Evolution of Air Interface Towards 5G
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Lecture - 17
Waveform for 4G & 5G (OFDM) (Contd.)

Welcome to the lectures on Evolution of Air Interface Towards 5G. So, we are at present discussing the Waveforms for the 4th generation as well as which is applicable to beyond 4th generation that is the 5G the foundation for the wave form for 5G and which is essentially OFDM. So, we have discussed some parts of OFDM in the previous lecture; let us continue with the discussions on OFDM and see more details about its futures properties and structures how the transmitter signal flows what happens at the receiver in this particular lecture.

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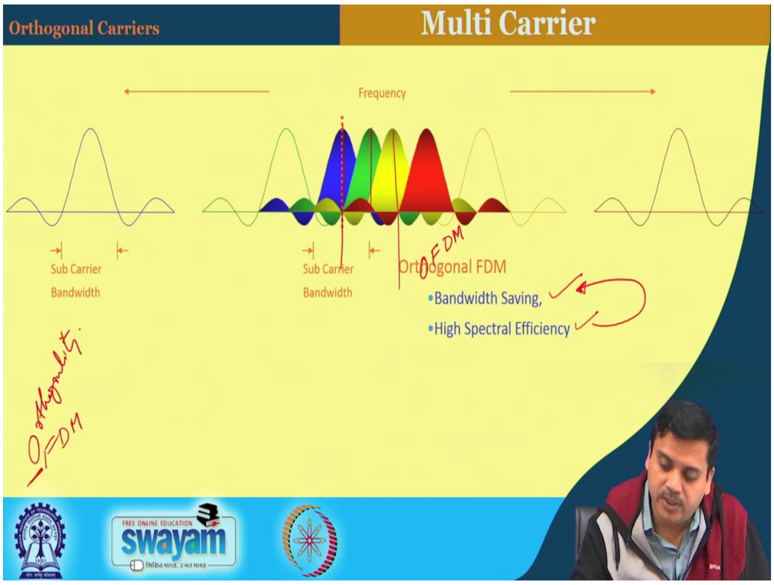

So, in the previous lecture we were discussing about the spectral efficiency that OFDM brings with this particular animation where we saw that it can give us a wide a huge increase in spectral efficiency. So, clearly there is a band width saving and high spectral efficiency, in other words I mean you can also think of high spectral efficiency would result in band width saving. So, it can be read in that manner as well alright.

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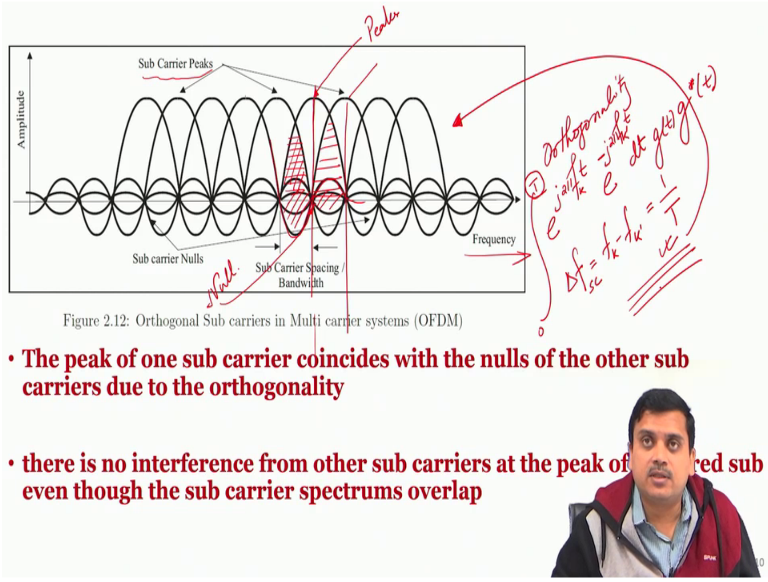

So, now we see the next picture which is about the orthogonality again because, in OFDM orthogonality is main thing and our interest is to visualize, picturize the things. So, that we can even create further things in future so, what we are trying to see is that in the time domain how does it look like. So, what we have over here are 3 wave forms and basically we have 3 different carriers first second and the third. So, they are essentially e to the power of j 2 pi f kt and rather the real part of it they would be the imaginary part as well. So, we have real and imaginary and which is seeing only one of the parts. So, this is one of the carriers, this is the second carrier, this is the third carrier.

