Evolution of Air Interface towards 5G

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Lecture – 12
Fundamental Framework for Waveform Analysis (Contd.)

Welcome to the lectures on Evolution of Air Interface towards 5G. Till the previous lectures we have seen a background of the way we have built up till the requirements of 5G.

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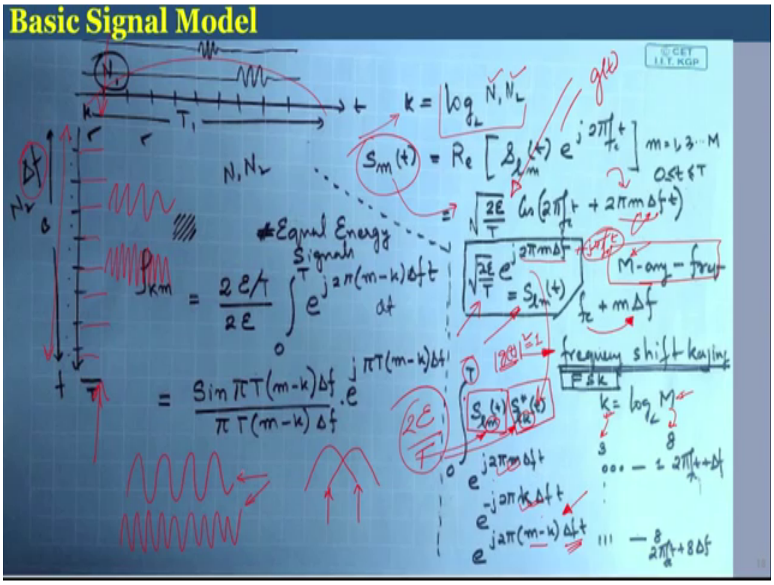

And now we have started discussing the signal model or the basic framework based on which we will discuss the first issue of discussion that is the air interface or the modulation or the waveform strategies. So, to do that, we have said in the previous lecture that we would look into the baseline model as well as look at the some of the earlier generation systems, because there lies the basis towards the future generation systems.

And we have developed the low pass equivalent as well as a band pass format of the signals, how they are connected. And we have looked at some of the very basic ways of modulating signals we have done it pretty fast assuming that you had some prior knowledge of it. It was just a reversionary approach. And thereafter we have moved to multi-dimensional signaling which forms the basis for waveforms even in imt advanced or the fourth generation system.

So, what we were discussing in the previous lecture is basically that you have a set of frequencies right, you have a set of frequencies, and in those set of frequencies you can choose one of the frequencies to indicate signal carrying information, and let there be a delta f be the separation between the frequencies. And along with that if you have time domain, so then you have basically N1 times N2 number of resource elements and log base 2 of that is the number of bits that are required to identify the selection of a basic resource element by which you are communicating information.

So, we move beyond that and we said that let us consider only one time slot that means, when there is multiple frequencies available to choose from, but for every time slot. So, in that case, we had also said that the amplitude remains constant that is one of the important features while we can choose the frequency by having a multiplicating factor along with the multiplicating factor along with the delta f which is the frequency of separation.

So, this kind of a signalling system is known as the M-ary fsk because you have M-ary pam. So, here you have M-arys electing M possible options and this particular signal which is in the pass band, you could find the equivalent low pass in terms of root 2 E by T and e to the power of 2 pi m delta f, because that is what appears over here and this, this form this one comes if we have j 2 pi fct, so this is due to the pass band conversion. We are left with only the baseband part. So, this is the low pass equivalent form of the corresponding signal.

So, what we see is that every fc, I mean for an f c, it gets modified by m delta f right. So, it could be plus minus around f c, and you could choose different frequencies and hence you are left with frequency shift key, this is what we said earlier. Now, if you have m possible frequencies log m base 2 that is the number of bits that would be communicated for every selection of a particular frequency. So, in the same manner, if we have 8 different frequencies, we will require three different bits to choose from them.

