

Evolution of Air Interface towards 5G
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Lecture – 23
Waveform for 4G & 5G (OFDM) Numerology Part-2

Identified the differences between 4G and 5G which are the most important differences in terms of the air interface, also we have been able to identify what are the common things that are present between the fourth generation and the fifth generation. And, primarily we have said that the air interface consists of OFDM with axis; that means, the multiple axes is based on OFDM and there is also support for DFT spread OFDM in the uplink direction as has been there in the fourth generation system. Along with it OFDM is also available for uplink direction transmission.

And, in the earlier lectures we have described that why OFDM could also be suitable and the primary reason we have said is that when a user sends data in the uplink direction the user need not send it over the entire stretch of bandwidth and then if it is sends over a smaller set of the bandwidth only 12 subcarriers let us say using one resource block, then the peak to average power ratio is not significantly high.

In that case doing a DFT operation to spread and reduce the PAPR is obviously, going to give some benefit, but it is not necessarily that the benefit will be huge and given the extra processing that one has to do because if one does an extra DFT processing there is also some amount of power and complexity that adds up to the system.

So, there is an overall balance which is necessary depending upon the application and scenario of deployment. So, there are two possible options which are available in the fifth generation, but it is highly compatible with the earlier generation system.

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38.211 : Symbols

$(k,l)_{p,\mu}$	Resource element with frequency-domain index k and time-domain index l for antenna port p and subcarrier spacing configuration μ ; $a_{k,l}^{(p,\mu)}$ Value of resource element (k,l) for antenna port p and subcarrier spacing configuration μ
β	Amplitude scaling for a physical channel/signal
Δf	Subcarrier spacing
Δf_{RA}	Subcarrier spacing for random-access preambles
κ	The ratio between T_s and T_c ; clause 4.1
k	Subcarrier index relative to a reference
l	OFDM symbol index relative to a reference
μ	Subcarrier spacing configuration, $\Delta f = 2^\mu \cdot 15$ [kHz]
$M_{bit}^{(q)}$	Number of coded bits to transmit on a physical channel [for codeword q]

So, we have discussed this particular slide in a previous lecture, where we have described most of the variables that are necessary and we have also seen what is a. So, in case of OFDM it is the complex constellation point if it is the DFT spread, it is a combination of complex constellation points.

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$M_{\text{sym}}^{(q)}$	Number of modulation symbols to transmit on a physical channel [for codeword q]
$M_{\text{sym}}^{\text{layer}}$	Number of modulation symbols to transmit per layer for a physical channel
$M_{\text{sc}}^{\text{PUSCH}}$	Scheduled bandwidth for uplink transmission, expressed as a number of subcarriers
$M_{\text{RB}}^{\text{PUSCH}}$	Scheduled bandwidth for uplink transmission, expressed as a number of resource blocks
$M_{\text{sym}}^{\text{ap}}$	Number of modulation symbols to transmit per antenna port for a physical channel
ν	Number of transmission layers
$N_{\text{BWP},i}^{\text{size}}$	Size of bandwidth part i ;
$N_{\text{BWP},i}^{\text{start}}$	Start of bandwidth part i ;
$N_{\text{CP},i}^{\mu}$	Cyclic prefix length;
$N_{\text{grid},x}^{\text{size},\mu}$	The size of the resource grid;

So, then there are other variables which are also available. So, we are not using this much except a few of them from here. It is the $N_{\mu} \text{CP}, l$ which is the cyclic prefix length and as you can see it is also configured with respect to μ ; that means, μ is

described in the previous slide as one which specifies the subcarrier spacing. We had seen earlier that if T_u is the T useful duration of the OFDM symbol then this is related to Δf as $1/T_u$, and we have also seen that T_{GI} is extension of the T_u .

So, generally this is some α fraction of T_u , some fraction of T_u . So, here what is done is T_{GI} is coupled with T_u through the same parameter μ . So, N_{CP} is the cyclic prefix length as you can see sub CP 1 indicating the length of it and μ indicating the subcarrier spacing indication which is parameterised by μ , and this is the resource grid side size we will see it at the later stage.