So, the first one we see in this snapshot there is only one fundamental part of the frequency, the second one which is this black one we see that there are 2 such cycles ok. So, it is an integer multiple of the first one that is what we are seeing that it is an integer multiple of the first one and the we have the third one that we are seeing there are 4 cycles 1 2 3 and 4, but you can have any other. So, primarily what we are seeing is that there are integer multiples of the fundamental frequency and if we do the integrands multiply each one of them, if I multiply this with this throughout and add them up you will find that the area under the curve goes to 0 and that is that is that is going to happen.

And one of the reasons that is happening which is kind of implicit in this picture is along with these things there is a g of t which we did not write in the previous slides when we are discussing, but it was already present when we are discussing the fundamental base line signal model of mrefsk and we discussed orthogonal mrefsk, then we discussed the OFDM structure in quite a few lectures back where we said that let this be the foundation on which we will build.

And we had XK in each of the frequencies so; that means, each of this carriers are going to carry a certain X of K which will be an amplitude and a phase term. So, phase term is going to simply the chase the starting point and the amplitude factor will simply change the amplitude, what we see over here is that all then have the same amplitude. So, this could indicate choice of using PSK kind of structure of QPSK kind of structure that could be imagined to be associated with this particular wave form ok. So, so essentially what it means is that if we have integer number of cycles of the fundamental frequency then they would be cancelling out each other when you are processing at the receiver side and one would get orthogonality and this is the corresponding time down picture of whatever we have seen in the frequency domain.

So, this gt that is what you were seeing this gt if it is rectangular this is very very important. Rectangular means that it is constant over this entire period then only we get the result of orthogonality which we have discussed earlier this result ok, because you can clearly see in this orthogonalitic picture equation we should have had a g t along with a g conjugate of t right. Now if these equations or if these expressions were kind of let us say a raised cosine then the condition of orthogonality would be different.

So, this is a sum important things which we should keep in mind that has been implicitly present in this and we may not have mentioned this explicitly, but that is what we should remember. Now, if we change these gt orthogonality criteria brakes down within this 1 by t criteria and it will be some other condition on which we get orthogonality or it will depend on the choice of gt as well. And when we said that we will get some opportunity to discuss some of the futuristic wave forms people have already worked on them one of the fundamental things that people have been thinking or spending their time upon is the pulse shape or gt along with this multicarrier.

So, these two things together have been investigated will get an opportunity later on to look at them. So, primarily this integer number of cycles of each sub carrier is present which is giving the orthogonality and this is how one should look at it when one sees the time domain picture.

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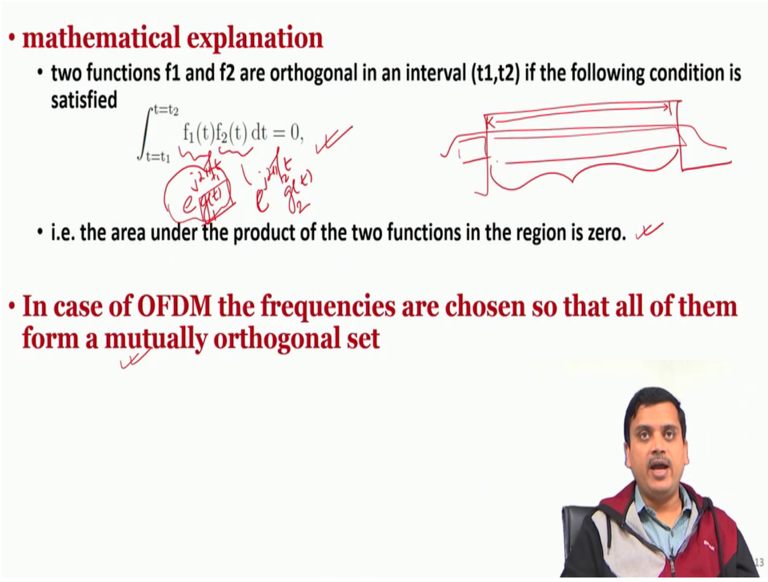

So, this orthogonality expression we have already discussed this where here this f 1 2 and f 2 t are chosen in a manner that they are the basis functions that is what we have said; that means, each one of them is e to the power of j 2 pi let us say f 1 of t and this is let us say e to the power of j 2 pi f to t right. And if one would like to bring in g of t over here and g of t over here then one would find the orthogonality criteria using the entire gt as well. And one would think of using a certain gt g 1 of t over here a certain g 2 of t of in the second one which is the more generic way of doing things and we will see some of the outcomes at a later stage, but which is not relevant for 4 G or 5 G, but potentially important for the future generation works.

