trying to find out your what you call that your reactive power, when you are measuring in mathematical equation that if series compensator line then you have to, you have also you have to also consider that your what you call that your a capacitive reactance and the susceptance right.

So, this is actually your megavar i j. I mean this way you can put, this way you can put in mathematical form.

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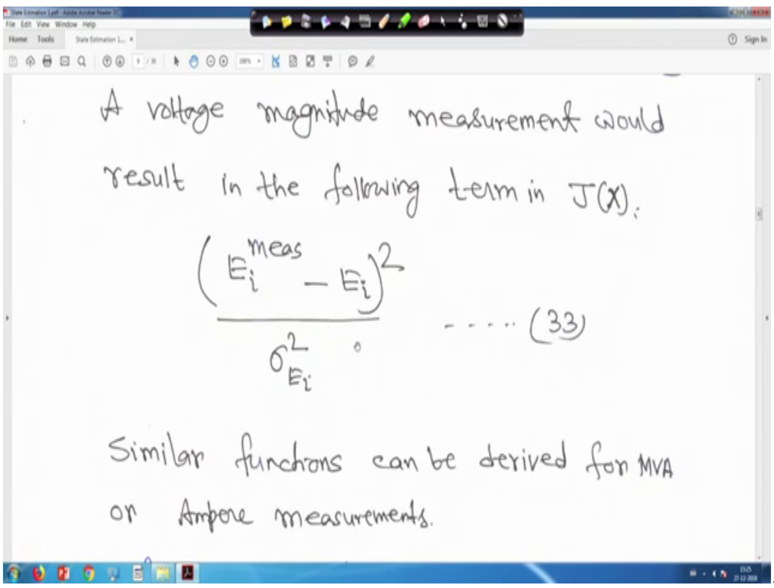

So, next one is that a if a voltage magnitude measurement would result in the following term in J X. That is it is E i measure course is magnitude minus E i whole square by sigma E i square because, you are measuring the voltage magnitude. So, measure will be there and this minus E i square sigma E i right from this you have measured, but from where you will get your what you call the E i.

So, you have learn, you need learn that load flow. Load flow right. So, similar functions can be derived from MVA or ampere measurement; if you want to measure MVA or if you want to measure the current flowing through any branch, so that ampere measurement right.

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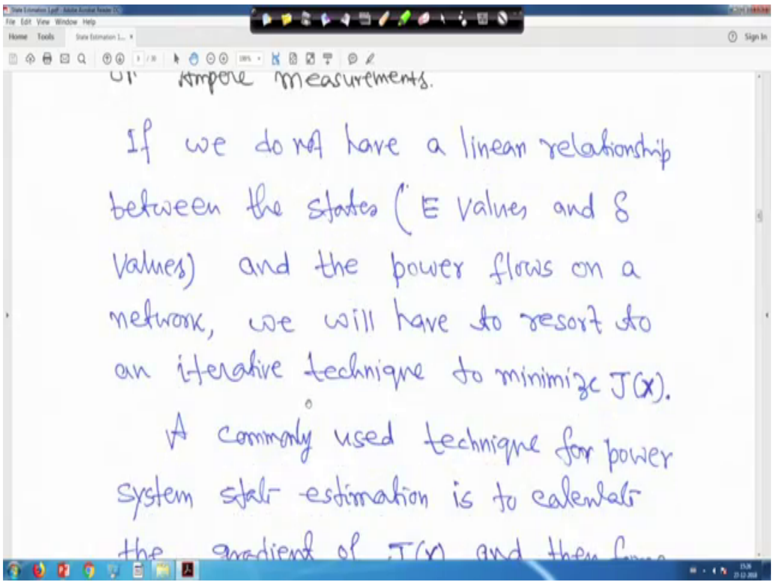

So, know if we do not have a linear relationship between the states right. That is your E values and delta values and the power flows on a network, we will have will have to refer to an iterative technique to minimise J X right because, that I mean ultimately this term that equation 33 and equation 31 and 30 here what you call 32 right; these terms are non-linear.

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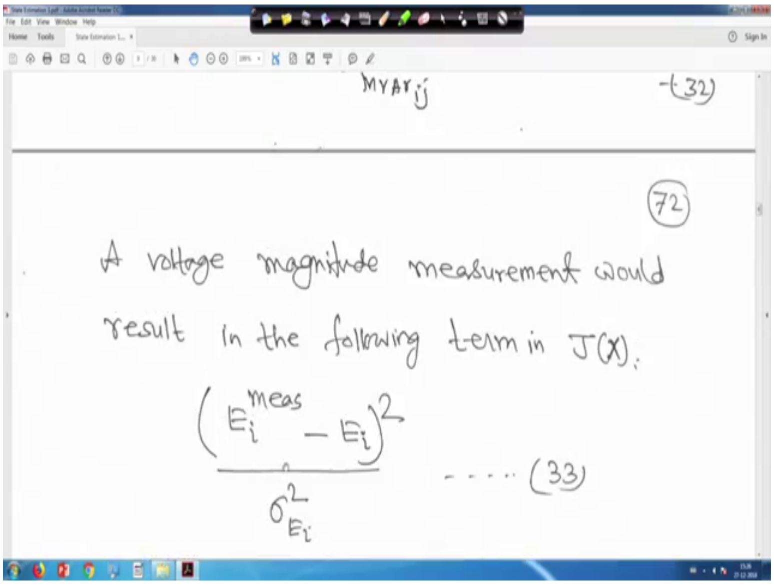