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$N_{slot}^{subframe, \mu}$	Number of slots per subframe for subcarrier spacing configuration μ .
$N_{slot}^{frame, \mu}$	Number of slots per frame for subcarrier spacing configuration μ .
$N_{symbol}^{CORESET}$	Time duration of a control resource set;
N_{symbol}^{PUCCH}	Length of the PUCCH transmission in OFDM symbols;
$N_{symbol}^{subframe, \mu}$	Number of OFDM symbols per subframe for subcarrier spacing configuration μ ;
N_{symbol}^{slot}	Number of symbols per slot
T_c	Basic time unit for NR;
T_f	Radio frame duration;
T_s	Basic time unit for LTE
T_{sf}	Subframe duration;
T_{slot}	Slot duration;

And, out of these definitions whatever we have we will again see them with example specific things that will be clearer. So, as we can see in all cases the number of slots per sub frame for a subcarrier configuration μ . In the earlier lecture we have described that the number of slots is a function of is the function of the subcarrier spacing which is different compared to the earlier generation where things were pretty much fixed, ok. So, we will see all of these descriptions in the next few slide, alright.

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Frame structure and physical resources

Fields in the time domain is expressed in time units $T_c = 1/(\Delta f_{\max} \cdot N_f)$ where $\Delta f_{\max} = 480 \cdot 10^3$ Hz and $N_f = 4096$. The constant $(K=T_c) T_c = 64$ where $T_s = 1/(\Delta f_{\text{ref}} \cdot N_{\text{ref}})$, $\Delta f_{\text{ref}} = 15 \cdot 10^3$ Hz and $N_{\text{ref}} = 2048$.

$T_c = 1/(480 \cdot 10^3 \cdot 4096 \text{ Hz}) = 1/(1966080000 \text{ Hz}) = 50.86 \cdot 10^{-9} \text{ s} = 50 \text{ ns}$ or 51 ns

Numerologies $T_c = 1/(15 \cdot 10^3 \cdot 2048 \text{ Hz}) = 1/(30720000 \text{ Hz}) = 325.52 \mu\text{s}$

Multiple OFDM numerologies are supported. Table 1 below describes the numerology where μ and the cyclic prefix for a bandwidth part are obtained from the higher-layer parameter subcarrierSpacing and cyclicPrefix.

Table 1: Supported transmission numerologies.

μ	$\Delta f = 2^{\mu} \cdot 15$ [kHz]	Cyclic prefix
0	15	Normal
1	30	Normal
2	60	Normal, Extended
3	120	Normal
4	240	Normal

Handwritten notes on slide:
 $\Delta f_{\max} \times N_f = B_w$
 $\Delta f = 2^{\mu} \cdot 15 \cdot 10^3 \text{ Hz}$
 $32 \rightarrow \mu = 5$
 $480 \text{ kHz} \times 4096$
 $T_c = 1/(\Delta f_{\max} \cdot N_f)$
 $T_s = 1/(\Delta f_{\text{ref}} \cdot N_{\text{ref}})$
 $K = T_c$
 $(K=T_c) T_c = 64$
 $T_c = 1/(15 \cdot 10^3 \cdot 2048 \text{ Hz}) = 1/(30720000 \text{ Hz}) = 325.52 \mu\text{s}$

So, when we go into the frame structure, the way things are defined is in a very systematic manner. So, fields in the time domain is expressed in units of T_c . So, T_c is the variable which defines the unit in terms of which things are defined. So, T_c is 1 upon Δf_{\max} multiplied by N_f . So, Δf_{\max} as you can see is the subcarrier spacing or the subcarrier bandwidth and the maximum value of it is taken in the denominators, so when you go 1 by Δf_{\max} if you get the chip duration and it is. So, basically if you look at 1 by Δf_{\max} it is basically the shortest T_u , our T_u which we have been describing as far as possible that is the OFDM symbol duration and this is further divided by N_f which is the number of subcarriers.

So, if you look at this entire denominator term denominator term is Δf_{\max} that is the maximum subcarrier bandwidth multiplied by N_f . So, we will see what is N_f . So, N_f is number of subcarriers. So, together this gives a bandwidth indication and 1 by bandwidth is the chip duration or the clock duration which is useful. So, here Δf_{\max} is specified as 480 multiplied by 10 to the power of 3 hertz, so, 480 kilo hertz. So, that means, what we will see is that if we relate it in terms of μ Δf is 2 to the power of μ into 15 into 10 to the power of 3 hertz.

