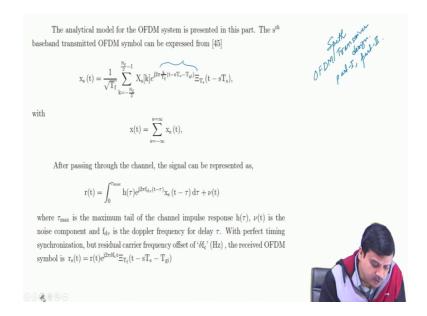
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Lecture – 18 Waveform for 4G & 5G (OFDM) (Contd.)

Welcome to the lectures on Evolution of Air Interface towards 5G. So, in the previous lecture we have been discussing about the structure of OFDM, the signal flow and we have mentioned over the diagrammatic representation of how things happen, what things go on we will briefly represent the analytical flow or way to write it down so, that when we are designing we can handle it in direct manner.

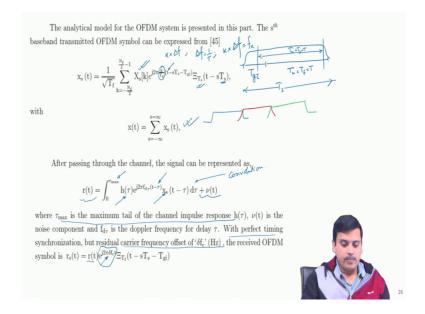
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So, here what we have is the analytical representation of OFDM. So, and I have said in one of the earlier lectures that I leave this particular part for you to find out and plot it in time and see how does it work out. This is not difficult and it will take some time for us to do it here. So, I am consciously avoiding that, but I guess it is doable by most of you.

We will provide the link which is there is a famous paper by Speth two papers OFDM transceiver design, fundamentals of OFDM transceiver design part – I and part – II where, one can find such a system model and it is necessary to understand such an integrated model. This helps us in writing one equation which can help consider all possible aspects.

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So, so, here we have X s of T, this module we have already discussed which is the constellation point, complex constellation point. This part we have also discussed this k multiplied by delta f, where delta f is equal to 1 upon T and k multiplied by delta f is equal to one can think of it as f sub k, and this is the time unit which is resulting in continuous generation of signal. This is the gating pulse we said it may be rectangular, but since this pulse is stretching over time T s and T s is now since you have seen guard interval T g I, T useful sometimes it is written as T f and as per our earlier notation it is T.

So, this entire duration is T s and we have said that one can easily do a pulse shape which is not rectangular, but slightly tapered at the end ensuring that a duration of T u is equal to T f is equal to T s per our notation is rectangular. So, in this period one will find it rectangular ok. So, that is what is ensured and this one is when we have a cumulation of OFDM signals OFDM symbols, this is what it is and it is getting pulse function as we have said is going to ensure orthogonality amongst the different OFDM signals next to each other just a diagrammatic way of representing. So, this is the sequence rather it is not a sum ok.

So, after passing through the channel what is happening is h of tau which is the channel coefficient and x s of t, we are looking at one particular signal they gets convolved. So, as you can see this is the equation of convolution, and what we have here is noise added. So, this is the received signal, alright and what we are seeing over here is the convolution

operation. So, here tau max is the maximum tail of the channel impulse response, ok. And, delta f c; so, with the perfect timing synchronization the residual carrier frequency delta f c of the receiver symbol goes to 0. So, otherwise with imperfect it is going to be represented in this manner, ok.

So, this is the overall signal structure. So, if we have received the signal with a certain amount of residual carrier frequency we have to whatever is received r t; r t has to be the model has to include this residual frequency offset and here what we have indicates the different tau delays that are present in the signal, .

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the signal portion without the noise part is $\int_{0}^{\tau_{max}} h(\tau) x_s\left(t-\tau\right) e^{i2\pi f_{ds} r\left(t-\tau\right)} e^{i2\pi \delta_{ds} t} \Xi_{T_t}(t-sT_s-T_{gi}) \, d\tau$ Since $h(\tau)e^{-j2\pi f_{d_{\tau}}\hat{\tau}}$ cannot be separated from $h(\tau)$, we re-write the above as $r_s(t) = \int_{-\tau_{max}}^{\tau_{max}} h(\tau) x_s \left(t - \tau\right) e^{j2\pi (\delta f_c + f_{d\tau})t} \Xi_{T_f}(t - sT_s - T_{gi}) d\tau.$ Considering ideal conditions of no synchronization error, i.e. $\delta f_c = 0$ and near static channel, i.e. $(f_{d\tau}, f_{d\tau})$ the received sub carrier can be expressed from [45],[46] as $R_{sk'}$ where $\omega[s, k']$ is the frequency domain noise component and H[s, k'] is the channel coefficient for k'th sub carrier of sth OFDM symbol. The above expressions hold when the CP is larger than the maximum tail of the channel impulse response