So, yeah I mean as we have discussed that the area under the region would be 0, if they are different and if they are the projection of one or only if they are equal to each other then will be non-zero right. So, this also we have discussed that in OFDM the frequencies are chosen. So, that they are mutually orthogonal, but I would always remind you to recall that we have chosen gt to be 1 in all the expressions then only we have got this criteria. Otherwise thus the result changes, this remains constant for all the OFDM all discussions that we think of OFDM would be valid only with gt is equal to rectangular or not changing within the period of interest.

Now one may think of a question about that how would one think of a windowing we will discuss windowing some people already know about windowing at this stage. So, there is a cyclic prefix which will also discuss. So, over the period of interest is the short discussion before we go into finally, there in the period of interest where we are actually doing the processing you will find that this pulse amplitude remains constant. So, if we are interested in this particular section over that period we can assume a rectangular shape and the spectrum that we have considered to be sink in all the discussion that we have had is primarily because of have been considered a rectangular pulse shape, all right moving ahead.

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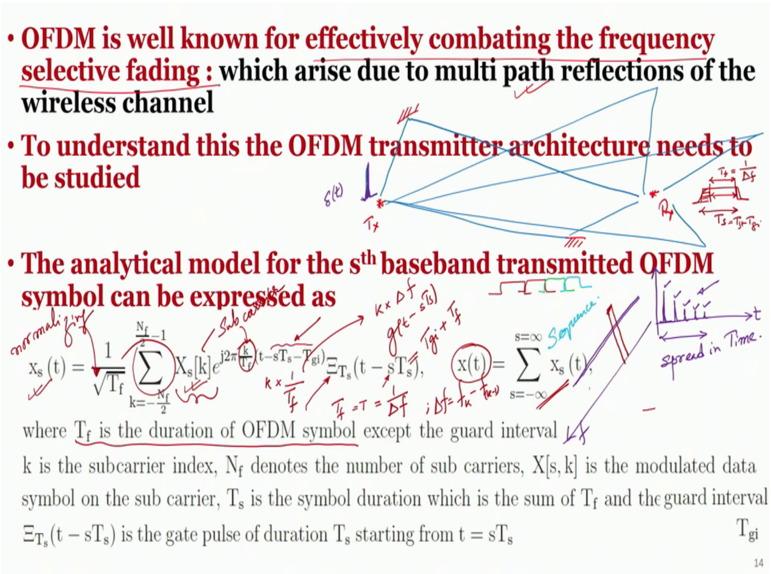

So, some other important aspects of OFDM why it is used for wireless communication is that it is well known for effectively combating the frequency selective fading, now this is very very critical and frequency selective fading which occurs due to multi path reflection. So, we have discussed earlier when we were taking about the rate receiver that if there is transmitter and a receiver. So, let us say this is the transmitter and this is the receiver and there are reflectors all around the place.

So, the signal from the transmitter the receiver would go via multiple paths right, it will come via multiple paths and if you send an impulse from the transmitter. So, let us take any other color. So, if I would send an impulse from the transmitter which is kind of delta t what we will find at the receiver is if we draw this as the timeline and this as the received signal power or amplitude. The first path we have discussed this thing, but we are doing it in short is kind of arriving after certain propagation delay corresponding of the delay of the 1st path then of the 2nd path 3rd path 4th path and so on and so forth.

So effectively the impulse gets spreaded in time right. So, when things get spread in time things in the frequency domain would get restricted or shorter. So, if you look at the Fourier transform of the delta function is kind of flat in frequency flat means it is a constant value across entire range of frequencies. But if you take Fourier transform of such a sequence and of course, there will be a phase associated with each of this pulses we will find that the frequency response in the frequency axis will not be flat this is flat in case of delta function a Fourier transform of the flat, but here what you will find is that the frequency response would be fluctuating something like this will see this pictures.

So, in OFDM we will again have a picture of that that when we are seeing this narrow sub carriers there is the design issue involved one usually chose it in such a manner that the band width of the carrier is such that it experiences near flat fading. So, that is what it meant that it is kind of very resilient to frequency selective fading. So, that is in short about the discussion we will see again later on.