So, this factor you will find that this goes into Δf_{μ} by factor of nearly 32 . So, 15 it will go to 32 times; that means, μ will take a value of 5 in this particular case, [FL] and N_f is 4096 ; that means, 4096 subcarriers are possible. So, you can multiply 480 kilo

hertz multiplied by 4096 to see the total span of sampling frequency as is available. Out of this entire bandwidth the entire bandwidth was not used, we know that from fourth generation system. There are certain guarding guard band at the edges where there is no transmission so as to avoid inter carrier interference.

So, here we have done a calculation of these things where we find out that the T_c chip duration is turning out to be around 50 nano seconds it is 50.86 or it is 50 nano seconds. So, this is equivalent to you can think of around 20 megahertz in time I mean in the bandwidth domain. The T_s which is again another parameter is with respect to some reference value and here what we find is the reference value is 15 kilo hertz and this reference value is the LTE value and 15 kilo hertz with 40, 2048 subcarriers. So, there you will find that T_s turns out to 325 micro seconds compared to this.

So, these numbers are from LTE. So, LTE system subcarrier spacing is 15 kilo hertz and in the 20 megahertz it uses 2048 point FFT. So, with respect to the LTE system the new fifth generation system is scaled in terms of time and bandwidth. So, this is a scaling factor or κ which is used to describe such parameters, ok.

So, now comes the one of the most important and most interesting terminologies which have been discussed for quite some time and that is called numerologies, ok. So, numerology is nothing, but the description of μ ; μ is we have said describes the subcarrier spacing. So, what we see is that multiple OFDM numerologies are supported that is one of this treatment and table below. So, basically this particular table describes the numerology, ok. So, this particular table as you can see this is table one which describes the numerology. It effectively tells us that μ equals to 0, you can see from this expression it turns out to be 15 kilo hertz and corresponding to this we will also find a cyclic prefix length which we will describe later on.

So, as you keep increasing the value of μ simply by using this factor like 2 to the power of 1 is 2 multiplied by 15 is 30 kilo hertz, then when μ is 2 this factor is 4 this factor is 60 kilo hertz. So, what we find is that subcarrier spacing is increasing with increasing value of μ . This is in contrast to the fourth generation system and since it is also OFDM only thing that changes is this subcarrier spacing. So, this subcarrier spacing change is controlled by this parameter μ . So, this configuration of OFDM for different subcarrier

spacing is termed as numerology in OFDM. And, we have described in the previous slide that the cyclic prefix duration is also coupled with μ which we will see shortly.

So, through the use of μ one can describe that T_{cp} or T_{gi} as per our description that is the cp duration followed by T_u which is equal to $1/\Delta f$, right. So, through one parameter that is μ one can describe the structure of a OFDM symbol and we had said in our notation earlier it was T_s here T_s is a different meaning. So, this is the total symbol duration as per our description of the OFDM symbol. So, we can say that T_s of indicating OFDM symbol duration. So, this entire value can change based on the choice of parameter.

Now, this choice of parameter μ and cyclic prefix are obtained from higher layer parameters. So, basically something which is above the physical layer can provide you choice of these parameters. So, if these two choices are given then you can describe an entire OFDM symbol. And, earlier we have described in several ways that how the choice of parameters are very important for successful deployment of OFDM system and what we see over here is that this particular aspect is exploited in the fifth generation, things are not kept constant.

So, there is a lot of potential to exploit the capabilities of OFDM, use it in a flexible manner as per the situation which is dependent on the propagation environment as well as the need or the environment. We have seen several scenarios and descriptions which have ended up in certain sort of requirements and we will see how these different requirements can be met with this flexibility provided by OFDM.

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Definitions

- A resource block is defined as $N_{sc}^{(RB)} = 12$ consecutive subcarriers in the frequency domain.
- Common resource blocks are numbered 0 onwards in the frequency domain for subcarrier spacing configuration μ . Common resource block number n_{CRB}^{μ} in the frequency domain and resource elements (k, l) for subcarrier spacing configuration μ is given by
$$n_{CRB}^{\mu} = \left\lfloor \frac{k}{N_{sc}^{(RB)}} \right\rfloor$$
- **Bandwidth part:**
 - A bandwidth part is a subset of contiguous resource blocks for a given numerology in bandwidth part on a given carrier.
 - A UE can be configured with up to four bandwidth parts in the downlink with a single downlink/uplink bandwidth part being active at a given time.

