Power System Dynamics, Control and Monitoring
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Lecture – 08
Power System Stability (Contd.)

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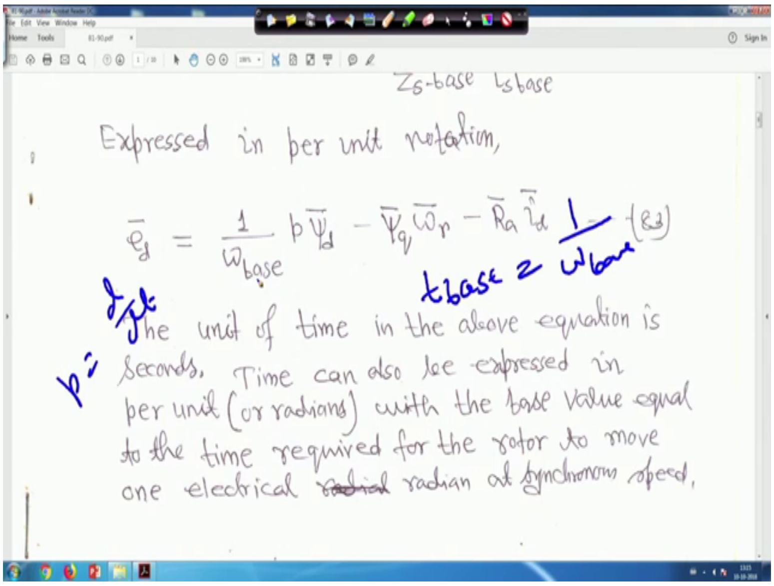

We are back again; so this equation how you got p bar right. So, it is actually this is we have taken that t base is equal to 1 upon omega base right, that is your omega base. Now question is p actually is equal to your d by dt and 1 upon omega base is there; that means, let me clear it; that means, this equation I can write like this,.

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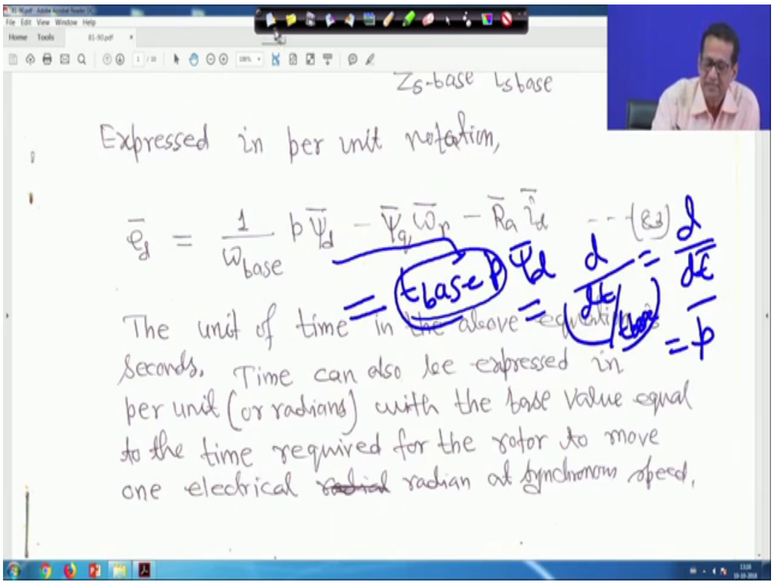

This equation, I can write like this is equal to your t base instead of 1 upon omega base only this term later you will know everything is this is p, then psi d bar only this term I am writing right.

Now, p is equal to d dt therefore, this term can be written as your d by your it is d dt. So, it can be written by dt divided by t base right. So, that is this term right. So, if you write like this then this term will become d by dt bar right. Because dt upon t base. So, that can be written as actually p bar right.

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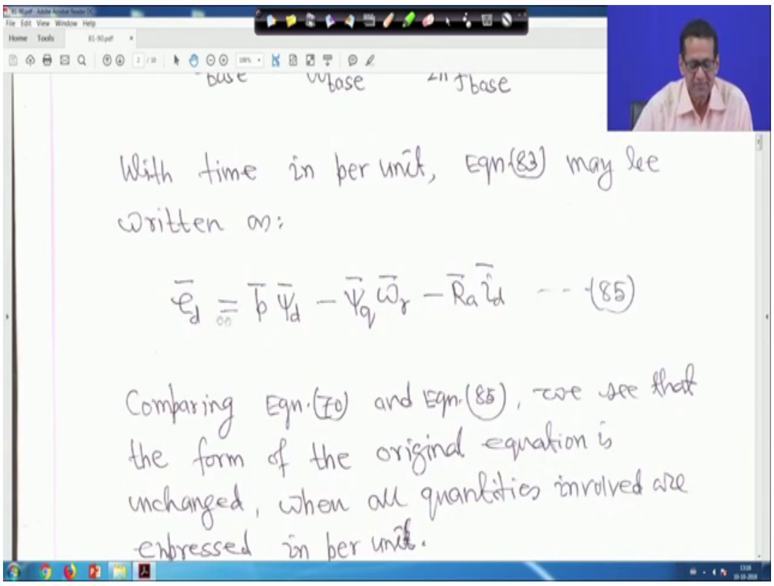

So, if you want to represent the time in per unit that is why this equation, this t base is written ed bar is equal to p bar psi d bar minus psi q bar omega r minus Ra bar i d bar these are all per unit values bar means per unit values. But throughout our analysis we will only study for your what you call to in a time in second right, but this is some representation in time in per unit So, equation 70 and an 85, compare equation 70 and equation 85 you will see that the form of the original equation is unchanged.

Only when all quantities involved expressing per unit so, this here equation 70 also they are also ed is equal to p psi d minus psi q omega r minus Ra id in per unit also it is like that right. So, ultimately equation is unchanged only that (Refer Time: 02:33) your 70 you have to consider all that here exact values and here they are dimensionless that is all otherwise expressions will remain same right.

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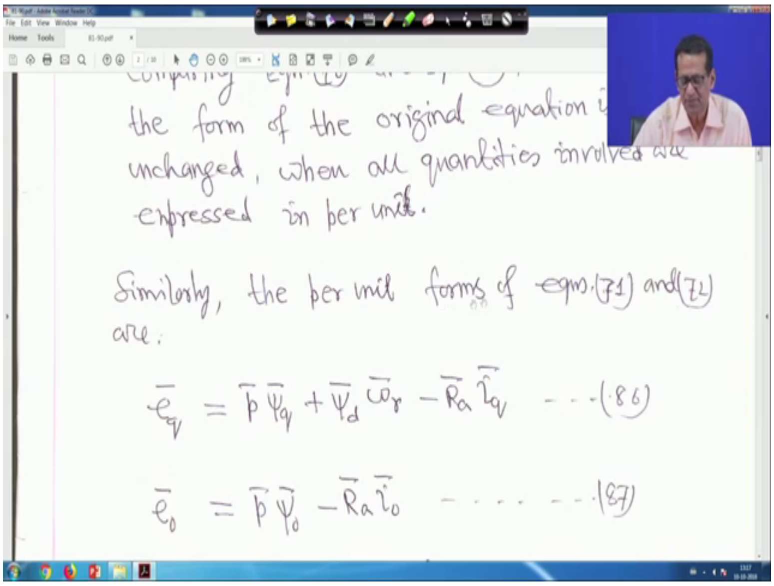

Similarly, per unit forms of equation 71 and 72 easily similar way you can do this similar way right. So, it will be e q bar will be p bar psi q bar plus psi d bar omega r bar minus Ra bar iq bar, this is equation 86. Similarly e 0 bar will be p bar psi 0 bar minus Ra bar i 0 bar. This is 87 bar means all are per unit values, but equation an unchanged only there in per unit values right.

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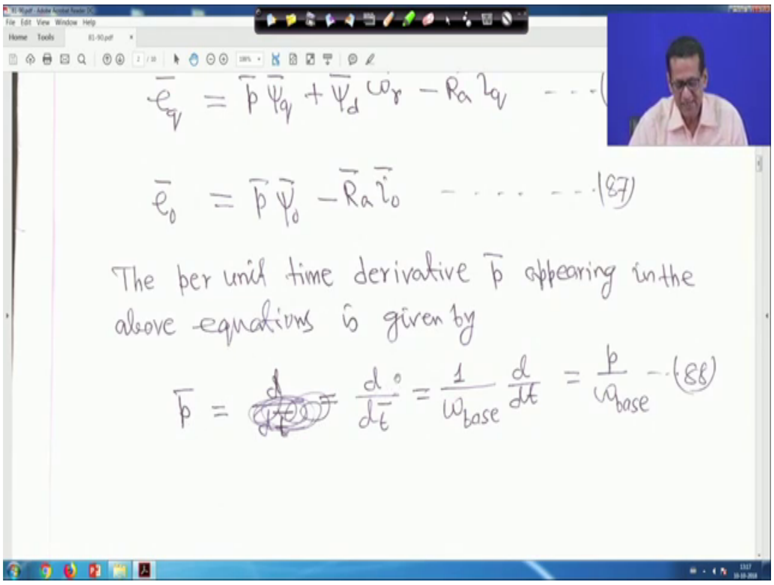

So, whatever I told you the p bar it is d by dt. So, it is 1 upon omega base d by dt. So, p by omega base right. So, this is whatever the way I showed you that how it will be p bar.

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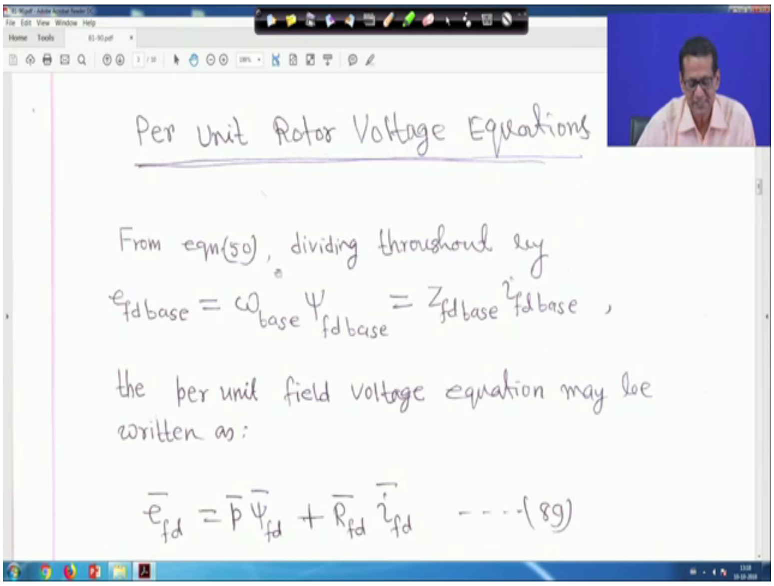

Next is per unit rotor voltage equations. Now from equation 50 your divide throughout by e field it is easy e fd base that is your field voltage field size right, is equal to omega base into psi fd base is equal to z fd base into i fd base same philosophy like stator here also it is rotor right So, the per unit feed voltage equation may be written as same thing both side you divide by e fd base left hand side then other terms you divide by omega base psi f d base and z fd base your i fd base the way we have done it for the stator similar meaning right.