So now at the receiver the received wave of the signal is now passed through the Fourier transferred the receiver and as a result you are able to get back your signal. So, what we have is the Fourier transfer operation is happening over here, ok. So, most of the time when we are discussing the basic structure we will assume that the residual carrier frequency goes to 0 and what we have over here is due to the Doppler associated with the tau-th path. So, f d is the Doppler associated with the delay of tau.

So, in that case this particular part which is a complex cannot be distinguished from h t which is also complex. So, together one is going to get complex coefficient and what we have here is effectively the residual carrier frequency and a certain amount of Doppler which is associated with the signal and as we said under ideal condition delta f c goes to 0 and in near static condition f f d tau is almost equal to 0,.

So, if we now write down the signal model under idealistic conditions at the receiver you are doing the integration operation going from sT s; that means, the previous entire OFDM symbol including the cyclic prefix and you have additional shift of guard interval and then this portion is as the denominator part and you only have integration over the T f. So, that means, the previous OFDM symbol then the cyclic prefix which is the T gi and then you are actually integrating the T f portion or the T u portion this is what we have been doing and we have mentioned them as T. So, if you do this operation with this signal model with whatever signal model.

So, r s we have you have to take from here if you take r s from there and you replacing the equations what you are going to get is that so, one minor thing what I would like to point out is we are trying to decode at the k prime-th sub carrier, right. So, we had sent we had the signal model in terms of k, but at the receiver we are using k prime so, that we can find out what happens when k is not equal to k prime and when k is equal to k prime, right.

So we know that when k is not equal to k prime because of orthogonality things are going to go to 0. So, only when k is equal to k prime we are going to get our desired result. So, if you put this equation inside this you will find out that we are able to write down the expression in a manner that the k-th sub carriers received signal is the constellation point sent on the k-th sub carrier; now, here we have k is equal to k prime.

So, k prime or k is the same multiplied by capital H s, k; s is the s-th OFDM symbol giving us a notion of time and k prime is the k-th sub carrier. So, s H s, k prime is the channel coefficient for the k prime-th sub carrier of the s-th OFDM symbol simply H s, k is the Fourier transform of h t h t comma tau. So, t I am taking it as constant or the particular desired time interval and instead of Fourier transform you can take this as the DFT operation at the receiver and then it would translate to the k-th sub carrier ok.

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	Table 2.3: WMAN system param	neters
Parameter	WiMAX /	3GPP-LTE
Carrier frequency	2-11GHz	2GHz
Bandwidth		Hat.25MHz,5MHz,10MHz,20MHz
Number of point in	128,512,1024,2048 201 ch	128,512,1024,2048
FFT	H = 1074 2018	\rightarrow
Sub carrier bandwidth	(10.94KHz)	(15KHz)
GI ratio	$\frac{1}{4}, \frac{1}{8}, \frac{1}{16}$	4,8
useful part OFDM	91.4µs	66.67µs
symbol duration	//	
Max transmit power	43dBm (10MHz)	> 38dBm (10MHz)
Minimum Frame size	2ms 6	0.5ms
Inter site distance in	2.8Km	1.7Km
macro cell		
Number of sectors	3	3

The maximum transmit power at the base stations is related to the cell radius, and hence the two systems have different values since the cell radius is different

So some system parameters of interest in OFDM system we have seen some of them earlier. So, we compare WiMAX with LTE because these are fourth generation system. What we are seeing is that there is a wide range of carrier frequency of operation. System bandwidth – we have different system bandwidth; scalable system bandwidth for different system bandwidth the FFT size differs, but what you will find is that with 20 megahertz this uses are 2048 point FFT. So, if I divide 20 megahertz by 2048 we are going to get the sub carrier spacing, ok. So now if we look at 10 megahertz it is divided by 1024.

So, what we see is as the total bandwidth scales down the FFT size also scales down leaving behind the same sub carrier spacing right. So, it scales down in a in a linear fashion same is what happens for the LTE systems. So, the sub carrier bandwidth here is roughly 10 kilohertz whereas, here it is 11 kilohertz by virtue of an over sampling factor, ok. So, there is a slight difference in the parameters as you can see the useful part of OFDM signal is different in both cases and they are usually designed for different cell radius now that is taken care of by transmit power variation which can be easily done. So, more or less these two are similar systems are not much variation, but slight difference in the parameter setting.