Handwritten notes on the slide include: "12" next to the resource block definition, "RB = 12" with a diagram of 12 subcarriers, and "Bandwidth part" with a diagram showing a subset of resource blocks. Logos for Swamyam and IIT Madras are visible at the bottom.

So, there is some definition of resource block we will see that we have described shortly earlier that the number of subcarriers in a resource block is 12. So, when we have different subcarriers stacked against each other a group of 12 subcarriers would be called a resource block, ok. The next consecutive will be another set of 12 subcarriers.

So, if there are if there are 10, if there are let us say 2048 subcarriers in all of which a certain fraction is used and we divide by 12 we will find the number of resource block that is available. Now, if we divide 2048 by 12, then we get the certain number, but not all those numbers are useful because out of 2048 in the entire spectrum we have said that certain numbers in the guard band are not used. So, only a smaller subset is used. So, that number has to be replaced in 2048 and we get the appropriate value of the resource block and that will depend upon particular spectrum and application scenarios, right.

Further, one more thing is that as we have said that in a particular bandwidth; suppose, there is a certain bandwidth one can have narrow subcarriers, we will see this; one can have wider subcarriers, we will see this. So, accordingly depending upon the value of μ the number of resource block is going to change, right.

There is also definition of bandwidth part which says that bandwidth part is the subset of contiguous resource block for the given numerology; numerology as we have said is the one which described through μ in a bandwidth part on a given carrier. A UE that is the user equipment can be configured with up to four bandwidth parts in downlink with the

single downlink or uplink bandwidth part been active at a given time. So, at any one given time although you can divide the entire bandwidth into four parts from the UE definition part perspective, but only one of them can be utilised at any one instant of time. So, this makes things a little bit easier from the UE implementation, ok.

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OFDM baseband signal generation

The continuous time signal $s^{(p,\mu)}(t)$ on antenna port p and subcarrier spacing configuration μ for OFDM symbol $l \in \{0, 1, \dots, N_{\text{slot}}^{\text{subframe}, \mu} N_{\text{symbol}}^{\text{slot}} - 1\}$ in a sub frame for physical channel except PRACH is defined

$$s^{(p,\mu)}(t) = \sum_{k=0}^{N_{\text{sc}}^{\text{size}, \mu} - 1} a_{k,l}^{(p,\mu)} e^{j2\pi(k + k_0^{\mu} - N_{\text{grid},x}^{\text{size}, \mu} N_{\text{sc}}^{\text{RB}} / 2)(N_{\text{CP},J}^{\mu} t - t_{\text{start},J}^{\mu})} \Delta f$$

by $k_0^{\mu} = (N_{\text{grid},x}^{\text{start}, \mu} + N_{\text{grid},x}^{\text{size}, \mu} / 2) N_{\text{sc}}^{\text{RB}} - (N_{\text{grid},x}^{\text{start}, \mu_0} + N_{\text{grid},x}^{\text{size}, \mu_0} / 2) N_{\text{sc}}^{\text{RB}} 2^{\mu_0 - \mu}$

where $t_{\text{start},J}^{\mu} \leq t < t_{\text{start},J}^{\mu} + (N_u^{\mu} + N_{\text{CP},J}^{\mu}) T_c$ is the time within the subframe,

$$N_u^{\mu} = 2048\kappa \cdot 2^{-\mu}$$

$$N_{\text{CP},J}^{\mu} = \begin{cases} 512\kappa \cdot 2^{-\mu} & \text{extended cyclic prefix} \\ 144\kappa \cdot 2^{-\mu} + 16\kappa & \text{normal cyclic prefix, } l = 0 \text{ or } l = 7 \cdot 2^{\mu} \\ 144\kappa \cdot 2^{-\mu} & \text{normal cyclic prefix, } l \neq 0 \text{ and } l \neq 7 \cdot 2^{\mu} \end{cases}$$

And Δf is given by clause 4.2; μ is the subcarrier spacing configuration;

So, from here we are at the middle of everything one can think of. So, this particular set of expressions describes the OFDM symbol that is being generated because we have said the 5G uses OFDM. So, what we find over here the continuous time signal S here we used such a s symbol earlier when are discussing the symbolic structure, l in time domain, p is the antenna port, mu is the numerology, t is simply showing the time unit.