Now, amongst all possible frequency shift keying, we are looking and looking at and are particularly interested in one special form where we would like that the frequencies are orthogonal to each other. Now, if they are orthogonal to each other, then we have special advancement towards from the original frequency shift keying in the form that the frequencies can be placed next to each other in the closest possible way that means, the separation between the frequencies will be the smallest possible separation and although the signals will overlap still, we will be able to decode.

So, with this basic premise one would like to have the orthogonality orthogonal frequency spacing, and by definition of orthagonality. What you have is the slm that means one of the baseband equivalent frequencies or the signals along with slk right. So, l indicating the low pass equivalent, and m is the mth frequency, k is the kth frequency indicating two different signals that is the mth signal or the kth signal that means either I choose one particular frequency over here or I select another frequency over here, so it is between these two.

So, in the time interval T, which is the symbol duration. We would like to see that these two signals are orthogonal. So, like we have two pass band signals which we would like them to be orthogonal. So, if we work this out this is the part of slm, this is the part of slk, we directly get from here, and in this equation we are not putting this in the expression I mean if you even if you put it will be 2 E upon T, where E is because of the energy of the signal. And here we have taken equal energy, so we are not considering the scaling factor in this expression.

So, what we are left with is e to the power of j m minus k delta f and integrated over 0 to T. So, if you solve this particular thing, what we will get is that in order for this to be 0, we get a certain condition on delta f, which will lead us to the answer of what is the separation that can be allowed. Now, in this situation usually two different ways are looked at. So, when we are looking at the situation where it is only a selection of frequencies, then we can take only the real component and, we take the real part of this, and set it equal to 0. And then whatever we get is the condition for orthogonality of signals, when we are only selecting the frequencies.

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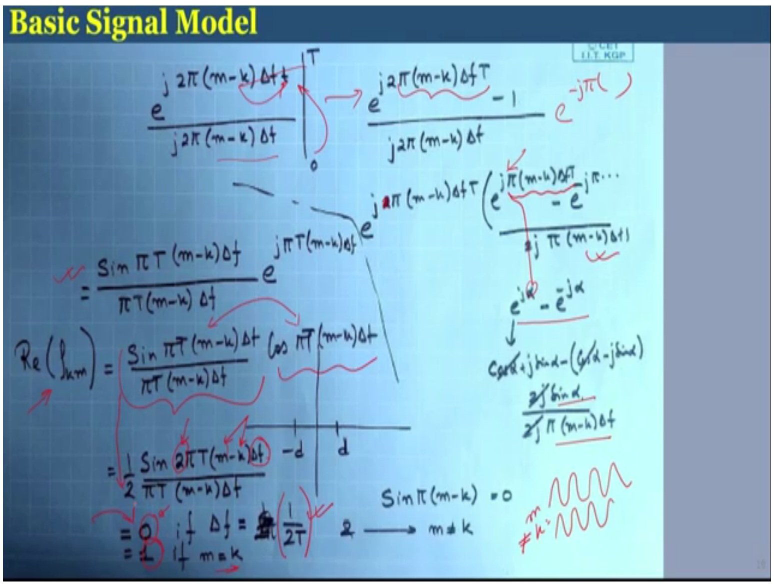

So, that is what we have set up over here, so you have e to the power of j expression divided by the term as a result of integrationintegrate over 0 to T. And substitute t into this, and substitute 0 into this, you are going to get the expression on the right, and then you solve it. Solve it in the sense that you take out away pi m k minus m minus k delta f T, which is over here this 2 is not here. And you are left with a term inside where from the first term you will be left with the pi, and the second term will obviously be minus j pi, and the rest of the term as it is, and the denominator remains as it is.

So, you have e to the power of j alpha e to the power of minus e to the power of minus j alpha roughly the expression or the expression turns out to be in terms of this where alpha is equal to this entire value. And if you work this out, it turns out to be sign of alpha whatever we have defined alpha upon this expression. So, basically in the denominator, you have alpha upon T, because if we consider T as part of alpha.