So, to make this term to zero right; I mean we have to make your what you call to minimize this objective function. So, it is not a linear. So, direct solution is not possible. So, we have to go for your iterative technique right.

So, therefore, a commonly used technique for power system state estimation is to calculate the gradient of J X right and then force it to zero using Newton's method. That means, what you have to do is this J X is actually non-linear function, you have to take its gradient right.

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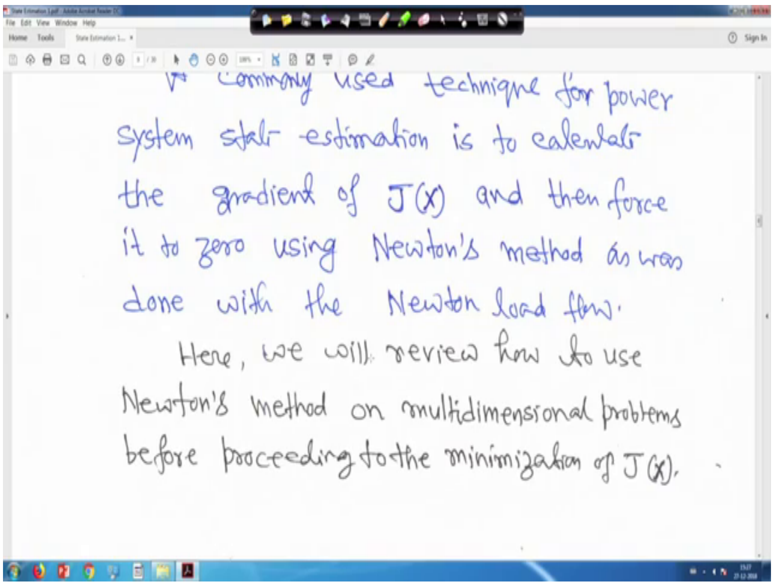

And then iteratively you have to force the gradient to zero. That means, you have to make to make that gradient to zero you have to go for your what you call iterative technique. So, I will show you how it is right. So, here we will review how to use newtons method and multidimensional problem before proceeding to the minimization of J X right.

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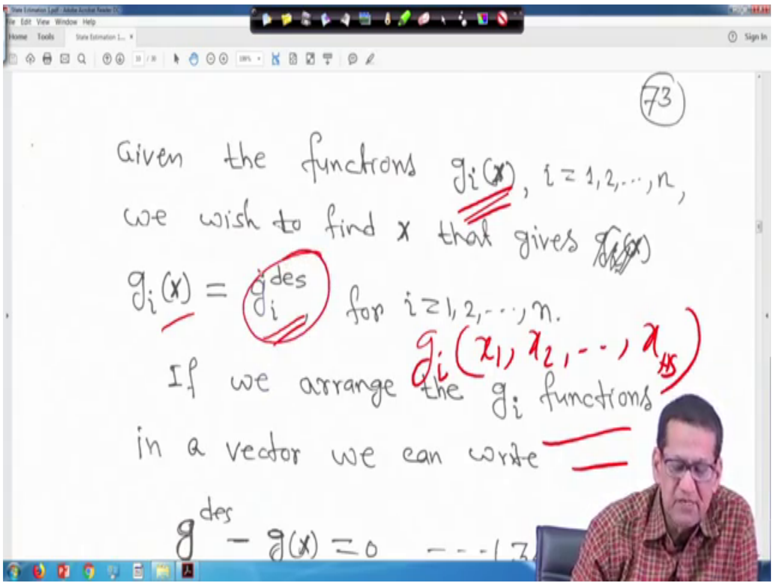

So, suppose given that function suppose you have a function g i X right. So, you have a function say it is g i X right. g i X means it is thick X, that is your g i function of x 1, x 2 like this up to x N s right.

So, this is actually g i. For i is equal to 1 to n right. So, we used to find x that gives actually g i X is equal to g i desired value I mean this is actually you want that when that solution will converge right for we have to obtain the; your different values of x right. x 1, x 2 like this such that that whatever desired value you want it will give right. So, if we are in the g i function in a vector we can write like this. Just hold on that we can write like this; that means, that means this one we can write g desired minus g X is equal to 0 right. So, this we want actually that g X should be having a when it will converge g X should be equal to g desired.