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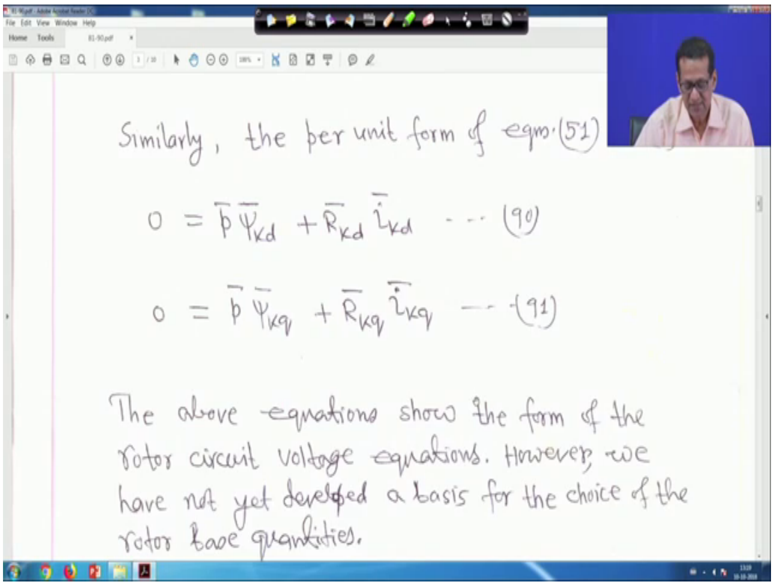

Then you will get e fd bar is equal to p bar psi fd bar plus R fd bar i fd bar this is equation 89 similar way. Similarly the per unit form of equation 51 and 52 it will be 0 is equal to p bar psi kd bar. Actually nothing is change just making bar to represent unit dividing those quantity and i kd bar this is 90. And another one 0 is equal to p bar psi k q bar plus R k q bar i kq bar this is equation 91 right.

Therefore the above this above equation shows the form of rotor circuit voltage equation. However, we have not yet developed a basis for the choice of the rotor base quantities right. Because we have to make things reciprocal, but earlier we have seen it is not reciprocal right.

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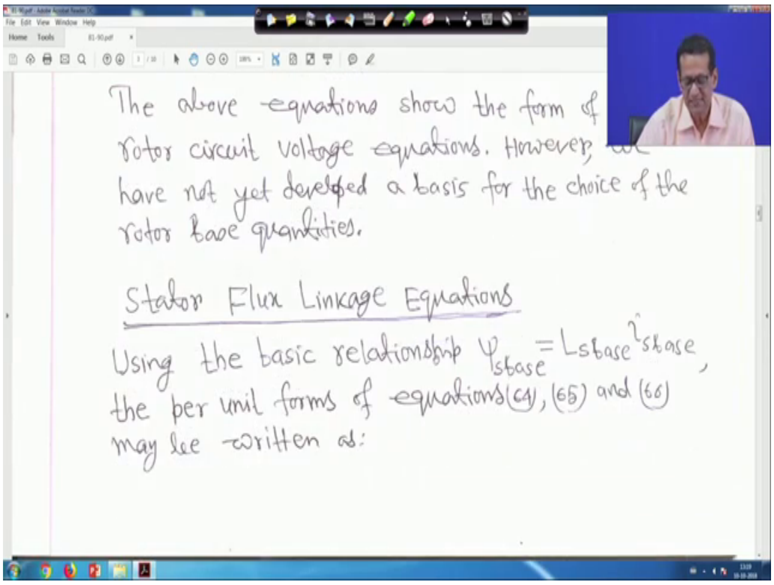

So, stator flux linkage equation using the basic relationship of psi s base is equal to Ls base i s base right the per unit forms of equation 64, 65, and 66 may be written as. So, I mean using this relationship psi s base actually in general we know psi is equal to L i right flux linkage is equal to inductance in current in general.

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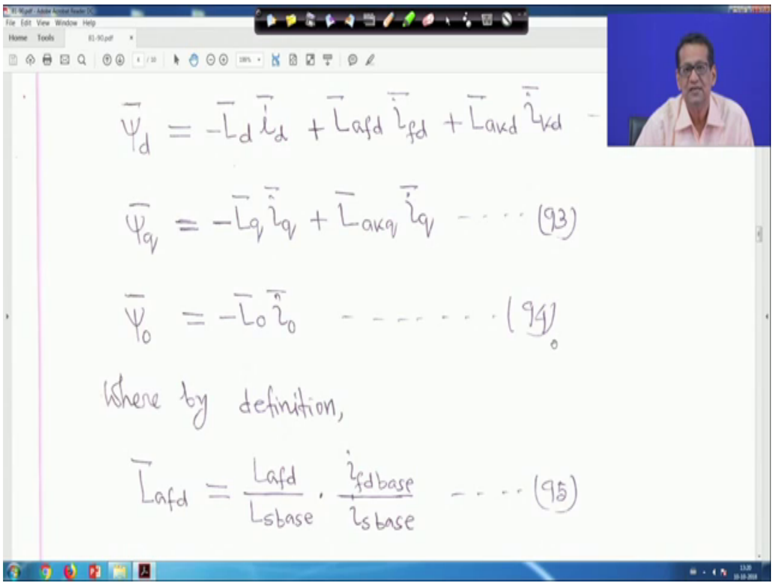

So, psi s base is equal to Ls base psi s base and it is psi d bar same way you could divide and simplify you will find it is minus Ld bar id bar plus L fd bar i fd bar plus L akd bar i kd bar this is equation 92. Similarly, psi q bar will be minus L q bar, i q bar plus L aq bar

L akq bar i q bar this is 93. And psi 0 bar will be minus L 0 bar i 0 bar this is equation 94 right.

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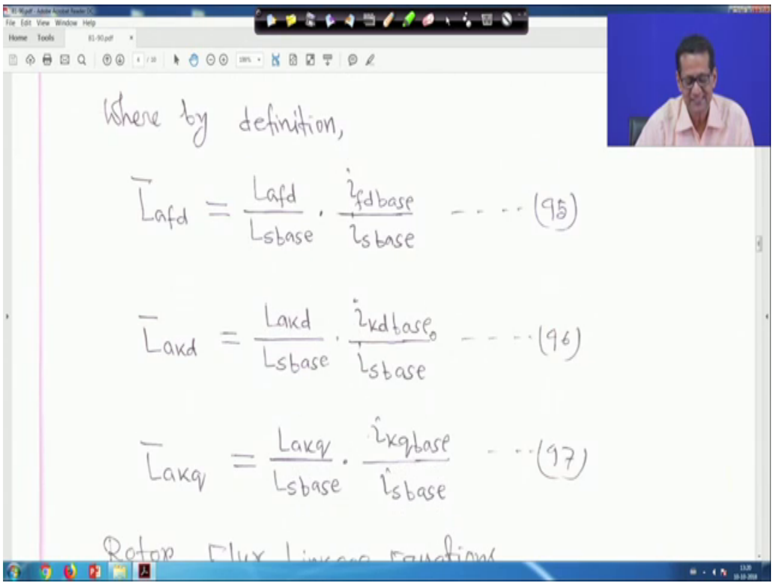

Now where by definition L fd bar will be L fd upon Ls base into ifd base upon i s base these are very simple thing, but I suggest that little bit you derive right little bit you derive right. So, L fd bar will be actually L afd upon Ls base into ifd base upon i s base right. Similarly L akd bar will become L akd upon Ls base into i kd base upon i s base this is equation 96. Similarly L aq bar will be L akq upon L s base into ikq base upon is base right so this is equation 97.

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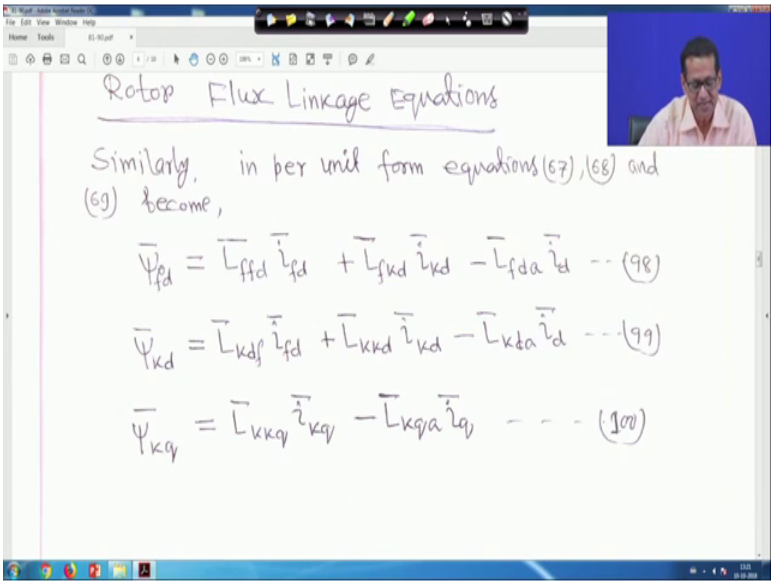

Now, rotor flux linkage equation, similarly in per unit form of equation 67, 68, and 69 same way will transform equation remains same only everywhere bar is there; that means, they are per unit. Therefore, psi f d bar will be L ffd bar I fd bar plus L l fk d bar i kd bar minus L fd a bar id bar this is equation 98.