And then we take the real part of it, because we are we are concerned only with the selection of the signal and nothing beyond that. So, when you take this you take the real part of the product of the two expressions, so you are left with sinc kind of an expression over here as well as a cosine term. And when you solve this and set this equals to 0, basically you have a term over here and here.

So, sin a cos b if you multiply that, you get two sin sorry sin theta cos theta you are going to get two sin theta and half the normalizing factor because of this comes in the denominator, so you get a sin two times this expression. And if we set this equals to 0 to maintain the orthagonality, then what we get is delta f that means this particular term which is the delta f should be equal to 1 upon 2T.

So, if m is equal to k that means when we are having the signal, and that means m is this, and k is also equal to this the same signal. We get the value of one, whereas when m is not equal to k, we will get a value of 0, because by setting this equal to 0, we get this separation of frequencies. so it in turn it means that if you let delta f take the separation values of 1 by 2T in that case the if the signals are not the same signals, you are going to get at the receiver a value of 0, when we are using correlation receiver.

So, if we see the way we have developed this construct is that we have chosen a particular signal to be transmitted, and at the receiver we are correlating against the kth signal. So, this is kind of matched filter what we do not have over here, of course you can already consider what we have missed out over here is the g of t should come in so gt will come in over here, g conjugate t would come in over here, and that would result in the mod of g square t mod of gt squared. So, mod of gt squared being a scalar number, so that would appear in each of the multiplicating factors.

So, in this particular expression we have taken gt to be rectangular that means, it is 1 in the interval 0 to T, you could also have it normalized, so that means we are as if saying that one particular signal is sent out of the different frequencies. And at the receiver you are correlating with some other signal as a result of typical mash filter operation. So if the signal is not same as the original signal that is if it is not matched to the signal, then what we get is a 0 right.

Whereas, if the signals are matched to each other the same signal, then you are going to get a 1. Indicating that only one of these will give a value of 1 at the end of the match filtering operation, and others are going to give a 0 right by virtue of orthogonality condition. So, if we set delta; delta f is equal to 1 by T which is by orthogonality condition, although the spectrum will overlap still we will be able to distinguish between the different frequencies by virtue of orthogonality criteria ok. So, this is the prime way of developing a orthogonal M-ary FSK, which is the basis for the next set of discussions that we are going into.

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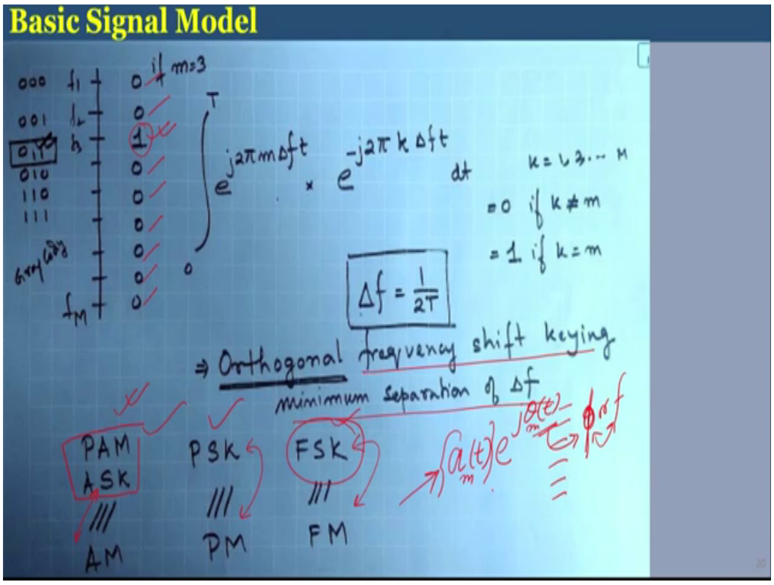

So, effectively this is what we have roughly explained that we would choose one particular frequency and if this matches, then the result is 1 else the result is 0. And what we have is orthogonal frequency shift keying with a minimum separation of delta f right. So, in digital as you all know this PAM or Amplitude Shift Keying is kind of equivalent or correspondingly, you can map to amplitude modulation for analog communications, PSK can map to phase modulation, FSK can map to frequency modulation, but these are completely digital. And we have seen one way of doing FSK with orthogonal frequencies, and with minimum separation ok.