So, one is to understand the propagation characteristics very well in order to understand what the frequency selectivity and the multipath and how does OFDM take care of it. So, if its experiences a flat; that means, nearly flat frequency response equalization is easier. So, that is again a huge advantage for wireless communications because already you have extra complexities this is going to only help reduce the complexity that is to be addressed.

So, this way we will proceed towards studying the architecture of the transmitter now that is one of the objectives, but before we get into that it is important to look at the signal model which will again help us in realizing the transmitter architecture. So, we have already written this kind of a expression before. So, X s of k, k is the sub carrier index this is the sub carrier index which you have been writing again which is being reflected over here. So, what you see in this particular part is that you have k multiplied by 1 upon T f and Tf is the same as T as we have used on the previous slides ok.

So, wherever you use T its actually Tf, but T f is the duration of the OFDM symbol right and Tf is equal to T whatever you have done is equal to 1 upon delta f and delta f is equal to f k minus f k minus 1. So, that is that completes the definition and what you see is that k multiplied by this is effectively this expression is basically k multiplied by delta f that is these frequency separation. So, if k is equal to 0 you have the DC carrier if k is equal to 1 you have the first fundamental frequency and so on all right.

And all of them are appearing simultaneously so, therefore, there is a summation and there is a normalizing factor this normalizing factor one can either consider or one need not consider this it depends upon ones choice because, this is not going to matter finally, except in mathematical notation. Because, if you are actually realizing a system you would usually normalize the power to a certain desired value and you will send it out only for mathematical convenience this kind of normalization factor is used. So, that one can get the normalized power of X s of t as 1, the X capital Xs of t which is the constellation can also be normalized to a unit power. So, this normalization is something which depends upon your actual realization and if you doing simulations otherwise it is kind of all right.

So, these are all things that have been described over here amongst this one should pay attention to this function which is the gating pulse or our g of t minus sTs this is basically the gating pulse s is the OFDM symbol index and Ts is equal to T g i which is the guard interval which will talk about plus Tf ok. So, we have over here certain other offset values. So, these offset values are present in order to create the signal in an appropriate manner, we are not going into the exact details of opening of the equation, but I cam leave it as an exercise you to work out the details of it and understand what is the meaning of this we will see that X t which is the summation over Xs its a summation over X of s ok.

So, what is the meaning of this in this particular expression, we see that gt is essentially an expression which is like this; that means, it is rectangular over a period of Ts ok. And Ts we said is equal to Tf plus T g i so; that means, if we say that this duration is the duration of Tf which is equal to 1 upon delta f right Ts is actually larger than that and the extra portion is the cyclic prefix, now one can easily see that we can have some shaping over here and some shaping at the end without effecting the rectangularness in within the region of a consideration all right.

So, we will discuss again some more things there. So, this essentially helps us in writing this notation over here whereby we have like the 1st OFDM symbols pulse shaped, the

2nd OFDM symbols pulse shape and then the 3rd OFDM symbols pulse shape, then the 4th OFDM symbols pulse shape and so on, so these are all orthogonal to each other in that sense. So, although it appears like a summation, but it actually a sequence; so, these are again some things of notation which one should remember to use appropriately so, that a things do not get mixed up.

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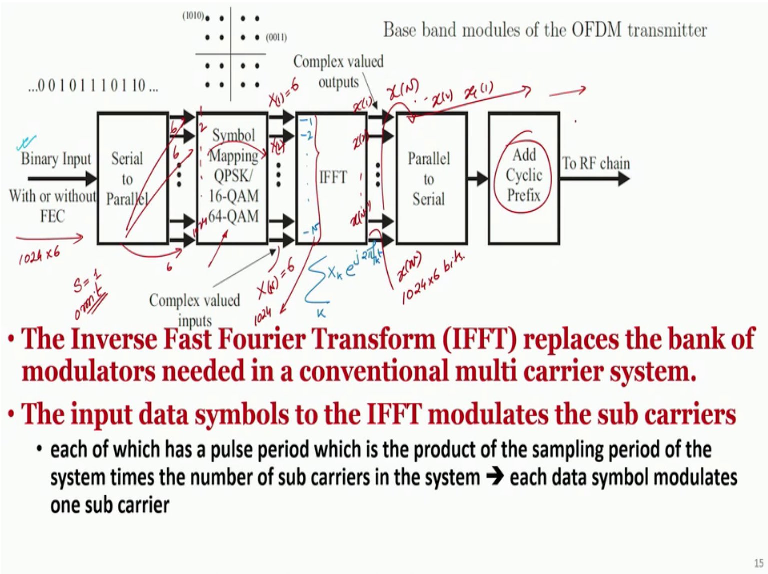