Similarly, psi k d bar will be L kd f bar i fd bar plus L k kd bar i kd bar minus L kd a bar i d bar this is equation 99. And similarly psi k q bar will be psi L will be L k q bar i kq bar minus L kq a bar iq bar this is equation 100. So, all these things all these things this per unit system later I will take your example also on per unit you will all calculations will be shown you will find the things are quite easy right.

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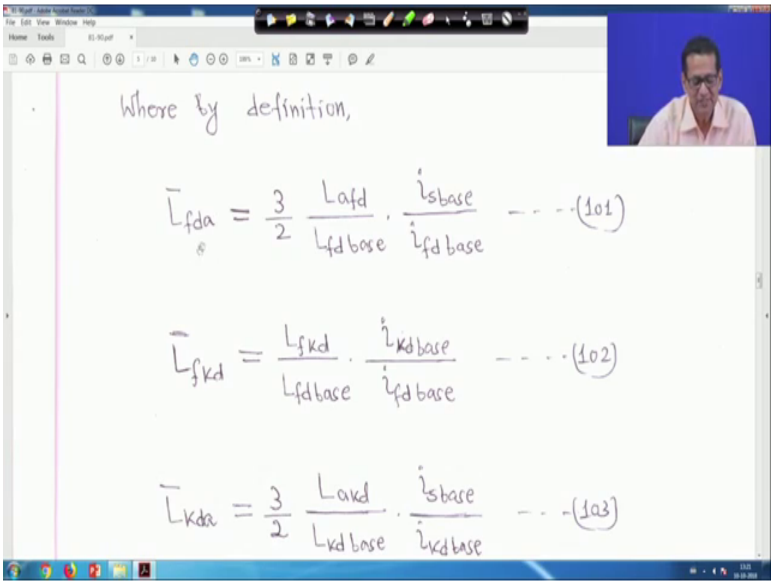

So, whereby definition again we know that Lf d a bar is equal to 3 by 2 Lf d upon Lf d base into i s base by ifd base this is equation 101. Similarly, L fkd bar is equal to L fkd upon L f d base into i kd base upon ifd base, right. Similarly L kd a bar is equal to 3 by 2 L a kd upon L kd base into i s base upon i kd base right.

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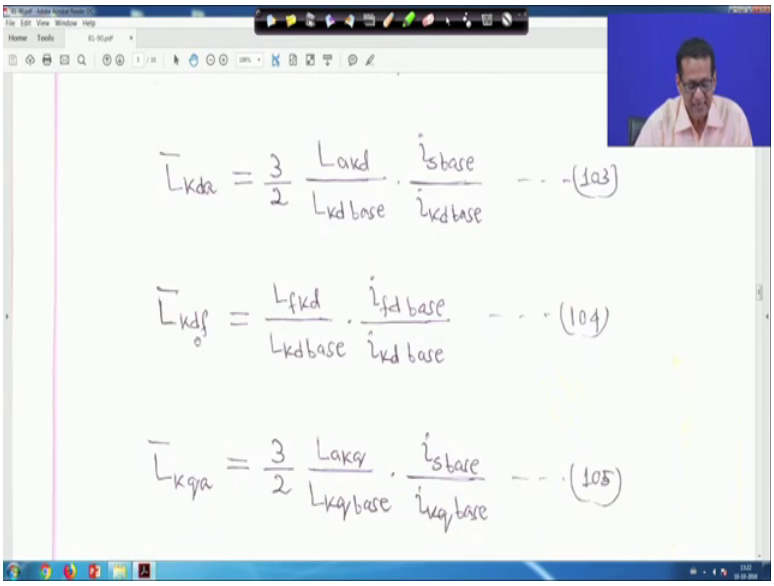

Similarly, L kd f bar will be L f kd upon L kd base into i fd base upon i kd base this is equation 104 right. similarly L k q a bar is equal to 3 by 2 L a kq upon L k q base into i s base upon i kq base. Now, all these things what you call if we see that here L fd a bar is equal to 3 by 2 L f d Lf d base i s base upon i fd base right. And just hold on and if you see here here L fd bar i s equal to L af d upon i s base ifd base upon i s base right and here your L LL f da bar is 3 by 2 L afd here L fd where multiplied by your 3 by 2 L fd upon L fd base is base upon i fd base right. So, similarly your L kqa bar also multiplied by 3 by 2, but we have to actually what we have to do is.

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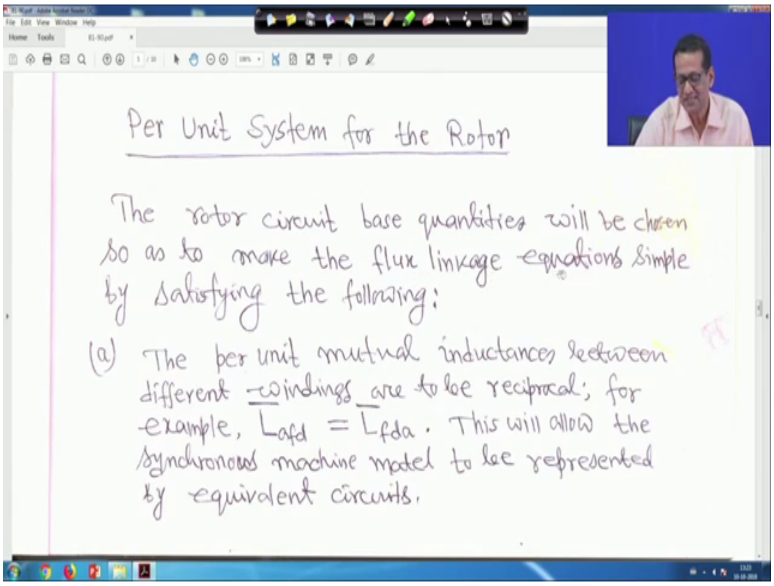

That we have to see that things are reciprocal; that means, per unit values either refer to stator or rotor side have to be same like transformer by refer to primary or secondary side. But one 3 by 2 factor is there we have to eliminate that So, the rotor circuit base quantities will be chosen first to make the flux linkage equations simple by satisfying the following. The per unit mutual inductances between different winding are to be reciprocal that is for example, L afd bar must be is equal to L fd a bar this should be equal right per unit refer to either side. This will allow the synchronous machine model to be represented by equivalent circuit right.

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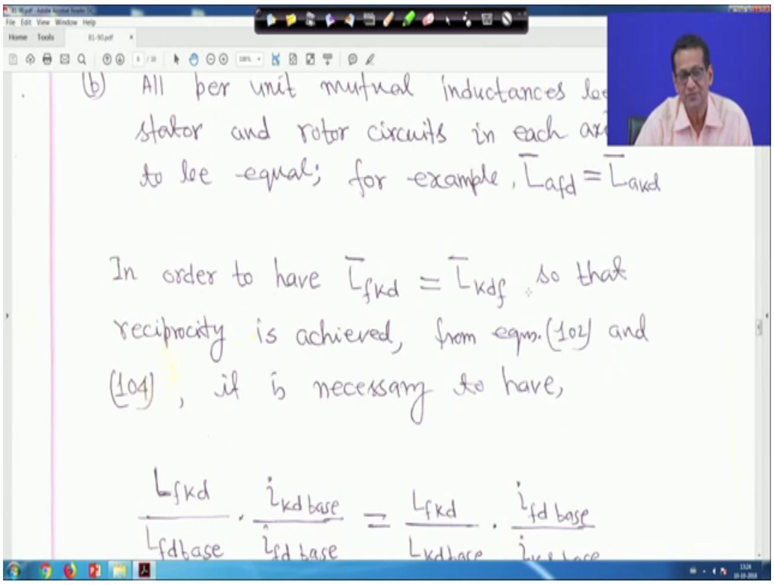

So, for that second thing is that all per unit mutual inductances between stator and rotor circuit in each axis are to be equal for example, L afd bar is equal to L akd bar right. In order to have L fkd bar is equal to L kdf bar, because it is what you call this has to be made reciprocal. So, that reciprocity is achieved from equation 102 and 104 it is necessary to have this relationship.

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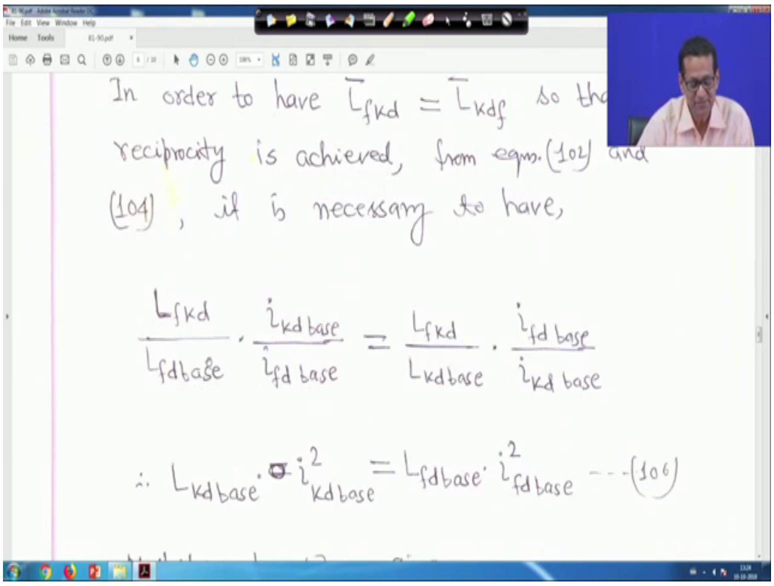

That L fkd upon L fd base into i kd base upon i fd base is equal to L fkd upon L kd base into ifd base upon i kd base just equation 102 and equation 104 if you do so if you do so, because this condition has to be made otherwise it cannot be reciprocal right.