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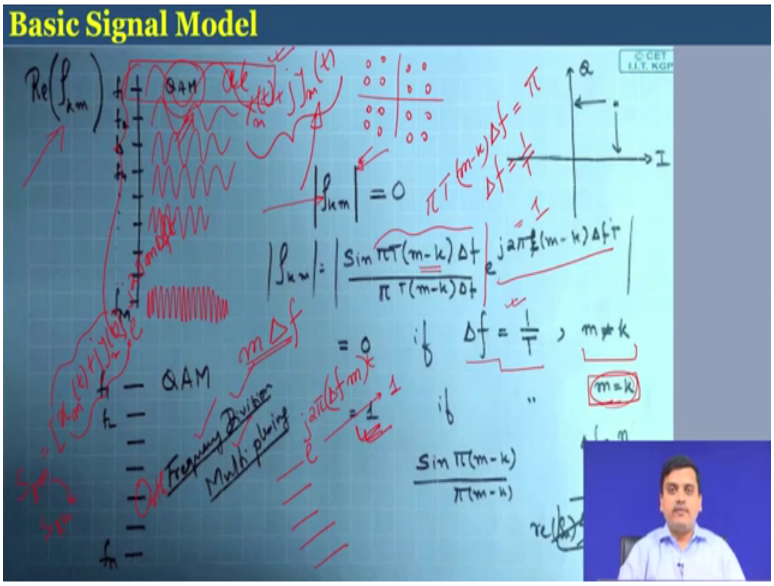

So, moving down forward, so there are if instead you are taking QAM kind of signaling, so this is a slight a different slightly different discussion from what we had before. So, what we discussed in the previous section that means just a few minutes back is that we are selecting one of the frequencies, and communicating the choice of selection whereas, one could also think of instead of selecting, since these frequencies are all orthogonal, since these frequencies are orthogonal to each other. So, at the receiver side or at the transmitter side, I can probably send them simultaneously. And each of the frequencies can be used to carry some data either via pulse amplitude modulation or via quadrature amplitude modulation.

Now, why we say this, because as we know if we have e to the power of a t e to the power of j theta m t, so theta m can be chosen by virtue of phase or by virtue of frequency rather by virtue of phase or by frequency and if we do both, then we will not be able to distinguish the information bearing signal whereas, if we are only handling this amplitude, and we use these phases or these frequencies which somehow separate the signals or which forms as the basis for the signal, then we are still able to decode the signal.

So, again since I have used the word basis, we can also go back. And see that all these different frequencies that we have used over here are can be thought of as basis functions, and they are orthogonal to each other, so these are orthogonal basis. So, now since they form the orthogonal basis instead of just selecting them, because their virtual orthogonal. We can actually use them to send data, and we have n number of orthogonal basis functions.

So, we can actually use n dimensional signaling and not only that we can go beyond that in each of the dimensions, we can go for amplitude modulation or a PAM pulse amplitude modulation or we can also go for quadrature amplitude modulation. To this will be the basis for the orthogonal frequency division multiplexing, which we will see in more details later on, but this particular discussion is the foundation for whatever we take up at a later stage.

So, now if one is using QAM right Quadrature Amplitude Modulation that means, we are going to have a we are going to have let us say X of t plus j y m of t, which is the low pass equivalent of the signal or which carries the information bearing signal. And they are chosen from the, or QAM constellation, so this is the QAM constellation.

So, in that case to find the orthogonality, we cannot use the real part of the orthogonal projection. So, when we are taking the orthogonality criteria, simply the real part is not going to help us, because there is a imaginary part which carries information. So, in that case once we find the correlation, you can say or the inner product of the two functions. We would like to set the modulus of it or the absolute value of it to 0, in order to get the points or the condition under which orthogonality remains.