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Transm	ission BW	1.25 MHz	2.5 MHz	5 MHz	10 MHz	15 MHz	20 MHz	
Sub-fran	ne duration			0.5	ms			
Sub-carr	ier spacing	(15 kHz)						
Sampling	g frequency	1.92 MHz	(3.84 MHz)	7.68 MHz	15.36 MHz	23.04 MHz	30.72 MHz	
	U	(1/2×3.84)	SM = DT a	Ƴ (2 × 3.84	(4 × 3.84	(6 × 3.84	(8 × 3.84	
		MHz)	2/2 3	MHz)	MHz)	MHz)	MHz)	
	T size	128	256 🕊	512	1024	1536	2048	
	of occupied	76	151	301	601	901	1201	
	rriers [†] , ^{††}		1/1 - 2 M 16-	-				/
	iber of symbols	1 E	2.84	71	/6			
per su	ıb frame /Long CP)		JUIPPE	superil			_	
СР	Short	(4.69/9) ×	(4.69/18) ×	(4.69/36) ×	(4.69/72)×	(4.69/108)	(4.69/144)	
length		6,	6,	6,	6,	× 6,	× 6,	
(µs/sam ples)		(5.21/10) ×	(5.21/20) ×	(5.21/40) ×	(5.21/80) ×	(5.21/120	/160)	
		1*	1	1	1	×1	<1	
1,		(16.67/32)	1 1 1 1 1 1	(+((-1++0))	(16.67/256)	1-6 6-1-6	(12)	1

In this particular slide what we have is an entire range of parameters that is used for LTE especially we are looking at the multi factor. So, what we have listed down is that LTE can operate or fourth generation system can operate in different sub carrier bandwidths and for each sub carrier bandwidth there is something called a sub frame duration which we will see at an appropriate time, but what is more important for us is the sub carrier spacing the sub carrier spacing which is very important. So, this turns out to be 15 kilohertz for LTE systems.

Now, how does it work out one can see that is there is a sampling frequency which is chosen to be of multiple of 3.84 megahertz. Now, one may recall that 3.84 megahertz is the chip rate for 3G systems. So, there is a similarity or there is some kind of a matching with the structure of that has happened in the past, ok. So, for 2.5 megahertz the, this sampling frequency is 3.84 megahertz and the FFT sizes 256. So, if one would divide 2.5 megahertz by 256 point FFT one would get the value of sub carrier spacing, but then this is the result that you are going to get over here as you can clearly see would be somewhere around 10 kilohertz ok.

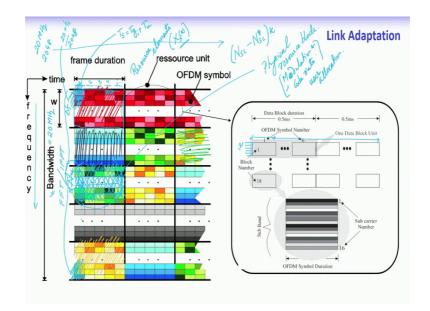
But, this is not what is used. Rather a higher sampling frequency is used instead of using 2.5 megahertz. Now, what does this give us? Effectively there is a wider bandwidth. So, if this is 2.5 megahertz if this is 2.5 megahertz we have let us say 3.84 megahertz which is wider bandwidth and then you divide this by 256; you are going to get 15 kilohertz ok.

And, now one would argue that 15 kilohertz sub carrier spacing if I multiply by 256 sub carriers then one is going to fill out an entire set of frequencies.

But, what is done is at the edge the sub carriers are set equals to 0 they are not allowed to be transmitted. So, if they are not allowed; that means, if it is 0, then effectively what happens is that one is using only a certain smaller set which fits into this 2.5 megahertz and then there is a roll off of the spectrum and it dies out by the time 2.5 megahertz finishes. So, there is a spectral mask which is very very clearly defined and one is able to effectively use the available bandwidth with having as low adjacent channel interference as possible while having sufficient guard band.

So, these are some of the interesting facts which have been brought into the LTE or the 3GPP generation of systems. So, what they have done is they have effectively used a larger sampling factor and they have not directly sampled the bandwidth, used the over sampling factor and then divided by the FFT size to get the sub carrier spacing and instead of using all the sub carriers they are using a smaller set of sub carriers.