So now we look into the transmitter structure it is very important to understand the structure because this is something which will be used throughout. So, there are binary input bits which are coming in at this point and these binary bits are sequence of 0s and 1s which are converted from serial to parallel now why it is done so, will be cleared once we look at it. So, here if we look at the output of OFDM what we have is X k e to the power of j 2 pi fkt right and there are sum over all this different sub carriers so we just take a look at this expression. So, these are the constellations and these are the carriers all right. So, that is what we have over here and these are happening in parallel that is why we have this kind of a summation ok.

So, now, what we should see is that each of these sub carriers should be loaded simultaneously right. So, if they have to load it simultaneously these 1s 1 2 upto N are the different sub carriers we have said that this summation this thing would be realized as the DFT. So, now, for clarity if you look at this particular expression we can clear all the markings that that has been put on this particular slide and things will be easier now ok.

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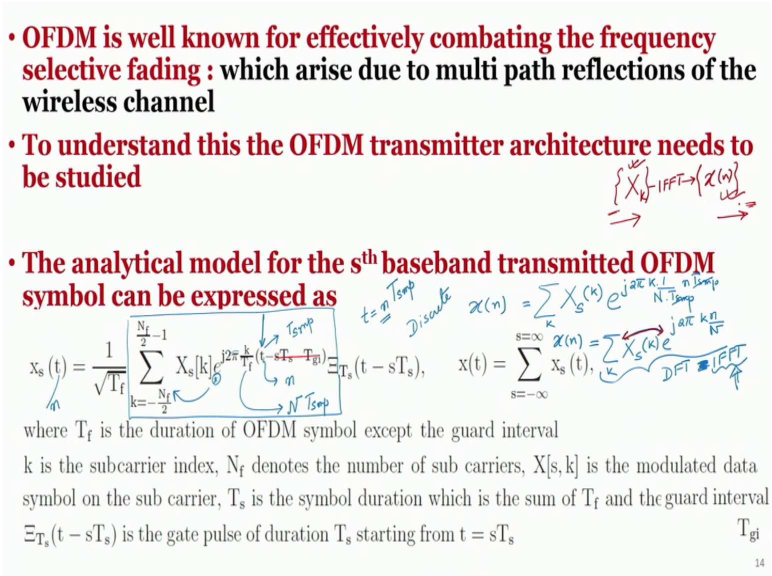

So, if you look at this t is continuous time, we can change this continuous time into sample time where let us say T samp where T samp is the sampling period and t is the time where it is equal to n times T sampling ok. So, if that is so, we can realize this entire thing in the discrete domain in the digital domain with index n. So, instead of putting t we can index this as n and here instead of putting t will be again indexing this as n and this Tf will be sum multiple of this T samp ok. So, effectively when we look at this particular expression one will be getting X k sorry notation sth OFDM symbols kth sub carrier e to the power of j 2 pi ok.

So, now, n multiplied by T samp so, we have this 2 pi of course, then k multiplied by 1 upon N times T smp and you have the t part with this n times T smp I am just only taking this particular t this offsets and kind of neglecting as of now in order to understand this offsets I am neglecting. So, one can of course, bring them back it is not going to affect your expression. So, this cancels out you have a summation over k and you are going to have x of n. So, now, if you look at this expression it is simply X s of k e to the power of j 2 pi k n upon N sum over k this is nothing, but the DFT which can be realized through an IFFT expression and this is what we have as x of n ok.

So; that means, this entire things can be realised in complete discrete manner and entire set of multiplex multiplication operation that was happening here could be replaced by multiplication and the summation this huge processing can be simply replaced by a DFT and the DFT can be also replaced by an IFFT operation because IFFT is very very low complex right. So, we have to now visualize this entire expression that is happening over here through the view of IFFT right so; that means, in IFFT all these Xks these are the inputs of the IFFT. So, we change the pen color to mark it better. So, these are the inputs. So, we have Xks as the inputs and xns as the outputs and in between we have IFFT right through IFFT it produces the time domain. So, now, why we call it IFFT of course, there is a direct similarity and then what we see is that these were actually modulating the frequency as the sub carriers and this is the time domain signal.