So, we do not have k over here, because it is OFDM symbol. So, OFDM symbol as we had said earlier we can connect over here x of t is equal to in our notation what we have described earlier X s of k sum over k; k is the subcarrier e to the power of j 2 pi k t by e to the power of j k t pi by N, one can think of this way or one can think of one e to the power of j 2 pi k n. So, when we are doing in the terms of discrete we are going to get it as X s.

So, our s would correspond to l over here k remains the k e to the power of j 2 pi k n upon N and of course, there is a gate which we should always have n minus s N s, right. This is something which always had. So, now, if we compare this we said earlier in one of the earlier descriptions that this a corresponds to the constellation point for the

OFDM, otherwise it will correspond to the combination of the symbols. See here we have the same thing again that X_s if we compare these two equations now x_t what we had written earlier corresponds to S_l of t , right.

There is certain antenna port parameterised by μ we will see that and here this particular symbol corresponds to the complex constellation, e to the power of course, we have e to the power of $j 2 \pi$. So, we have $j 2 \pi$ and then we have this k which is also k over here. So, this k is also have been a k summation over there. Now, what we find is Δf being specified which is 1 upon N in our case or you can have k multiplied by Δf in the other way. So, t corresponds to t the only difference we have is that Δf is specified as 2 to the power of μ multiplied by 15 into 10 to the power of 3 hertz, right. This is the only difference that is available in the system. So, so here you can have Δf over here. So, which would indicate the spacing.

So, here our description was for any given Δf ok, but here what we find is that this particular Δf can take different values which is specified by the parameter μ . Rest of the parameters are constant parameters. These are all the constant parameters. This is the shift which we had indicated earlier in our expression and we had said that it is for you to figure out how does these equations fit in. So, what we see over here is there is N μ CP description and T_c is also given in time and there is also an offset of $t \mu$ that is also described. So, rest of the parameters are constant. So, what we essentially see from here is that it is the same OFDM symbol, but with a Δf description which is 2 to the power of μ 15 kilo hertz.

The $N \mu$ CP description is given over here which is which can be used in order to calculate the value of Δf and in this description it is said that the Δf is given in clause 4.2 of the document 38 dot 211. So, if you go to clause 4.2 in the document, Δf is mentioned and Δf is described exactly over here in this particular class. So, this is the primary foundation on which the fifth generation system stands. So, it is very important, let we take a look at it.

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Variable Sub Carrier Bandwidth (OFDM Numerology)

Lets take a closer look at the Equation, repeated below for convenience

$$s_l^{(p,\mu)}(t) = \sum_{k=0}^{N_{\text{grid}}^{\text{size},\mu} N_{\text{sc}}^{\text{RB}} - 1} d_{k,l}^{(p,\mu)} \cdot e^{j2\pi \left(k + k_0^{\mu} - N_{\text{grid},x}^{\text{size},\mu} N_{\text{sc}}^{\text{RB}} / 2 \right) \left(t - N_{\text{CP},l}^{\mu} T_c - t_{\text{start},l}^{\mu} \right)}$$

$$k_0^{\mu} = \left(N_{\text{grid},x}^{\text{start},\mu} + N_{\text{grid},x}^{\text{size},\mu} / 2 \right) N_{\text{sc}}^{\text{RB}} - \left(N_{\text{grid},x}^{\text{start},\mu_0} + N_{\text{grid},x}^{\text{size},\mu_0} / 2 \right) N_{\text{sc}}^{\text{RB}} 2^{\mu_0 - \mu}$$

*, where Δf is defined in Section 4.2 as $\Delta f = (2^{\mu}) * 15$ [KHz]. If we substitute this value of Δf into the above equation of $S_l^{(p,\mu)}$, we will get in the argument of the exponent function $\{ \underline{e}^j \}$ the product of sub-carrier index 'k' and Δf as $k * \Delta f = k * (2^{\mu}) * 15$ [KHz], which can be written as $k * \Delta f_{\mu}$, where $\Delta f_{\mu} = (2^{\mu}) * 15$ [KHz].*

*Therefore the sub carrier spacing (SCS) can be easily read as $\Delta f_{\mu} = (2^{\mu}) * 15$ [KHz].*

This means that depending on parameter μ , the SCS Δf_{μ} will take values which are multiple of 15 KHz.