And, regarding the CP length: so, there are two options of CP length available and one is the short CP length and one is the long CP length, and these are fit into the frame structure in a certain manner that one is able to have an integral frame size. So, we will see them when we are discussing the fifth generation frame structure.



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Ok. So, this is how the overall time frequency picture looks like. So, we have the frequency axis we have the frequency axis or the bandwidth over here. So, it is written over here frequency grows on this side time grows on this side. So, now, if we study this particular thing carefully we have FFT operating in this entire region. So, this is your FFT operation or IFFT operation and the number of sub carriers N sc is basically defined over this, ok.

So, that means, one is having n number of sub carriers, so, when we said 2048 and we said 20 megahertz of bandwidth so this entire bandwidth is let us say 20 megahertz it can be different values as described in the other picture and FFT size would be whatever is the total number of sub carriers in this. And, then each of these elements each of these elements over here this element (Refer Time: 15:16) is the sub carrier spacing which is obtained by dividing the corresponding bandwidth factor. So, I write it as f s indicating the up sampling factor divided by the number of sub carriers. So, one gets a sub carrier bandwidth,.

And, this is the OFDM symbol duration which is T s which is equal to T gi plus T u sorry T u or T f it is the same thing, right. So, now, we have to send signals over this. So, as we have said that each one of these each one of these rectangles small rectangles they are usually called resource elements resource units or they are called resource elements each one is called the resource element because each one can carry a constellation point X s of k ok. So, now, it will be clear why we need to use the term s.

So, these units in time correspond to different OFDM symbols, ok. So, let us say this is the OFDM symbol number 1, number 2, number 3, number 4 number 5 and so on that is how it is numbered. So, this is one of the sub carriers of a OFDM symbol. This is the second sub carrier of the same OFDM symbol, this is the third sub carrier of the same OFDM symbol, now if we go down further we are going to get the n-th sub carrier of the OFDM symbol, ok.

Now, not all sub carriers as used as we have described in the previous slide, the edge sub carriers which are at the edge of the bandwidth at the both the edge of them they are made 0 and only a smaller subset is used in order to avoid the out of band radiation in order to avoid adjacent channel interference,. Clean these things up ok.

So, now, each of these elements they carry complex signals. So, effectively if I have N sc number of sub carriers, we will be reducing the number of sub carriers which are used for guard band and then we have the total number of sub carriers which can carry bits. If each of them carry let us say k bits this is the number of bits that can be sent in one OFDM symbol.

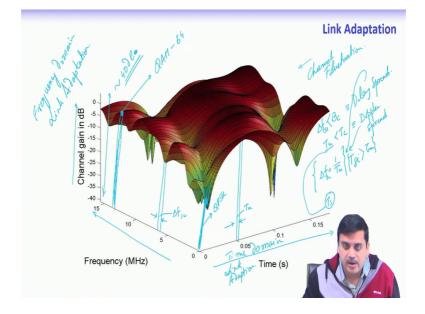
Now, if one has to allocate these different OFDM symbols to different users then one has to assign resource in order to do the allocation. So, it is not possible to allow each individual resource element to be addressed because if you do that the amount of signaling overhead is huge to reduce that the neighboring sub carriers are grouped together, a block is formed. So, neighboring sub carriers are group let us say we talked about this group they are grouped together not only that a sequence of OFDM symbols are also grouped together. So, these are sequence of OFDM symbols they are grouped together. So, this entire block of OFDM symbols along with the along with the sub carriers are grouped together to form one addressable unit.

Now, one should also remember that there is forward error correction code. So, one has to have a certain larger number of bits to allow the error correction code and one can choose different modulation techniques and also want us to allocate these resources different users. So, there is a certain minimum size of this block that includes a certain number of sub carriers and certain number of time domain signals to form a resource block. So, one can think of a physical resource block and allocate the same modulation and code rate to this particular block and also one can allocate this to a user as well.

So, this is the minimum resolution with what which one can do it, ok. So, these are. So, then in each of the blocks in LTE systems there would be 12 number of sub carriers and they will be grouping up to 1 millisecond of signal in order to 1 millisecond of OFDM symbols. So, 1 millisecond of OFDM symbols to form one physical resource block and these resource blocks are grouped together in order to do any allocation of modulation code rate or user allocation.

So, whenever we do any resource allocation, any link adaptation we would be doing it over this entire block of symbols simultaneously. This will only save reduce the amount of overhead. Further, the channel which fluctuates in both time and frequency kind of remains constant over one such period and hence it does not make much difference to allocate them in a different manner. So, we have the particular representation in this picture.