Although this is the common way of explaining such things one can even visualize the entire thing as time domain operation, but since there is a huge similarity with IFFT one can take frequency domain as input and time domain as output, that is also another way of viewing things. So, whatever way is comfortable you can see that end of the day one has to ensure or one knows that its finally, a time domain signal that is one is handling with. But more popular description is that this is the frequency domain and this is the time domain and that is how you will find it most of the notation in most of the discussions ok.

So now we are back to the transmitter structure. So, what we see is that each of these are the index of the sub carriers. So, they must be filled with these different constellations. So, it is Xn. So, rather as per the notation in this particular slide it is one and s we are kind of is equal to 1 or we are kind of omitting s your omitting s a for ease. So, we are just noting the sugates for this one particular OFDM symbol and symbol mapping can be anything depending upon the signaling from higher layers or the choice and as said the symbol mapping can be different in different sub carriers. So, now, if you are using 64 QAM this would be having 6 bites if let us say we are using 64 QAM in all there will be all having 6 bits and if the value of N is let us say 1024. So, you will be sending 11024 multiplied by 6 bits simultaneously correct.

So, so, many bits from here 1024multiplied by 6 bits have to be read and they have to be sent 6 bits to each of the branches and there will be 1 2 up to 1024 such branches they will be doing symbol mapping fitting into IFFT and here we are going to get their outputs small x of 1, small x of 2 up to small x of N minus 1 and finally, we are going to get small x of capital N, N number of samples. Then we will be reading then out in a manner x 1 x 2 up to x capital N and they will go out to the analog section of course, there is an addition of cyclic prefix we will discuss the addition of cyclic prefix and it goes to the analog part and then to the RF chain.

So, this is how the signal flows and the equations that we have written maps to this kind of a structure.

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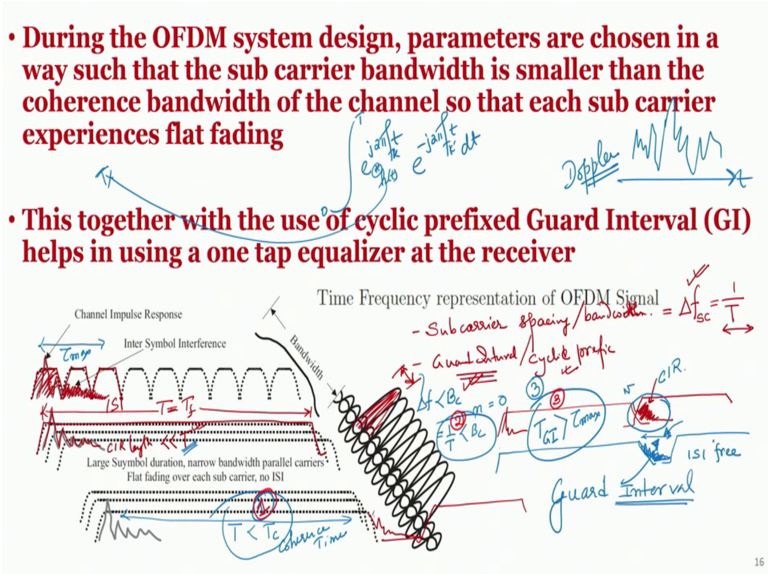

So, moving ahead what we see is that there are some critical parameters one of the parameter is the sub carrier band width that is or sub carrier spacing, sometimes it is also called the sub carrier band width and the other parameter is the guard interval which we will discuss sometimes or most of the time it is also called cyclic prefix. So, guard interval is a more generic term cyclic prefix is a very specific implementation of the guard interval and one would remember that this sub carrier spacing which is effectively or essentially written as 1 by delta f sorry its delta f one could rather specify it by delta fsc indicating sub carrier spacing and this was equal to 1 by T or 1 by Tf whatever is the notation we have been using.

So; that means, one chooses the sub carrier spacing the symbol duration gets decided this is very very important to understand and there is also another thing the guard interval. Now why we are saying that these are important parameters for 2 2 reasons or multiple reasons one is that this is the diagrammatic representation of the channel impulse response.