So, in the next particular slide we have actually given the exact description which you have provided in the previous slide that is we have identified the exponential term. We have identified it very very clearly that there is a k which gets multiplied with delta f and t in order to complete the OFDM signal generation and delta f in section 4.2 we know that it is given as this expression.

So, if we combine all of them we get the OFDM signal generation in the fifth generation system. So, one could also replace this entire set of values with an other variable delta f mu delta f sub mu to be defined in this manner and then one gets S l mu which is the parameter over here being directly influenced in the equation.

So, whatever we have described primarily tells that this subcarrier spacing or SCS can be easily read of as this particular value, ok; that means, it tells one can choose different values of subcarrier spacing depending upon the parameter mu and which is a multiple of 15 kilo hertz as we have been saying, ok. So, this is the primary you can say the most important information or meaningful information in terms of the air interface for the fifth generation, ok.

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CP length (OFDM Numerology)

$$N_{CP,l}^{\mu} = \begin{cases} 512\kappa \cdot 2^{-\mu} & \text{extended cyclic prefix} \\ 144\kappa \cdot 2^{-\mu} + 16\kappa & \text{normal cyclic prefix, } l = 0 \text{ or } l = 7 \cdot 2^{\mu} \\ 144\kappa \cdot 2^{-\mu} & \text{normal cyclic prefix, } l \neq 0 \text{ and } l \neq 7 \cdot 2^{\mu} \end{cases}$$

$$[\kappa = \Delta f \cdot 4096 / (\Delta f_{ref} \cdot 2048) = 2 \cdot 2^{\mu} \cdot 15 \cdot 10^3 / 15 \cdot 10^3] = 2^{\mu+1}$$

μ	κ	$N_{CP,l}^{\mu}$ {Normal ($l \neq 0$)}	$N_{CP,l}^{\mu}$ {Normal ($l = 0$) or $7 \cdot 2^{\mu}$ }
0	2	288	320
1	4	288	352
2	8	288	416
3	16	288	544

μ	$\Delta f = 2^{\mu} \cdot 15$ [kHz]	$T_{sym} = 1 / \Delta f$ [μs]	Cyclic prefix	CP length [μs]
0	15	66.66	Normal	4.69
1	30	33.33	Normal	2.34
2	60	16.66	Normal, Extended	1.17
3	120	8.33	Normal	0.57
4	240	4.16	Normal	0.29

Handwritten notes: 4.16, 4.69 + 4.16, 8.33

So, when we look into the CP length we have presented a calculation of CP length over here. So, from which what we find is that for different values of mu we will get different CP lengths, and what we see is that we have two different cases; that means, when l is not equal to 0, ok. So, what do we mean by l not equal to 0, let us see that. So, when we look into the description we find that CP is described in various manner; one is the extended cyclic prefix length which is slightly longer as you can clearly see there is a multiplicative factor 512 compared to 144, right. So, this is a larger multiplicative factor this clearly means that CP length is longer compared to other CP cases.

So, if the channel duration or the channel impulse response is large enough. So, one then can use the extended cyclic prefix otherwise one would use the normal cyclic prefix. But, l which is the time index so, for l equals to 0 or l as the last symbol one will find that there is a particular length of CP which is again larger than the case when it is not the boundary OFDM symbols. If it is a boundary OFDM symbols then the length is larger whereas, if it is not the boundary OFDM symbols it is a different value.

What it means is that there is a consecutive set of OFDM symbols that form a slot which we will see short shortly. The first OFDM symbol and the OFDM symbol index for which it is equal to 7 into 2 the power of mu. The value of CP length is given by this number in the middle for all other values, it is given by this a typical OFDM symbol is

given by this. So, we have actually listed down the values of CP for the normal case in in this and for the case where it is l is equal to 0 or 7 into 2 to the power of μ over here.

So, what we find is that in all cases the number of samples in the CP has remained constant. However, what we find is that in the other cases the length of CP is different,. But, what we also see is that when we calculate CP there is a T_c which comes into play, right.

So, this is just a number of samples which are being defined in the CP length, but one should also get a T_c which is coming into play when one describes the CP length. So, as μ changes the T_c would change the subcarrier spacing would change effectively one will find that this product will give a different number and the values are going to be different which we will see shortly.