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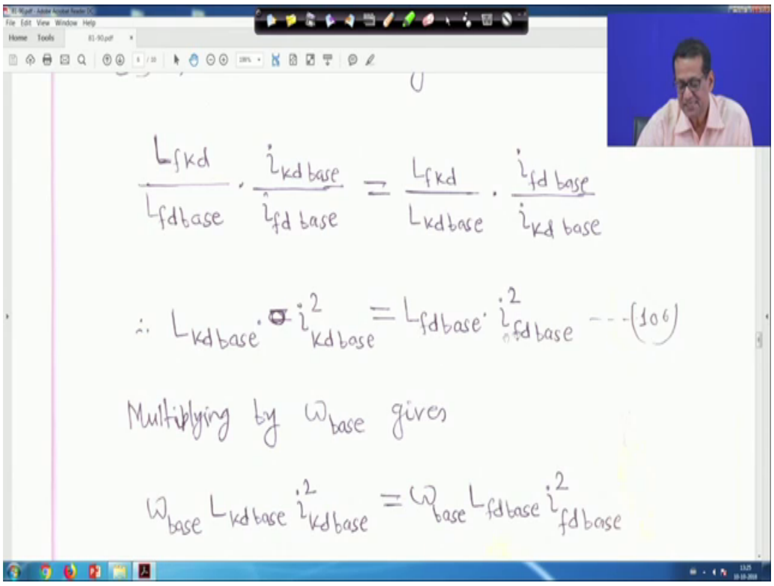

So, in that case what will happen this side is becoming L kd base into i kd base square is equal to L fd base into i fd base square right. So, just what you do see cross multiply cross multiplication right. So, in this case that means multiplying both sides by omega as if if you do omega base.

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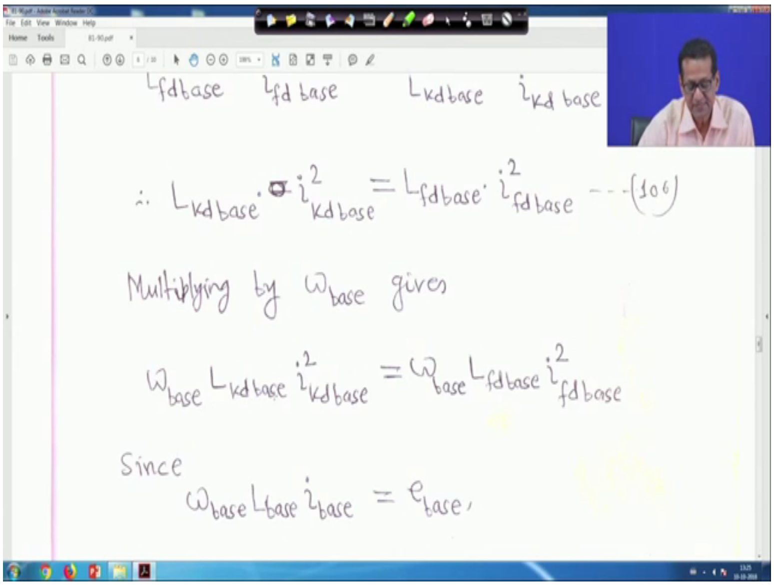

Then you will be omega base L kd base I k d square base is equal to omega base L fd base i fd square base right.

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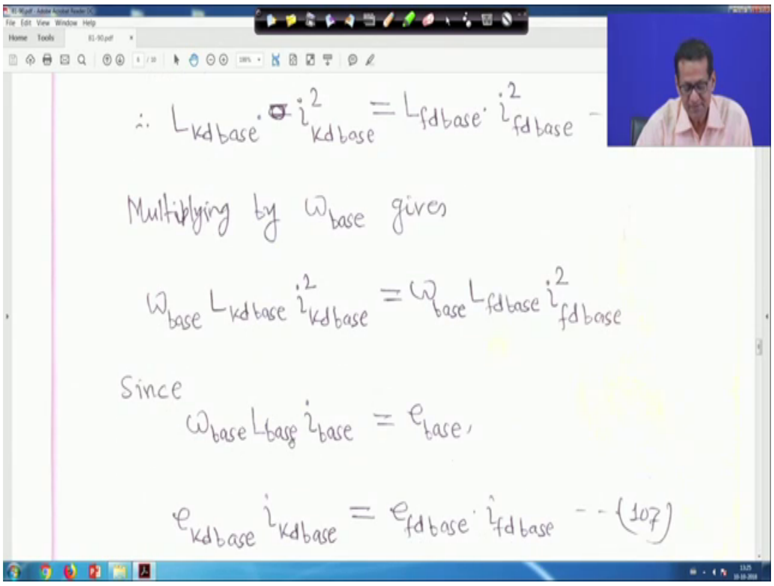

But we know that omega base L base and i base is equal to nothing, but the e base right this we know. Therefore, this side we can write left hand side it will be e kd base i kd base from the dimension analyse of the per unit and this dimension your what you call that earlier we have seen this right from its dimension only right.

So, it basically it will be ek d base i kd base because it is i kd square is there. So, naturally it will be e kd base is equal to your this thing this base rate this 1 e that is your this is the general formula for the dimension thing as far as per unit thing. So, instead of that we can write is e kd base i kd base and these are right hand side will be e fd base i fd base this condition has to be satisfied for the reciprocal thing right.

So, only thing is that apparently it looks little difficult, but it is not difficult. If you face any question if you face any problem you when you will go through these you put the question in the forum, we will answer all the questions right. And while at that time if you want you fail to derive one or two thing you will try to provide you what you call the derivation. But I did not bring it here it will kill lot of time right, but you have to keep this mind only keep it in your memory only right.

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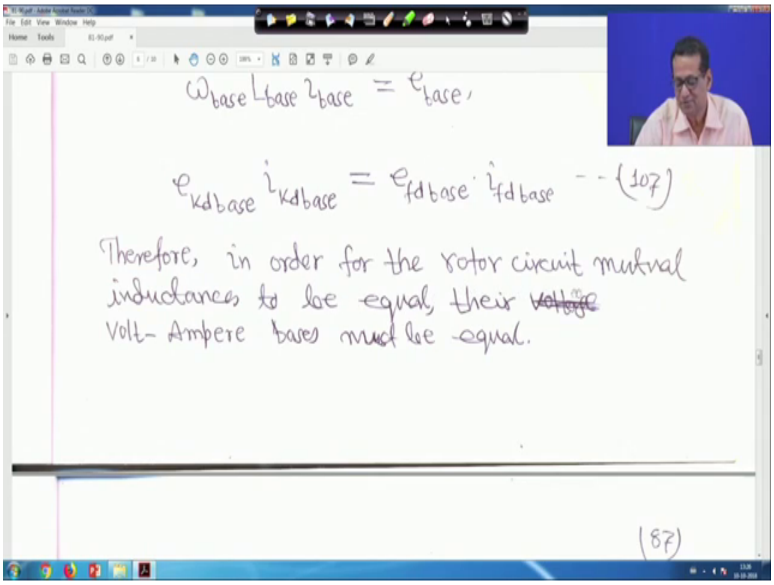

Therefore in order to for the rotor circuit mutual inductances to be equal their volt ampere base must be equal. Because this side is volt ampere b into i basically e into i this side also e into i. So, volt ampere base must be equal right.

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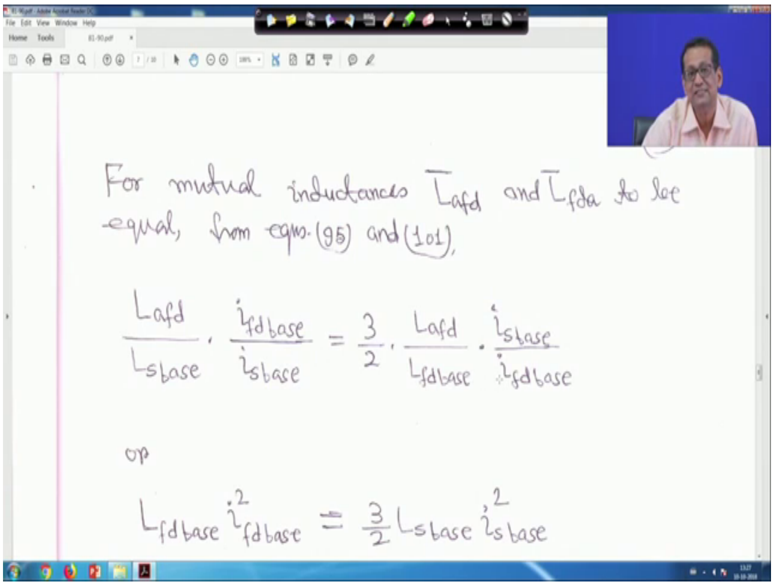

Similarly for mutual inductances that is L afd bar and L fda bar to be equal from equation 95 and 101 they have to be equal. Otherwise you cannot make per unit or refer to either side is equal you have to make this condition right.

So, if you do so then we are making L f d upon L f d upon i s base into i fd base upon i s base is equal to 3 by 2 L afd upon L fd base into i s base upon i f d base. And you cross multiply and simplify it will be L fd base into i fd base square is equal to 3 by 2 L s base into i s base square right. So, both side you multiply by omega base.

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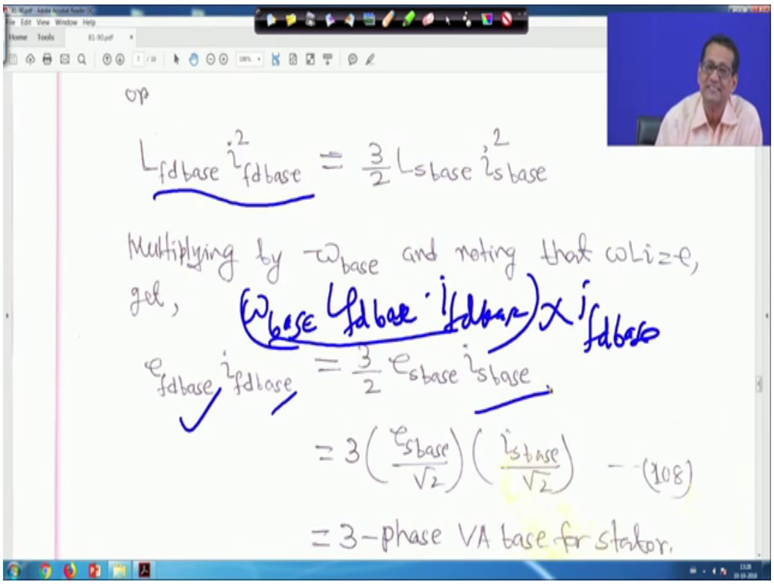

If you multiply both side by omega base and same as before same as before you will find that this side you multiply both side by omega s then omega base Lf d base and it is i fd square so, into i fd base into i fd base. So, first three terms will give you actually efd base. So, it will be e f d base in to i fd base. Similarly the right hand side it will be 3 by 2 es base into i s base I mean it is something like this. I mean both side you multiply by omega base then left hand side only I am writing right hand side similarly you can get it. It is omega base then lf d base then ifd base right it is i fd base square. So, into your i fd base. So, this term is nothing, but e fd base into i fd base similarly the right hand side same meaning right

So, then we right hand side also you can write that your 3 by 2 then es base into is base. Now this 1 you can write 3 and 2 these two is there you can write es base upon root 2 into i s base upon root 2 this is equation 108 or I mean or you can write it is basically 3 into it is 3 phase volt ampere base for the stator this is basically erms and this is basically i rms we have taken earlier we have taken earlier.