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So, now we look at one very important aspect which is used over LTE and other systems is called link adaptation. So, we briefly look at that. So, let this be the frequency access and let this be the time axis and this access be the channel gain given in decibels. So, if we look at the joint time frequency picture the channel fluctuation would appear as is given in this particular image.

So, what we see is the channel fluctuates both in time direction and in frequency directions and we have said that sub carrier bandwidth should be such that the channel experienced is flat and the time duration of the OFDM signal should be such that it experiences nearly flat fading in time or slow fading in time. So, this is your T u and this is your delta f, right. So, and one remembers that delta f is equal to 1 upon T u.

So, what we see is that the these factors are primarily influenced by the channel gain and the other factors which are of consideration that B c which is the coherence bandwidth is greater than the sub carrier spacing we can write this as sc to be more precise and the T u should be less than the coherence time which is due to Doppler. This is the due to Doppler we can write Doppler spread and this is due to delay spread, right.

So, these two parameters influence the choice of sub carrier spacing and T u as well as T gi should be greater than tau max, ok. So, not only T u this we should replace by T s to include the guard interval. So, that is the overall picture one should remember and what one sees is that the fluctuation level has nearly 40 dB of signal strength variation over a short distance or over a short unit of time.

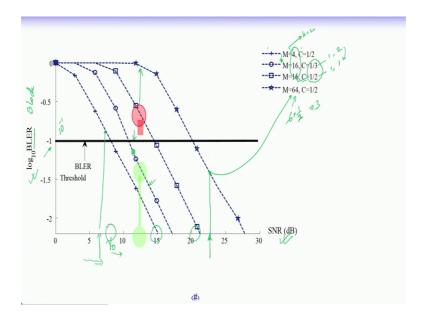
So, a 40 degree fluctuation is a huge fluctuation and what is important is to maintain a certain quality of service or certain bit rate. So, if the signal strength is low we know that we will use a lower bit rate, if the signal strength is high will use a higher bit rate. So, in this situation what we see is that OFDM gives us the opportunity to allow certain sub carriers for example, here which are facing low SNR conditions and certain sub carriers here which are facing high SNR conditions to use different modulation orders.

So, let us say this is using 64 QAM and this is using let us say QPSK because this is in lower SNR condition this is at a higher SNR condition, right. So, OFDM by virtue of splitting the frequency domain picture into smaller-smaller chunks we can do frequency domain link adaptation as well as we would obviously, do time domain link adaptation, right.

So, if we now go back to this grid picture what we will realize is that each of these frequency bands; so, we can use different colors and say that this frequency band may choose a certain modulation and code rate whereas, this particular band may use a different modulation and code rate where as this particular band in the middle might choose a different modulation and code rate and same would happen in the time domain also. So, we can choose a different color and indicate that this particular block may use a different modulation code rate compared to this.

So, every resource block can be given a different modulation and code rate because the channel has fluctuations in both time and frequency domain.

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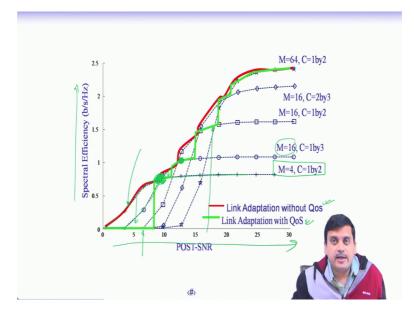
So, what we have just explained is we have the graphical representation of a similar thing that if this is the SNR axis and this is the error probability curve and when we have the BLER we mean the block error rate probability of an entire block going into error,. So, these are for different modulation and code rates and generally, as your SNR increases as depicted by this your error probability decreases. So, when your SNR is high one can increase the bit rate by simply changing the modulation from M equals to 4 to M equals to 16 because M is equal to 4 means k is equal to 2 2 bits per signal and symbol and 16 means 4 bits per symbol one can increase the data rate ok.

So, similarly one can even think of going higher up; that means, one can think of going to a different code rate. So, we erase this. So, not only not only does the modulation increase the spectral efficiency, but also code rate increases spectral efficiency what we see over here is the code rate has changed from 1 by 3 to 1 by 2 and by 1 by 3 means one information bits and 2 parity bits; here means 1 information bit, 1 parity bit, so there is increasing spectral efficiency.