So, what we find is that the CP length accordingly takes different values as the μ value changes. So, what we see over here is as μ increases, subcarrier spacing increases. So, if subcarrier spacing increases the T OFDM symbol duration would decrease and if T OFDM symbol would decrease since CP length is a fraction of the T OFDM symbol and hence that would also decrease.

Now, very simple although it appears to be very very silly, but still what we see is if we would have kept the CP length of this value whereas, OFDM symbol duration has become this value and have combined this, we would have find that this spectral efficiency would be less than 50 percent because this is useful part which is 4.16 divided by 4.69 , sorry it is it is plus 4.16 . So, what we find is that this fraction is less than 50 percent and spectral efficiency is less than 50 percent.

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CP length (OFDM Numerology)

$$N_{CP,l}^{\mu} = \begin{cases} 512\kappa \cdot 2^{-\mu} & \text{extended cyclic prefix} \\ 144\kappa \cdot 2^{-\mu} + 16\kappa & \text{normal cyclic prefix, } l = 0 \text{ or } l = 7 \cdot 2^{\mu} \\ 144\kappa \cdot 2^{-\mu} & \text{normal cyclic prefix, } l \neq 0 \text{ and } l \neq 7 \cdot 2^{\mu} \end{cases}$$

$$[\kappa = \Delta f \cdot 4096 / (\Delta f_{ref} \cdot 2048) = 2 \cdot 2^{\mu} \cdot 15 \cdot 10^3 / 15 \cdot 10^3] = 2^{\mu+1}$$

μ	κ	$N_{CP,l}^{\mu}$ {Normal ($l \neq 0$)}	$N_{CP,l}^{\mu}$ {Normal ($l = 0$) or $7 \cdot 2^{\mu}$ }
0	2	288	320
1	4	288	352
2	8	288	416
3	16	288	544

μ	$\Delta f = 2^{\mu} \cdot 15$ [kHz]	$T_{sym} = 1 / \Delta f$	Cyclic prefix	CP length
0	15	66.66 μ s	Normal	4.69 μ s
1	30	33.33 μ s	Normal	2.34 μ s
2	60	16.66 μ s	Normal, Extended	1.17 μ s
3	120	8.33 μ s	Normal	0.57 μ s
4	240	4.16 μ s	Normal	0.29 μ s




So, what is done is instead combination of values of T CP and T OFDM symbol is tied; that means, they are coupled you can say that they are fixed you cannot change them, right. So, once you choose the value of mu your value of subcarrier spacing and your CP gets defined; in other words your entire OFDM symbol duration gets defined, right. So, by choosing a single parameter you can change the entire OFDM structure and we have said earlier how do these parameters effect each other, ok.

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Numerology Summary

- **Variable Sub carrier Spacing / Bandwidth**
• (OFDM Symbol duration)
- **Variable Guard Interval (Cyclic Prefix)**



So, in terms of numerology what we find is that effectively it means that there is a variable subcarrier spacing or variable subcarrier bandwidth either of the terms which would otherwise mean that it is a variation in OFDM symbol duration and along with it there is a variation of the guard interval. Now, let me tell you that, it is not necessary to have these things coupled together, it is not necessary to have this. This can be chosen independently.

So, although we have said that a combination of this CP length and this OFDM symbol is kind of meaningless, but technically speaking one may choose to do so depending upon the situation, but whether it gives appropriate benefit in terms of actual throughput is something one has to see.

So, accordingly one may find that this combination may not end up being fruitful in a particular situation in that case it may be wiser to have a wider a longer OFDM symbol than the one that is given over here. However, it is not true that this OFDM symbol should be always coupled with the fraction of T CP as given by this number one may also decide to do with another number where efficiency is lower, but it serves the purpose. But, in this particular standard they have fixed up this combination of CP along with the OFDM symbol duration.

So, at this point we stop this particular lecture because in the next lecture we start to look at the exact frame structure based on which we have the entire air interface for 5G. So, what we have effectively done is describe the OFDM signal generation equations. So, if one is interested in generating the signals one should follow them and now, using all the descriptions that we have given so far we will fit them into the frame structure and see how does it fit together.

Once we are done with the frame structure then we will be interested in looking at the philosophy or design issues, why this have been chosen, what are the advantages and how what kind of gains one can get, what are the possible architectures, what is the genesis, when where from things have started, in the subsequent lecture.

Thank you.