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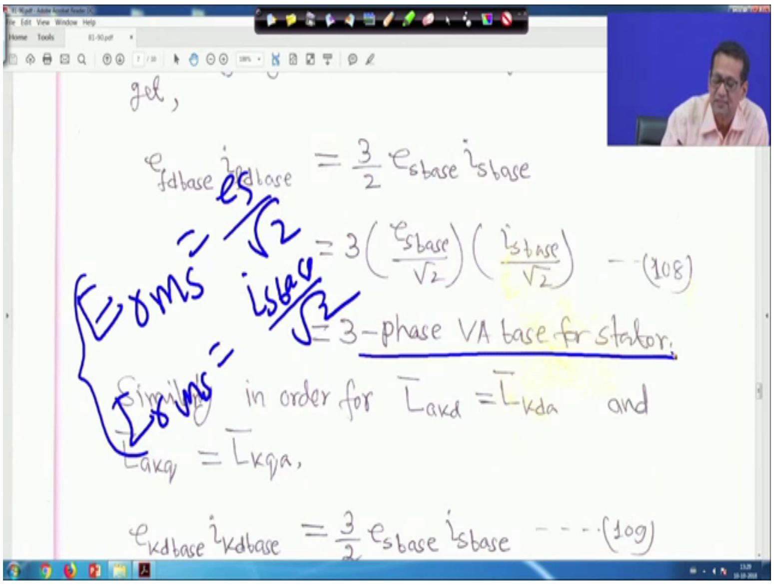

That your e rms is equal to es upon root 2 similarly i rms is equal to is base by root 2 right that is actually nothing, but 3 into e rms into I rms that is nothing, but 3 phase volt ampere base further stator right.

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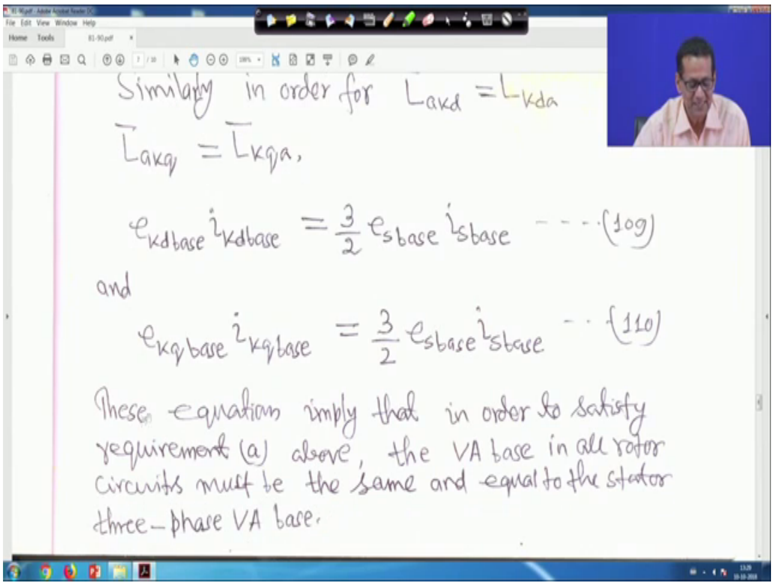

Similarly, we have to make this thing La kd bar is equal to L kd a bar and L a k a k q bar is equal to L kq a bar because you want everything should be in reciprocal right. So, the similar way you can make it ek d base if you equate if you equate this 1 and this 1 you will get ek d base i kd base go back to those referred equation and just equate it just equate it I did not write it here if you do it it will be e kd base i kd base is equal to 3 by 2 es base into i s base.

Similarly, ek ek q base you you are you have to call into i kq base is equal to 3 by 2 e s base i s base this is equation 110 right. So, these equations imply that in order to satisfy requirement a above the volt ampere base in all rotor circuits must be the same and equal to the stator three phase volt ampere base right. This we have to might be a quantities right I mean this side is the right hand side is the stator base quantities and left hand side is the rotor side right that have to be made equal.

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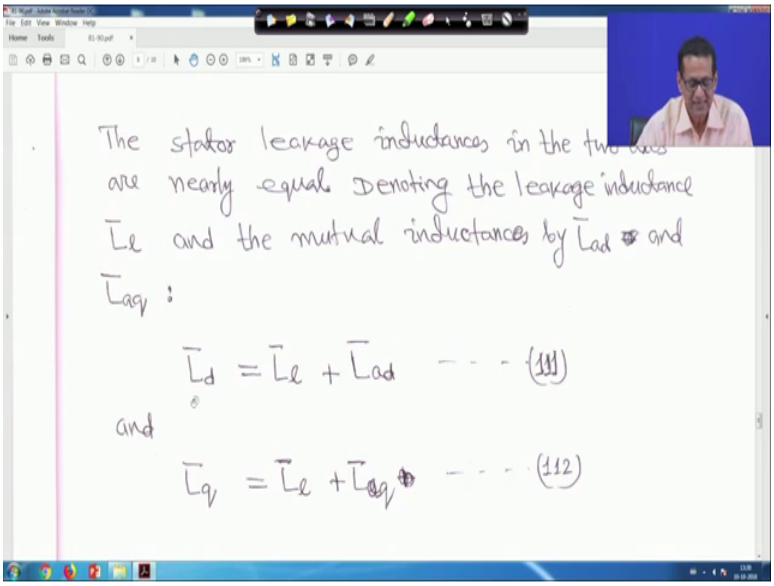

So, stator leakage inductances in the two axes are nearly equal right denoting the leakage inductance L L bar and the mutual inductances by L ad bar and laq bar right. Therefore, L d bar is equal to L L bar plus led bar directly you can write and L q bar is equal to L L bar plus L e q bar directly you can write this 111, this is 112 right.

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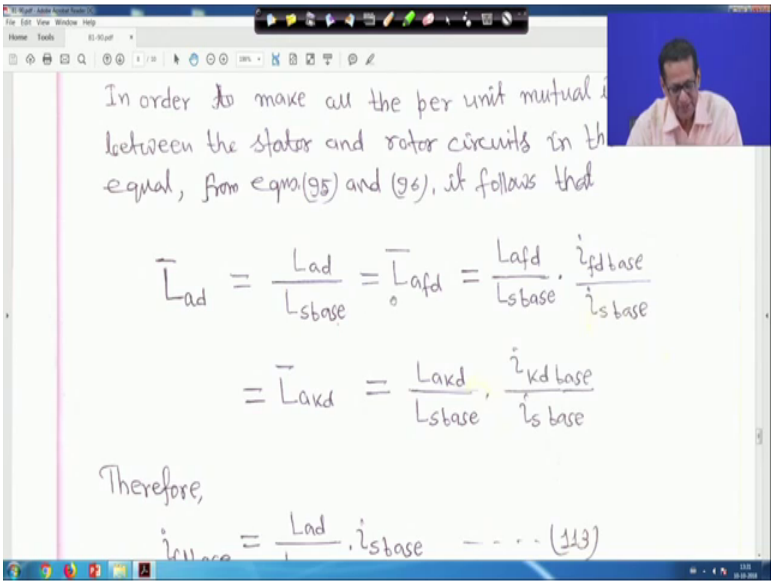

In order to make all the per unit mutual inductances between the stator and rotor circuit in the dx is your dx is equal from equation 95 and your 96. It follows that you have to make everything is equal because you have to make a primary thing either rotor side or stator side they should be equal. Therefore, L ad bar you can write L ad upon i s base is equal to you have to equate it that L fd bar that is also we have seen L fd upon i s base into i fd base upon i s base or this can be is equal to L akd bar is equal to L akd upon i s base into i kd base upon i s base right.

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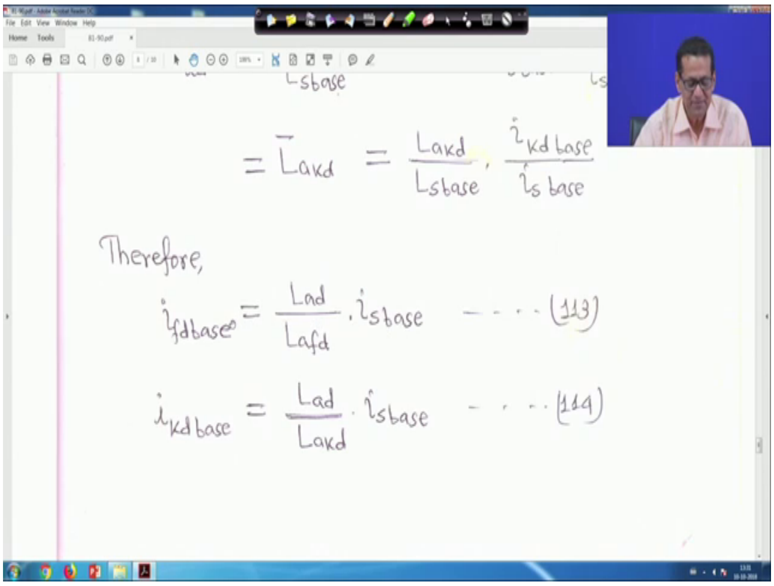

Similarly, your i fd base will be L ad upon L fd into i s base and similarly your i kd base from that L i kd base will be L ad upon L akd into i s base this is equation 114. Just my solutions to all of you when we will go through this course know all every week notes everything will be uploaded. But you please do it little bit of your own if you stuck somewhere you put the question in the forum immediately we will solve your problem right.

But nothing is there actually initially perhaps you may think that what things are, but just practice one or twice we will find actually everything is easy it is per unit system only per power system analysis course in your undergraduate course you have studied per unit. But in that synchronous machine case for dynamic stability studies this per unit representation is slightly different right, all the meaning is same.