But, then what we will find is if there is a threshold for block error rate; so, here we have taken the threshold to be 10 to the power of minus 1 ok. So, then not all higher modulations can be used; so, one has to restrict oneself to that modulation code rate which is below the threshold and as the SNR increases one can use higher and higher modulations.

So, we find there are SNR switching points; these are called SNR thresholds, ok. So, if the SNR is somewhere here; if the received SNR is somewhere here, one would definitely choose this modulation and this is 64 QAM with rate half as given in this particular picture. 64 QAM means 6 bits, rate half means 1 by 2 so, which means three information bits per second per hertz one can use over here. Whereas, if the SNR condition is below the minimum threshold; so, this is let us say gamma 0, then one does not find any modulation code rate to have a BER which is lower than this threshold and hence one does not find it a good condition to transmit the signal.

So, only if the signal is higher than gamma naught which is the smallest SNR threshold then one can start transmitting data and one would choose the maximum modulation code rate which satisfies the SNR threshold condition and this happens for every such resource block that we have been that we have explained over here. For every resource block this thing operation happens, not on a sub carrier basis.



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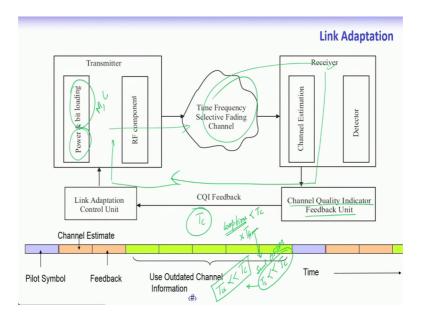
Ok. So, at the end if we look at the SNR axis and if you look at the spectral efficiency access what we see over here is that if we choose a fixed modulation code rate as SNR increases error probability decreases. So, spectral efficiency throughput increases and throughput saturates after a certain point where no longer there is notable decrease in the error probability, ok. So, if one chooses higher order modulation one will find that a spectral efficiency is very low up to a certain point near to 0 because error probability is

very high then it starts increasing and it also saturates at a maximum point where after the error probability decrease does not improve this spectral efficiency.

However, if one chooses to dynamically adjust or switch between these different modulation and coding rates then what would happen if one is at a certain point in the SNR axis one would choose that particular modulation and code rate where one is getting the highest spectral efficiency. So, what we see we have two kinds of curve over here; one is link adaptation with QoS; that means, the one with this threshold one with using this threshold and the other one where threshold is not used, but only the highest spectral efficiency curve is used.

So, if you are using the highest spectral efficiency curve the transmission throughput is higher in that case, but QoS is not maintained. So, one can get higher and higher error probability. But, if one maintains the QoS then the spectral efficiency at a particular SNR is lower compared to what one would get, but one would maintain the desired error probably performance. So, these are some important things which happen in this modern communication systems and one has to remember this.

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And, there is a flow of things that happen. So, the receiver measures the channel condition. After measuring the channel condition it feeds back the channel through something known as channel quality indicator there is a feedback. Through feedback the transmitter adjusts the bit loading; bit loading means the modulation index and the code

rate as well as it can adjust the transmit power and then it would send back signal. This entire flow of things should happen within a duration of coherence time again, right because the channel should not fluctuate by the time the feedback has been given to the transmitter and it uses the feedback towards forward transmission.

So, that means, your loop time should be less than the coherence time. So, loop time is of course, a multiple of frame time and a frame consists of several OFDM symbols ok, rather several OFDM blocks and so that means, the symbol duration or T s must be significantly smaller than T c and not simply just smaller than T c that we had discussed earlier. So, as we go deeper into more requirements and advanced mechanisms we find more and more restrictions coming into the system. So, T s being less than T c would also boil down to T u or T f being less than much much less than T c. So, this is something which we should remember and it is not simply that T u is less than T c that is used in the design.

So, we stop our discussion over here and we will move on to other things for OFDM from this point. However, it is very important to remember whatever we have discussed because this is the foundation based on which we will discuss several other things. The entire physical layer or even the entire communication protocol stands on this basic grid architecture. So, we must remember that there exists a bottom layer grid architecture on which all these things happen and there is a frame structure we will discuss the specific details of frame structure of the fifth generation system at a certain point, but overall this is the skeleton on which the entire signal protocol stands.

So, it is very important that we should understand this and whenever different things happen we should be able to connect back to this frame structure to realize how signal is flowing, how bits are flowing, what is happening at each stage, how resources are being allocated. So, we must understand the basic element of a resource unit which is very critical for all these kind of broadband multi carrier communication systems.

Thank you.