Evolution of Air Interface Towards 5G
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Lecture – 30
Waveforms Beyond 5G (GFDM)

Welcome to the course on Evolution of Air Interface Towards 5G. So far we have been discussing about the various waveforms, which are potentially future generation waveforms, and we have seen various structures namely FBMC and UFMC. So, will briefly revisit UFMC in a few minutes, and then will proceed on to seeing the next important structure, which is the generalized frequency division multiplexing as we had said in the previous lecture.

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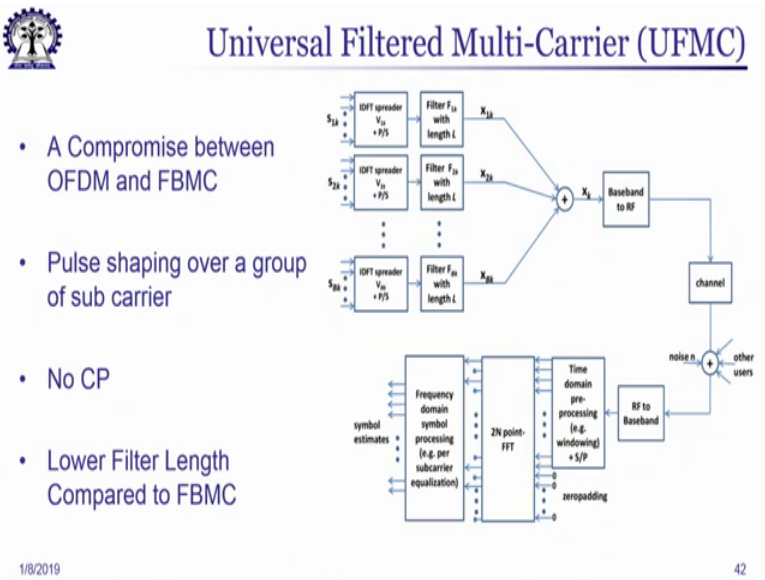

So, in the last discussion we had presented the generalized structure for UFMC, where we had said that there is initial DFT block and group of subcarriers are filtered right that is what we had mentioned.

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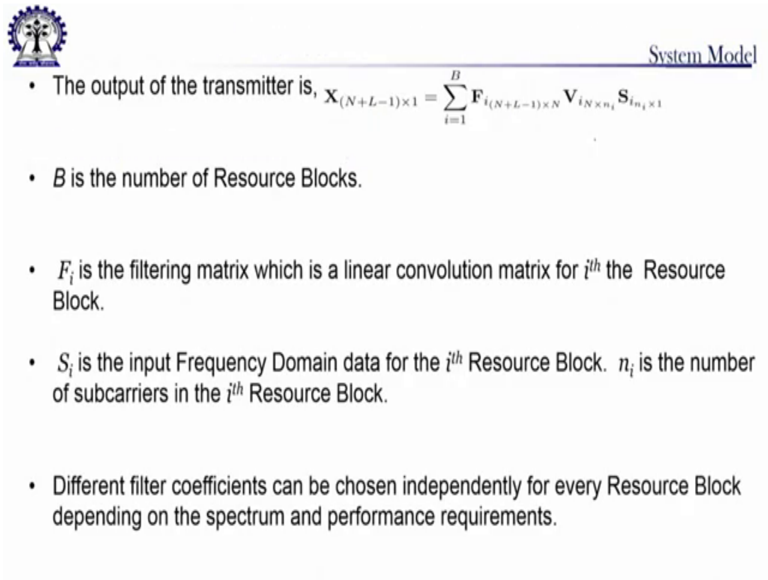

And if we look at the notations which have been carried forward, we do it over here that means there is the filtering which is reflected over here. The IDFT operation which is over here; and S are the data vectors which are here right, so that is how these equations frame up. And B is the block is the number of resource blocks that are to be used. And then we had also identified that for each of the resource block there is a certain size.