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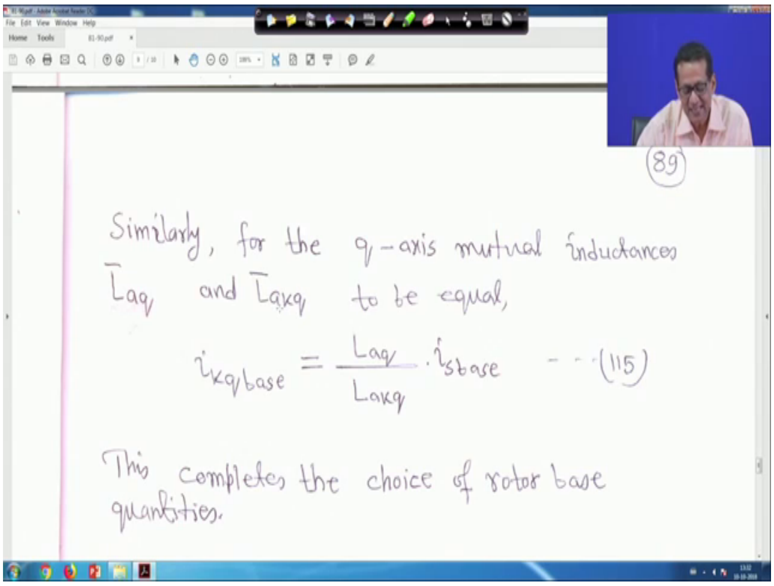

Similarly, for the q axis mutual inductances L aq bar and L a kq bar to be equal.

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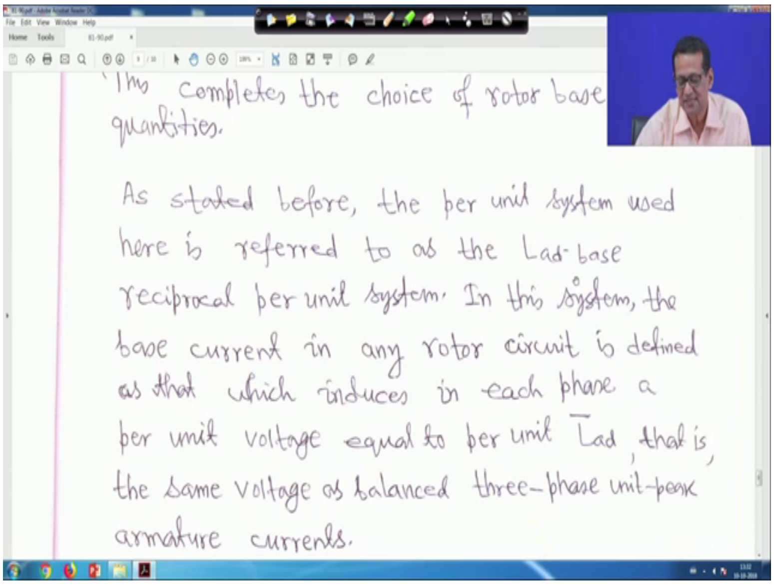

So, in that case we can write same as before i kq base is equal to L akq by a sorry L a q by L a kq into i s base this is equation 115 right. This completes that choice of the rotor base quantities right. So, as stated before the per unit system used here is referred to as the L ad base reciprocal per unit system in this system the base current in any rotor circuit is defined as that which induces in each phase a per unit voltage equal to per unit L ad bar that is the same voltage as balanced three phase unit your three phase unit peak armature currents right. So, this is what you call that is per unit system.

So, things are actually my experience show over the year; this per unit system initially student they have I will take I will take example also. Student may feel initially that things are difficult, but it is not at all difficult just to derive one or twice you will find actually the way you have done per unit analysis for power system courses in undergraduate studies it is the same thing, only the representation is slightly different right. Because of your Dq transformation right so you should practice. And if you have any problem then use this thing. I have over this over the years I have experienced on this initially is maybe little bit of problem, but after that when you will go through it will find things are quite easy right.

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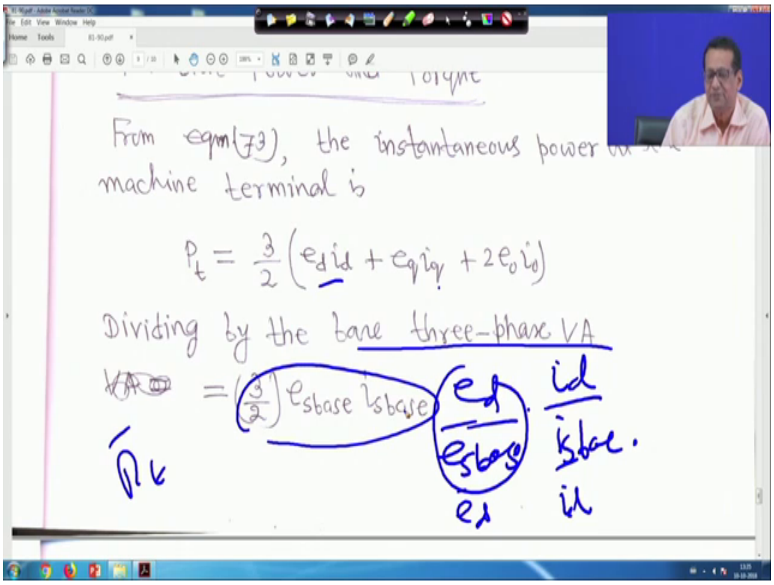

Now, per unit power and torque; here also that we have derived this equation from equation 73 that Pt is equal to 3 by 2 ed id plus eq iq plus 2 e 0 i 0. So, divine the dividing by the base three phase volt ampere both side. That is 3 by 2 es base is base if you do. So, it will be it both side you divide by 3 by 2 e s base. So, this side it will be per unit left hand side 3 by 2, 3 by 2 will be cancel right hand side then it will become e d upon es base into i d upon i s base eq upon es base into iq upon i s base it will be 2 into e 0 upon es base into your i 0 upon i s base.

So, this is actually writing Pt bar is equal to e d bar i d bar plus eq bar i q bar plus two e 0 bar i0 bar there in per unit I mean it is like this because we know this dividing by your because it is power. So, three phase volt ampere of both side if you divide this 1 left hand side right it will be your Pt part Pt it will become actually Pt bar that is Pt upon 3 by 2 e s base i s.

Now, right hand side if you do. So, 3 by 2, 3 by 2 will be cancel es base i s base the first term will be your e d by es base into your id upon i s base right. So, this term actually nothing, but your ed bar and this term is nothing, but your i d bar

Similarly, this term also eq upon es base will be es eq bar and iq upon i s base will be iq bar similarly 2 into e 0 upon es base will become e 0 bar and i0 upon i s base will become i0 bar we because we are representing things are in per unit.

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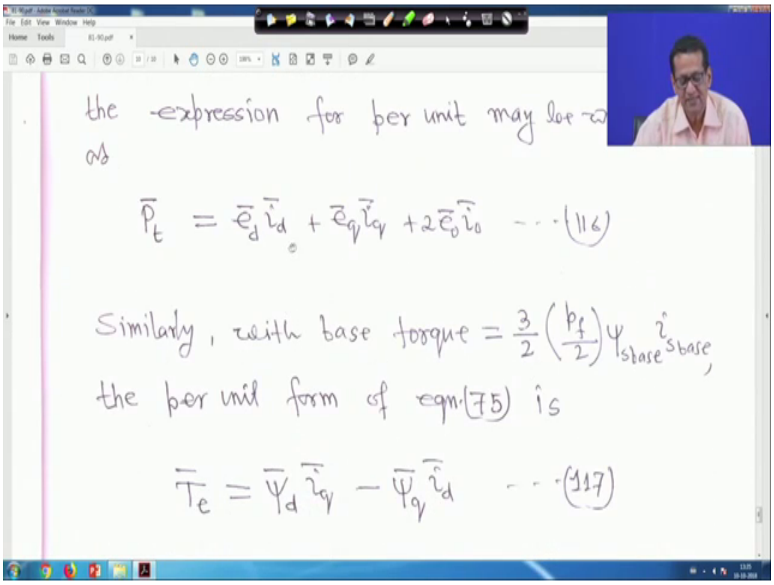

So, that is why this equation is written as your Pt bar is equal to only expression even same only things are per unit values right. And all numerical rather things from real value to per unit of transformation we will see and all numerical will do in per unit value right. So, Pt bar is equal to ed bar i d bar plus e q bar i q bar plus 2 e 0 i 0 this is 116.

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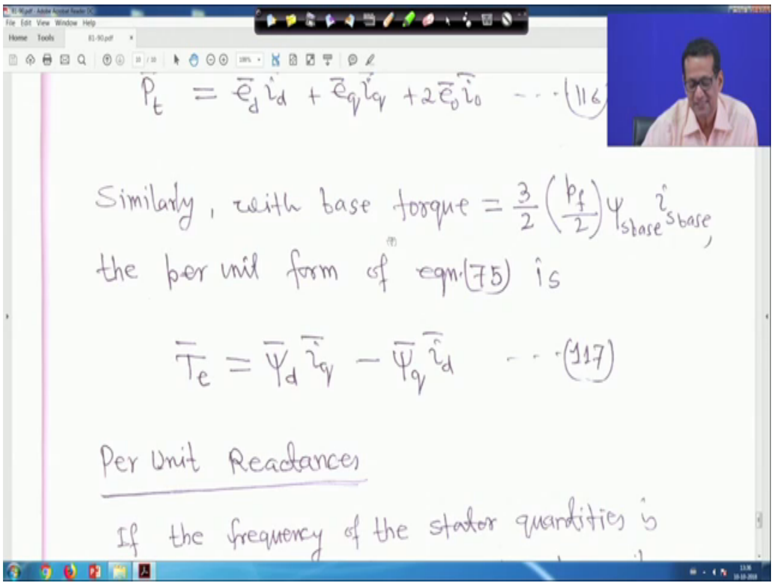

Similarly, with the base quantities with similarly with sorry base torque that is 3 by 2 p f by 2 psi s base i s base this also you have seen; so per unit form of equation 75. Now you divide that your equation 75 by this 1 you will simply get t bar will be psi d bar iq bar minus iq bar id bar same as before it will come psi d upon psi s base and it will become your iq upon i s base. So, all will bar similarly for psi q upon psi s base and this term will become id upon is base. So, all are bar bars per unit everything remains same all are per unit right and this is equation 117.