Now in a typical single carrier system single carrier system if it has an large a certain amount of band width pulse duration which is indicated over here clearly there is inter symbol interference, the eco is larger whatever symbol is being sent in this part gets stretched to this part and to this part. So, there is ISI. So, in OFDM what we are doing is that the symbol duration is usually made large and this is that T which is equal to Tf in the notations that we have been using till now, I am just warning that we will probably have some change of notation later on is made large enough. So, that this channel impulse response CIR length is much much smaller than T or T is much much larger than channel impulse response ok.

So, we said at some point that these sub carriers individual sub carriers their width should be decided in such a manner that they are smaller pretty much small. So, that it experiences nearly flat fading. Now we have to go to the details of the propagations characteristics which we will do later on that these 2 are high inter collect connected. Now a flat fading effectively means that the channel impulse response is perceived as an impulse, it is not a stretched version of the impulse response, its not a stretched version of the impulse; that means, ecos are not present.

So, to allow ecos to vanish one and to make this T sufficiently large, now although you make T sufficiently large what do you will find is that end of it will still stretch and there has to be a next symbol that comes in. So, basically this is one OFDM symbol you have made it pretty at large compared to channel impulse response this is already being made, but still because of the channel impulse response this will become the end will become something like this.

Now the next OFDM symbol is suppose to start from this point. So, there will be some amount of ISI which is present over here to avoid that what one can usually do is 1 one can choose a design where the symbol duration whatever is present and the next symbol there is some guard interval. So, we are using the term interval instead of band over here right. So, this is the beginning point, this is the end point or small n and one can see it I mean; that means, there is this guard interval between the two, what is done is this signals which is coming afterwards they put some kind of information which we will see what is called this cyclic prefix, but effectively there is a filler or a gap between these two signals.

So, this channel impulse response is going to die out over here of it dies out over here there is ISI free system and one can have a communication. Now, one can argue that one can design orthogonal system in time like one have designed an orthogonal frequency system in frequency, but we must be very very careful at this point to note that when we design the orthogonality in frequency orthogonality within a symbol we had used integrate 0 to Te to the power of j 2 pi fct e to the power of minus j 2 pi sorry f kt f k prime t dt correct.

We did not have any other thing between, but now what we have is the channel impulse response. So, orthogonality can be brought in if we bring this channel impulse response in to the picture, if we bring this channel impulse response into the picture then we must also have a convolution with the channel impulse response as the basic signal and overall we can bring in an orthogonality. But that would mean that this channel impulse response information is required at the transmitter it requires to be available at the transmitter. So, that being made available and then being used kind of makes things very very difficult, that is one of the aspects we cannot avoid and hence its better to choose a guard interval a loss which you cannot help.

But then there are mechanism which people have working on to reduce this guard interval we will get an opportunity to see that. And now coming back to the other important thing if we make this T very very long then because of mobility there is a factor called Doppler which comes into play which we will see again later, because of Doppler or because of mobility we have said that channel fluctuates in time. We also require that the channel remains constant in this entire period so; that means, T or symbol duration must be smaller than something known as the coherence time of the channel.

And the sub carrier spacing delta f must be less than the coherence band width; that means, the band width across which channel remains flat. So, these are the 2 important conditions that are required to be satisfied, now delta f as you can see it is equal to 1 by T which is from this and this is another condition. So, we have 2 conditions over here 1 by T must be less than coherence band width and T must be less than T c or 1 by T must be less than 1 upon Tc. So, these 2 conditions have to be satisfied simultaneously in order to make the system work as well as the other thing which is the TGI which we have been referring to while looking at the equations.

So, now, this is clearer TGI should be greater than the maximum channel impulse response length or tau max as we call it. So, tau max is the maximum length of the channel impulse response in this picture it is tau max over here. So, this is the 3rd important condition that one should consider. So, one let us change the color of the pen. So, this is 1st condition, this is the 2nd condition, this is the 3rd condition that one must keep in mind while designing such a system.

So, we stop this particular lecture over here and we will continue more on the fundamentals of the OFDM in future lectures because based on this as we have being saying the 4th generation system stands the 5th generation system stands. So, it is essential that we understand it thoroughly. So, that things becomes easy whenever we discuss the air interface for the next generation systems and its although kind of not required to mention, but you should see that OFDM in the wireless system has been popular over last 2 decades it has been used even beyond that it is going to be there for another decade.

So, it is going to live for a very long period compared to the history of wireless communication. So, it is essential that we understand the details and we hope that with this you will be able to design future things improve upon this basic structure that exists to an extent that we can have even more advance system in future.

Thank you.