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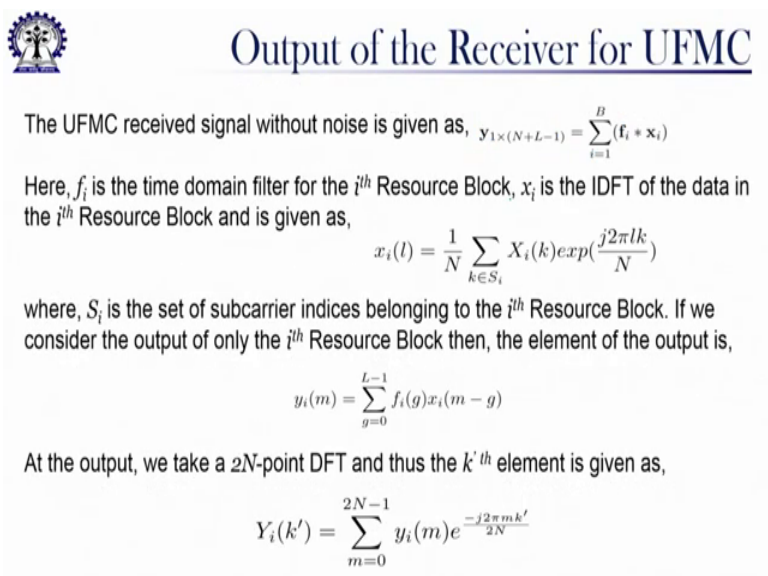

Of the resource block, which we had also indicated here, I mean in this case the S i indicates the size of the resource block, whereas in the previous case it was indicating the data, but rest of it is fine. And we also summarily said that the output is usually done in a manner that there is 2N-point FFT, and the reason is that the output is longer than N, there is N plus L additional things because of the filtering operation that happens. So, one has to take a 2N-point to FFT.

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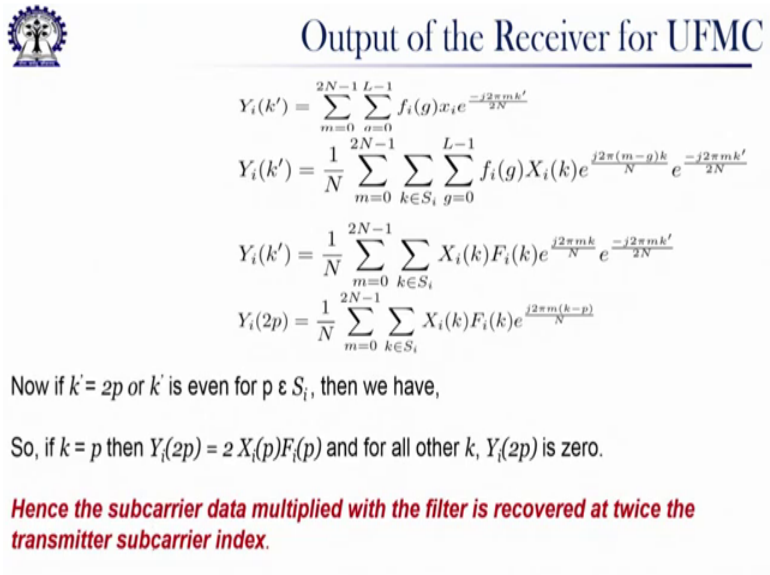

And when one takes a 2N-point of FFT, if one takes the even subcarriers right; so, when one takes the even subcarriers for k prime equals to 2p, then one finds the desired signal along with the filter coefficients. Now, since the filter coefficients are known, one can easily recover the desired signals; for all other values of k, it was shown to be 0.

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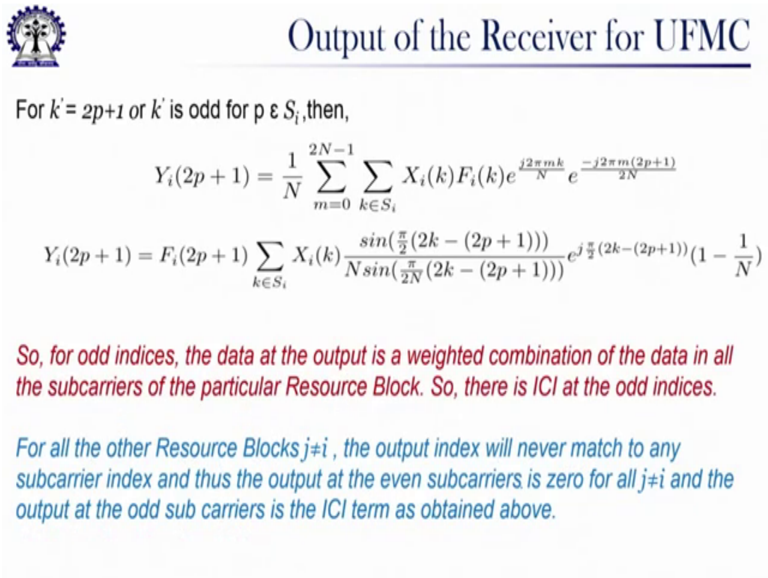