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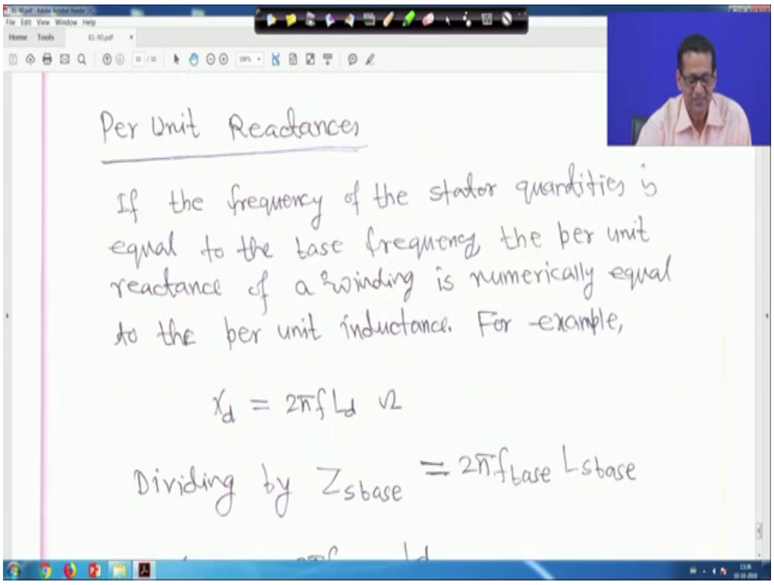

Now, per unit reactance if the frequency of the stator quantities is equal to the base frequency per unit reactance of winding is numerically equal to the per unit inductance right sometimes in the numerical. So, it will per unit reactance it can be taken as per unit inductance also for example, Xd is equal to say two pi f L d ohm right d by because dividing Zs base because Zs base is my impedance base and that is nothing but two pi f base into ls base right is omega is equal to your that is Zs base Zs base is nothing Z is equal to L omega. So, omega base will be 2 P i f base into L s base

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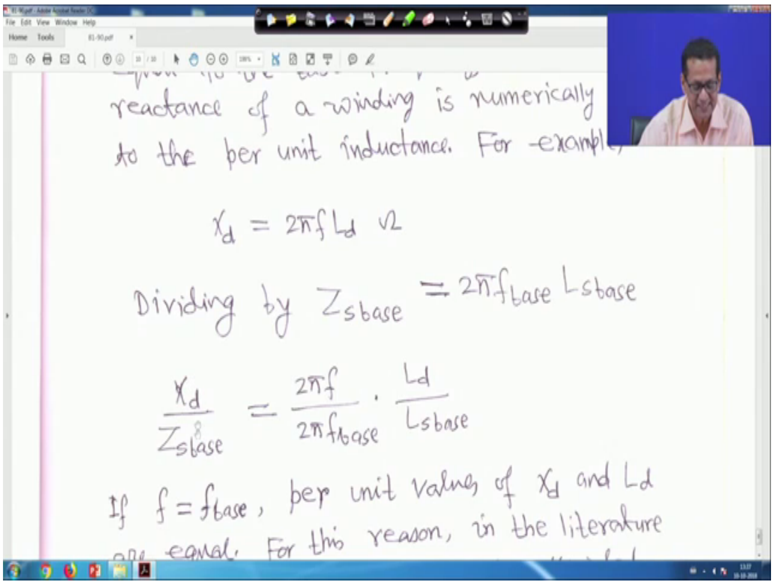

Now, if you divide both side Xd upon Zs base. So, it is 2 pi f upon 2 pi f base into Ld upon L s base right.

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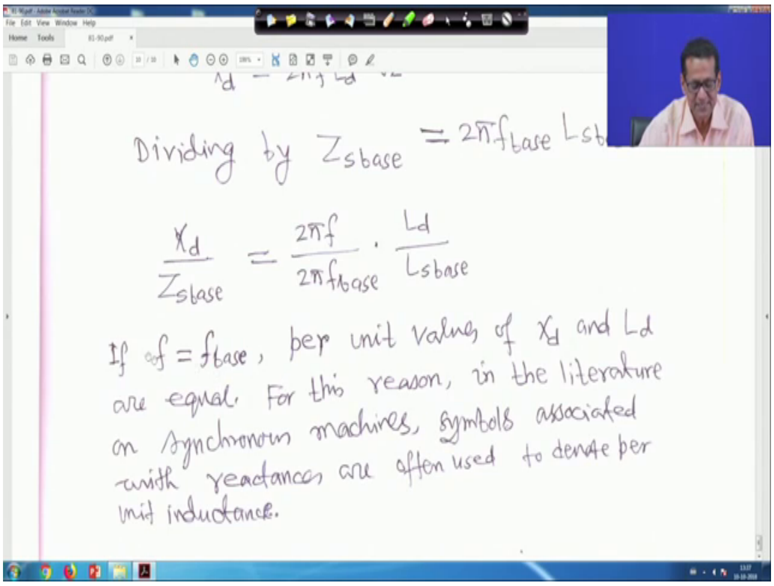

But f is equal to f base right per unit values of Xd and Ld actually are equal because f is equal to f base because f base here may synchronous machine rotating at a say your synchronous speed and f base is nothing. But if it is 60 hour it will be 60 hour if it is 50 hour it will be 50 hour. So, f is equal to f base. So, this term will be cancel two pi f is equal to f base and this is 2 pi f base. So, ultimate it will become Xd per unit is equal to

Ld per unit. So, in per unit whenever say per unit reactance means it is per unit your inductance also right.

So, so for this reason in the literature as your, and since what you call on synchronous machine symbols associated reactances are often used to denote per unit inductance right So, numerically if it is given per unit reactance when it is per unit inductance also if it is given per unit inductance means it is per unit reactance also right so right

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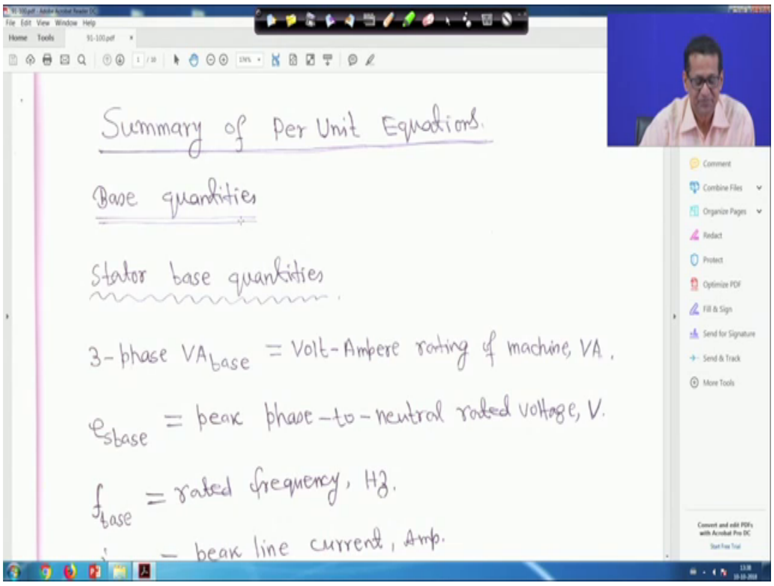

So with this, what you call this per unit this is now summary of this per units equation right. So, base quantities now stator base quantities three phase volt ampere base is equal to volt ampere rating of machine volt ampere base is equal to peak phase to neutral, what you call p phase your sorry peak phase to neutral rated voltage in volt.

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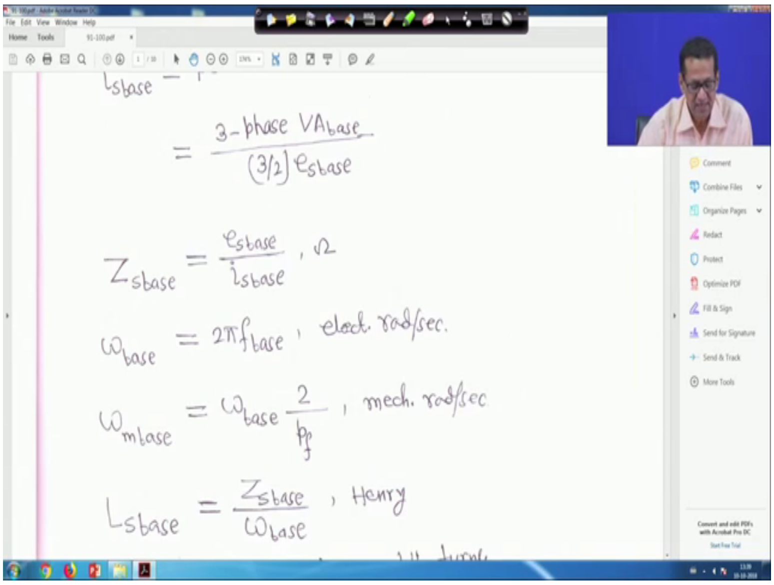

So, whatever we have studied the summary I have made a summary for you f base is equal to rated frequency in hertz i s base is equal to peak line current ampere that is 3 phase volt ampere base divided by 3 by 2 s base right. Similarly Zs base is equal to es base upon i s base ohm right. Omega base is equal to 2 pi f base that is electrical radian per second right.

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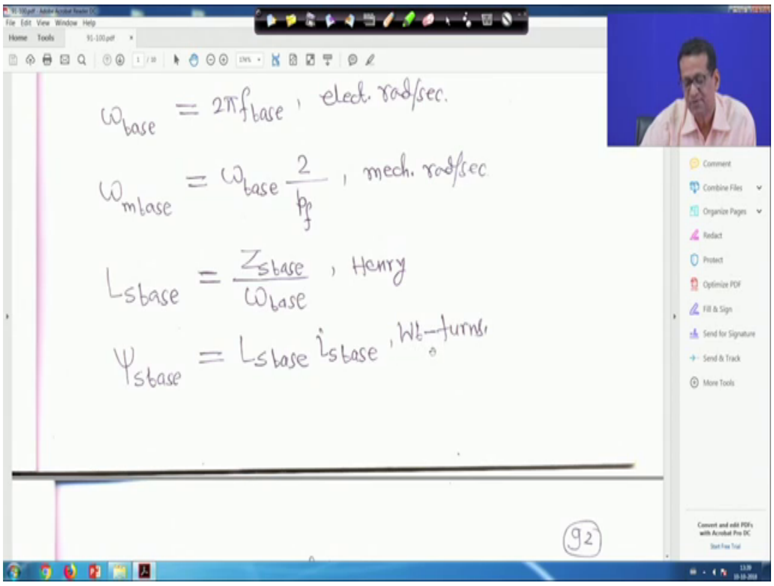

And omega m base is equal to omega base into 2 by pf that is mechanical radian per second. Then L s base will be Zs base upon omega base this is in Henry. And psi s base will be L s base into i s base that is turn that is y bar turns right.