So, here what we had seen is the rho k m is an expression, which is given by this. Now, instead of taking real part, if we take the mod, because we are not interested only in selection but also in the real and imaginary part of the data, we cannot simply use the real we have to let the real and imaginary both go to 0. So, if you have to let the real and imaginary both go to 0, then we can set the modulus of the same to go towards 0.And you can have the rho k m expression, which was shown in the previous slide the whole thing the absolute value of that going to 0.

So, we clearly see that this expression will go to 0, if this numerator goes to 0. And this numerator can go to 0, when delta f is equal to 1 by T, and m is not equal to k correct. Whereas, even if delta f is equal to 1 by T, but m is equal to k we are going to get a value of 1. Now, this is very very important. And we can also see from the point of view that if you take the modulus of this, you are going to be left with this entire term. So, if I take the modulus, since this is e to the power of j, it goes out we are left with the modulus of this and the modulus of this is equal to one effectively. So, it is a product of the two.

And what we see is that this term is a sinc expression, and this sinc expression is equal to 1, for the argument of sinc being equal to 0 right. An argument of seeing can go to 0, if m is equal to k correct. So, under m is equal to k condition, this will give us 1. And under the condition that m is not equal to k, if we set the additional condition that delta f is equal to 1 by T or in other words what we see inside over here is pi T m minus k delta f being equal to some integer multiple of pi.

In that case, we clearly get and the minimum separation is m minus k equals to 1, we clearly would get delta f is equal to 1 by T as the condition under which this would go to 0. So, again if we try to realize how this is working at the receiver end is that all these signals they are going simultaneously right, and as your frequency as your m increases, since we have m times delta f. Since, we have m times delta f, we have pictorially represented as higher and higher frequencies.

So, if at the receiver we are correlating with m is equal to k that means, if we are related correlating with this, we are going to get a peak value for this, whereas others will be 0. So, in other words the receiver even though it gets all the signals in this case, we have to have m number of correlators, each correlator will be correlating with e to the power of j 2 pi delta f times m times t ok. And hence each of them is going to produce a magnitude of 1, because our correlation expression we have taken as slmt, slkt.

But, now we would also have this additional x m plus jy m term along with that that means our s l m should have x m of t plus j y m of t multiplied by e to the power of j 2 pi m delta f t right, so that should be the term for slk. All the m terms get replaced by k. And what you are left with is this particular term at the receiver. So, this term you can now decode or d map using typical QAM demodulation procedure.

So, now if we observe, what we are doing carefully. We are actually sending all the signal simultaneously that is the additional discussion that we had in the previous few minutes. Now, if you are sending all the signals simultaneously, so you are kind of using all the frequencies. And you are sending the signal using frequency division multiplexing, but in this case it is orthogonal frequency division multiplexing right.

So, what we see is that from the same basic framework, we are able to look at FSK as well as OFDM. Whereas, FSK is variant of this particular orthogonal, FSK is what is used in very earlier generation that is the second generation, which we will see very soon. And the other format, which has the same background or the same fundamental basis is used for the next generation system, so that is why whatever we have analyzed over here forms the foundation for whatever we are going to discuss in the future lectures ok.

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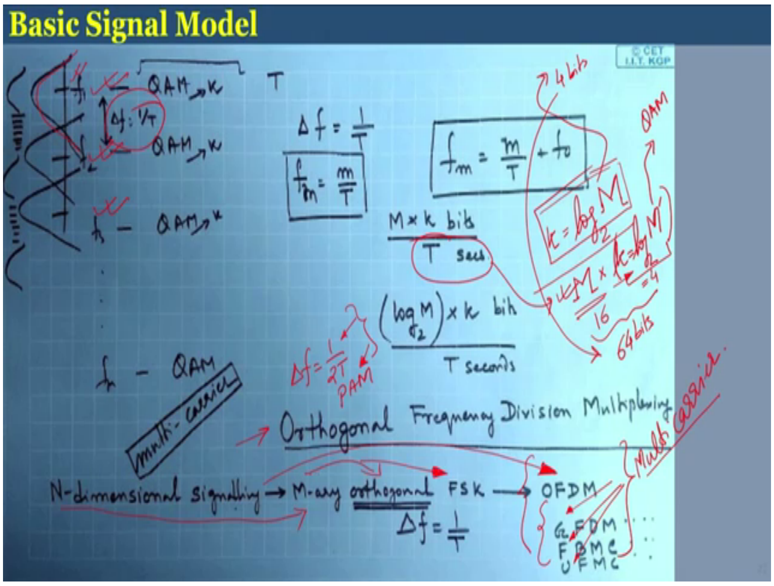