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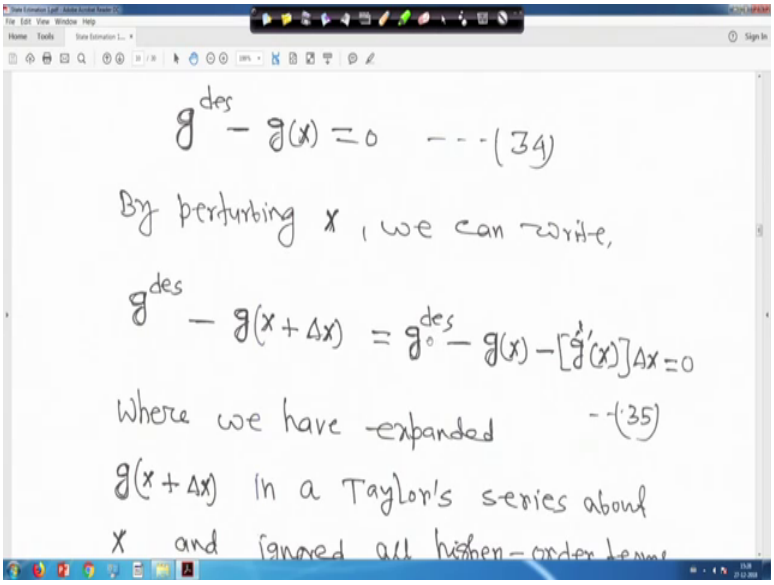

So, by perturbing X, we can write, that suppose you make perturb X right, from say some initial value initially at zero. So, now or initially it was x say I have written like simply x right. So, g desired will be there that is your specified right. We know that, minus g then x plus g delta x right. And therefore, this g desired minus your g X of delta x x plus delta x you can write g desired then you, then you can do this term, you expand it in Taylor series right. That is g desired minus g X minus your g dash x into delta x right. So, this way only up to first order term. We are ignoring that your higher order term right.

So, this way you can make it like your in Taylor's series. Therefore, where we have expanded that your g X plus delta x in a Taylor series about x and ignored or higher order terms right.

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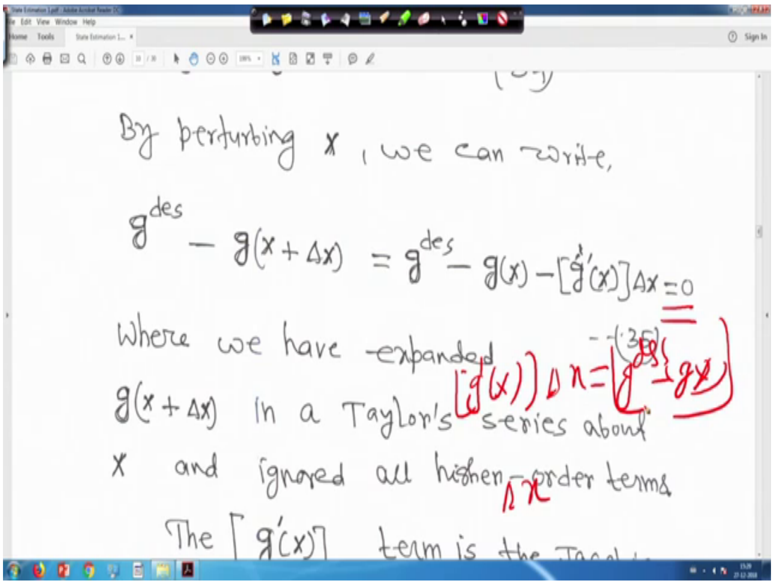

Therefore the g dash x term is Jacobian matrix of first derivatives of g X. Then what we can do is, that this equation that your this equation that g desired minus g X minus it is equal to 0. Therefore, if you look into then this equation you can write I mean this equation it is is equal to 0 right. Therefore, this equation, so in bracket I will just write it in g dash g dash X right delta x is equal to g desired minus g X right this is the thing. Therefore, delta your what you call delta x will be g dash x your what you call inverse then g desired minus g X right.