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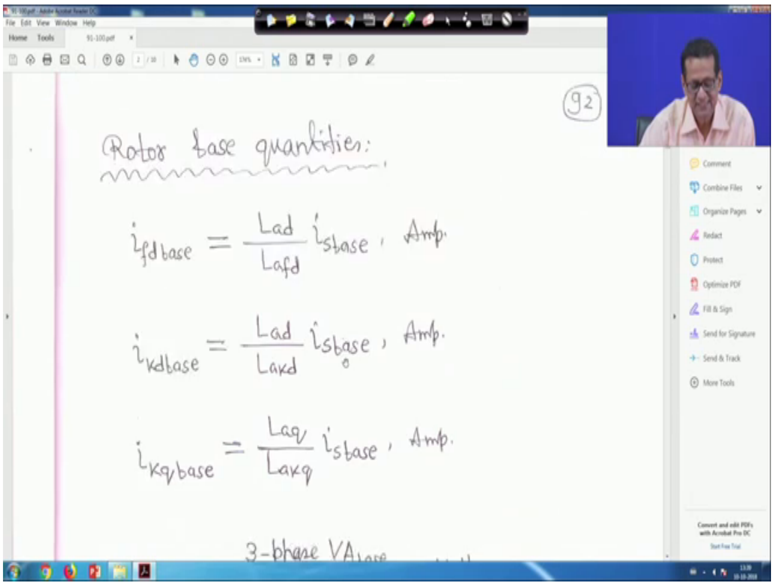

So, next rotor is base quantities this is a summary. So, ifd base will be L ad upon L fd into i s base ampere this will help you to solve numericals right. i kd base will be L ad upon L ak L akd is base ampere. ikq base will be L aq upon L akw L akq i s base ampere. Ef d base will be three phase volt ampere.

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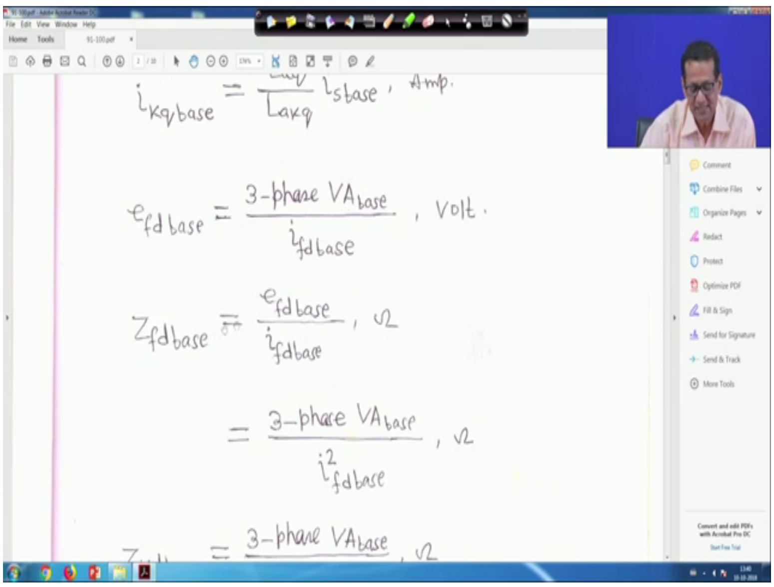

Based upon ifd base this is in volt right. z fd base is equal to ef d base upon ifd base this is ohm right. Is equal to three phase volt ampere divided by i fd square base right so, this is the summary.

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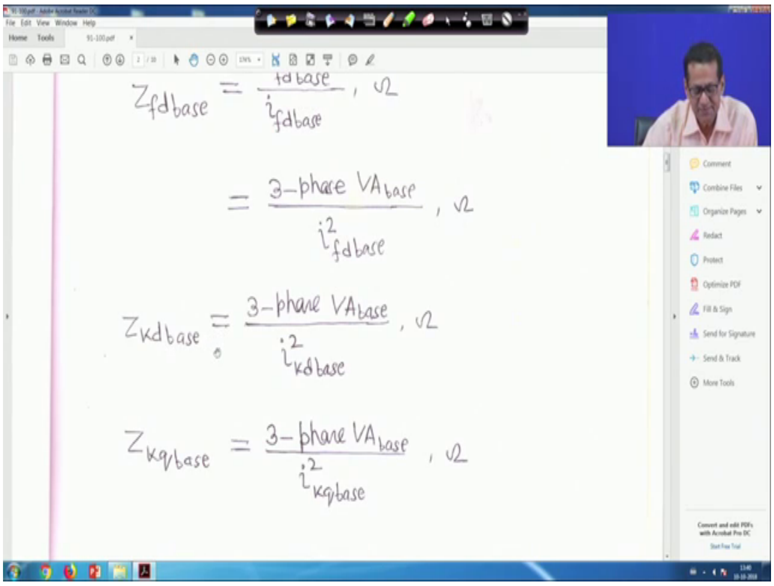

And zkd base is equal to three phase volt ampere base upon i kd square base this is ohm right. Z k q base is equal to three phase volt ampere base upon i kq square base. So, this is your ohm right.

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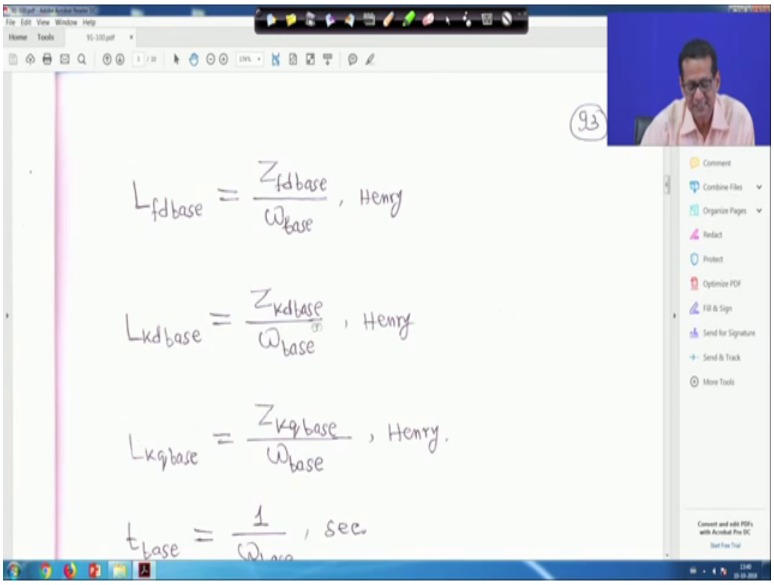

If d base is equal to Z fd base upon omega base this is Henry. Then L kd base is equal to Z kd base upon omega base, this is also Henry. L k q base is equal to z k q base upon omega base this is also Henry.

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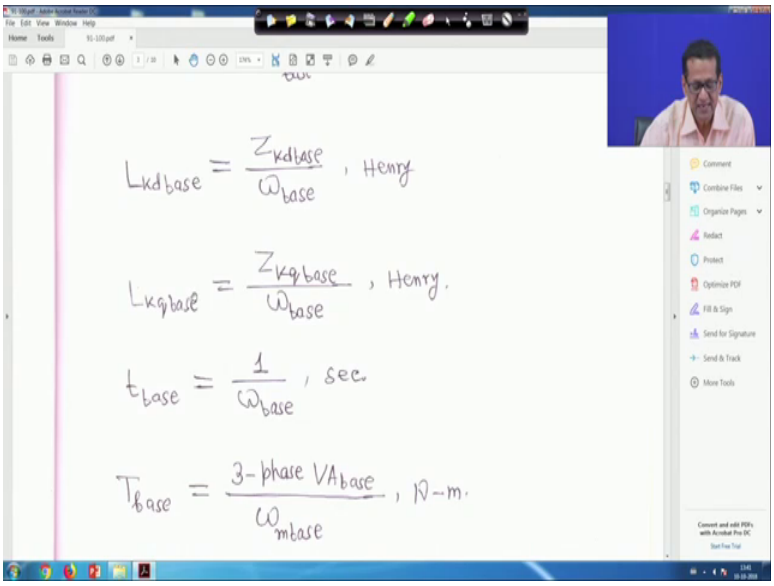

T base 1 upon omega base this is second right. And therefore, t base is equal to nothing three phase volt ampere at torque this is multi time this is torque base.

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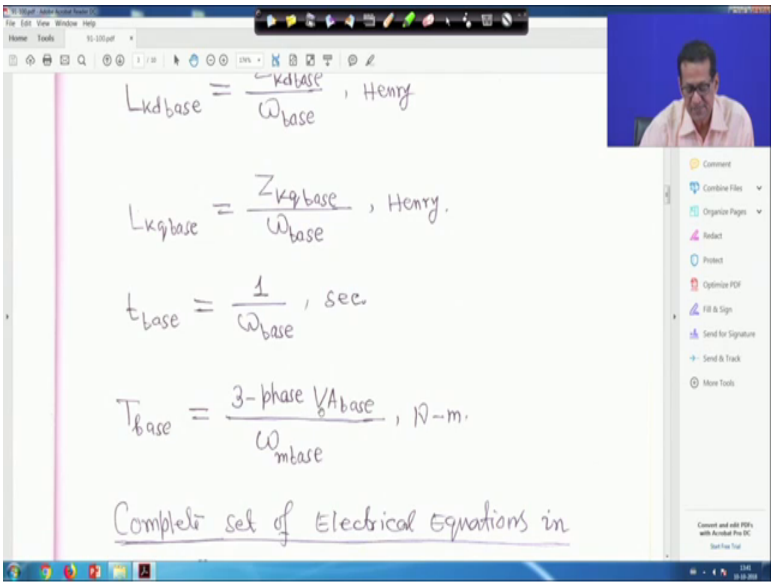

Is equal to three phase volt ampere base upon omega mechanical base, this is Newton meter right. Complete set of electrical equations will be seen the next lecture.

Thank you very much. We will be back again.