So, now at the receiver side as we have clearly said, we are going to do this operation. And although these frequencies are overlapping right, still because they are orthogonal to each other by virtue of orthogonality criteria, what we are seeing is that they can be decoded without any interference right. So, this is orthogonal frequency divisions multiplexing, which have just explained in the previous few minutes.

So, now what we see is that at any one interval that is T period earlier, when we are doing FSK, we were sending k bits which were equal to log base 2 of N or log base 2 of M, which was the number of carrier or the sub-carriers that we had. Whereas, here what we have is at every time instant T, you are sending M number of parallel signals. And each parallel signal is going to carry k bits, which is equal to log base 2 of some M prime which is in this discussion by virtue of QAM signaling.

So, if we do 16 QAM in that case, this value would be equal to 4. And if we have let us say 16 carriers, then this value is 16, so together what we have is 64 bits being sent simultaneously. Whereas if we compare the earlier system of FSK, in that case if M was 16, then k would be only 4 bits.

So, what we see there is a huge improvement in data rate that is possible that means, because here you are simply selecting one possibility of the M possibilities. Here you are letting all the M possibility is getting used, and in each of the parallel signaling, you are allowing some kind of modulation to go along. And mostly it will be the amplitude domain modulation. And we are using QAM, because it is very compact. And you can send higher number of bits in that, so this is the way.

Of course, I should mention that instead of sending QAM, you are also free to go for some real signaling like pulse amplitude modulation. In that case, what is going to happen is your separation in frequencies delta f will be as per our earlier discussion that it will be 1 by 2 T. And in each case this M prime, which we said am you are going to use a PAM signaling over there right, so that means although we are going for a different lower spectrally efficient signaling format. But, since we are using a narrower bandwidth, so we are packing a larger number of carriers in that situation. And hence we are not losing any spectral efficiency in this particular case as well right.

So, going ahead further, what we see is that N-dimensional signaling a framework that we have studied gives rise to the M-ary orthogonal FSK, which is a basis for discussion of waveforms for the second generation system as well as it forms the basis for discussion of OFDM, which is again a fundamental waveform for the fourth generation system not only that based on this signaling structure. The other signaling structures can also be studied, which were also investigated as a forerunner towards 5G.

And they are known as the generalized frequency division multiplexing, which we will see in due time also filter bank multi carrier, which will also see in due time as well as unified-filter, unified-frequency multi-carrier, unified-filtered multi-carrier systems which also we will see in due time, so that means we have looked at a fundamental framework on which a large set of waveforms are standing in today's context. And these set of waveforms which we have listed down over here, and we will see in details later on a form the basis of multi-carrier waveforms ok.

So, when we say multi carrier, what we mean is that you are actually sending all these carriers simultaneously while maintaining the orthogonality criteria that they are although overlapping, they have 0 correlations amongst them. So, this is the basis for multi-carrier signaling, and in fact in some form of 3G there are multi-carrier CDMA was also proposed. Whereas, how do you place this multiple carriers was also investigated in quite deep details, there are a huge amount of literature available in terms of multi-carrier signaling.

So, what we see is that we have looked into the premise or the framework based on which the 2nd generation even some forms of 3rd generation, 4th generation as well as the next generation waveform systems can be studied. We conclude this discussion over here we will take up the next set of discussions starting with the second generations system using this framework from the next lecture onwards.

Thank you.