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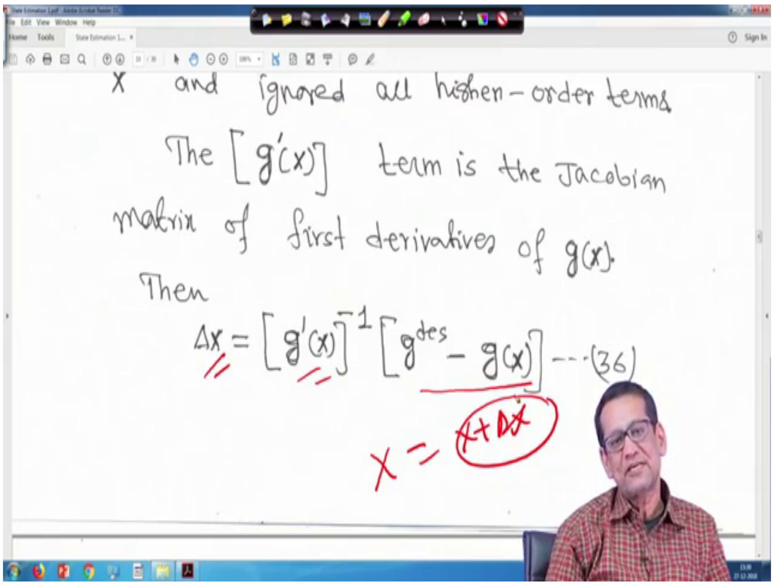

Therefore your this term delta X can be written as g dash X inverse right into g desired minus g X this is equation 36; that means, what we have to do is that, when you will start with some initial values right. When you will start with some initial values then every time that you have to force that your what you call g X should become g desired right. When g X is equal to g desired right then what will happen? That solution has converged right. At that time delta x will become 0.

So, in that case first you have the function g X takes it isderivative and try to iterate and in general that you have to update X is equal to X plus delta x in every iteration. That means, that in general that you have to when you later you will see that we have to force that guardian to become 0 iteratively.

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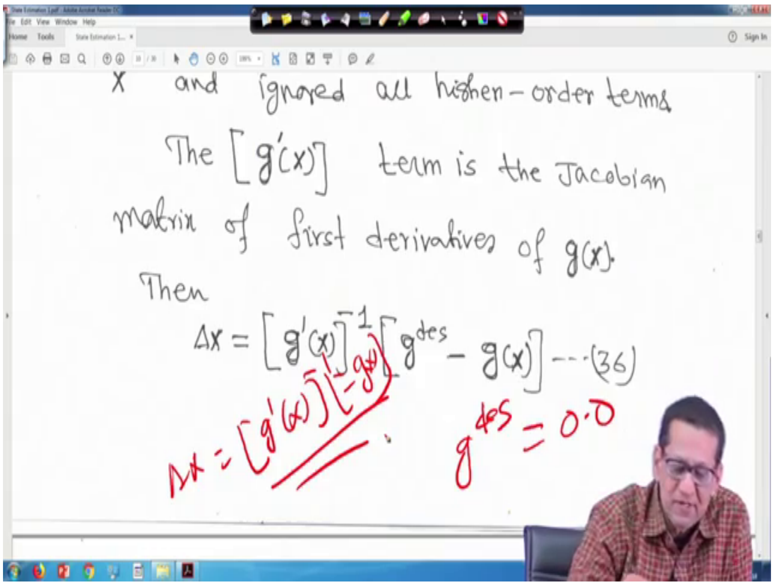

And if say our g desired say is equal to 0 right. In that case what will happen? That delta x will be nothing but g dash X inverse right. And if it is g desired 0, then it will be multiplied by minus g X right; so if g desired is 0 right. So, that is why we are writing this one right.

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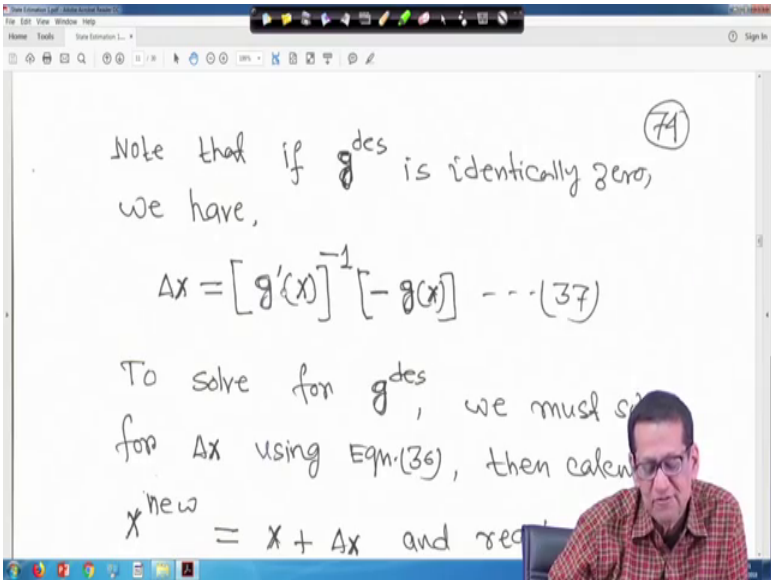

Note that, if we if that is why I have given if g desired is identically 0. Then we have delta x is equal to g dash X inverse into minus g X right. So, this is equation 37 right.

Thank you very much we will back again.