And for any other block, where the resource block is not the desired block, the index are not going to match and therefore, there is no issue about it, so that is how the UFMC is done. And we also said that it is somewhere between FBMC and OFDM it is between them, because like FBMC there is filtering operation, but unlike FBMC it operates on a group of sub carriers, because in FBMC it operates on every sub carrier. So, this a more generalized form you can think of it that way.

And it is more towards OFDM because of the complexity, one can use FFT architectures in this processing and the complexity is lower, where is an FBMC it is per subcarrier filtering, so such facilities are not directly available. And hence complexity wise, it is between them; performance wise also, it is between them; especially, in terms of out of band, leakage, that is one of the main reasons why we are going for this.

And we have said earlier that given a spectrum band this is f, one would like to use narrow bands, wherever their gap exists. So, if there is a certain gap exist, one would like to fit in a waveform within this structure as efficiently as possible and if rather possible one would like to have a multi carrier structure also within that, so that is the requirement. And these different waveforms have the capability to do it, complexity is an issue. So, UFMC what we said earlier is one, which compromises between them, provides the facility to sneak in small section of bandwidth, yet have low out of band leakage and have comparable complexity in with OFDM and much lower than that of FBMC.

But when things come out to be very very narrow bands available and very sharp transition bands required then, FBMC is more suitable. FBMC's also has the feature of orthogonality by which it has been designed, which we have discussed in the lecture before that.

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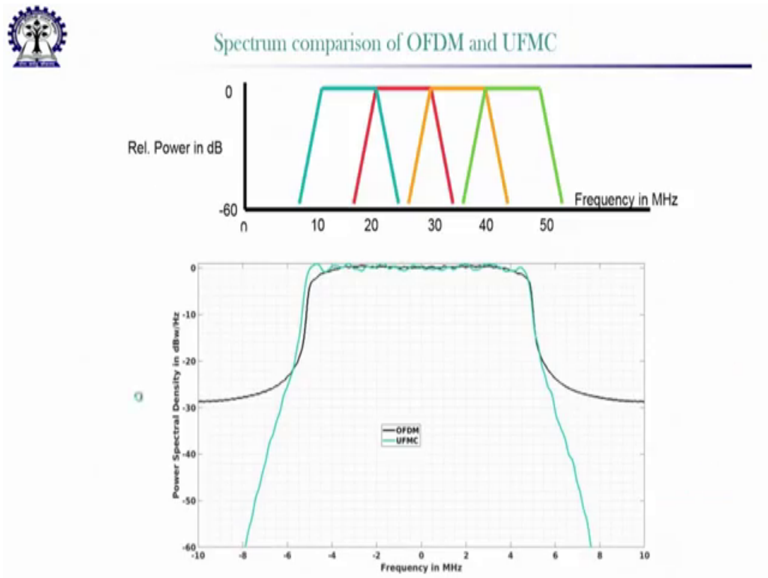

So, moving further lastly we had actually compared the out of band radiation leakage for OFDM and UFM UFMC where we find that significant reduction in out of band leakage is possible.

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Then we move on to the desired discussion, which is due for today that is the generalized frequency division multiplexing. And this will be the last waveform that we considered in this particular series of lectures, where we are looking at future potential waveform structures, especially multi-carrier structures. And we had said initially that all these waveform structures, they are some variant of OFDM in some sense. Now, we also would have said at that point that you may not agree with this, but these are all generalized multi-carrier techniques.

OFDM is more popular. So, one can view them as variants of OFDM, but rather if one sees them as a generic multi-carrier and these are variants of multi-carrier techniques, then it is rather better. The previous class of waveforms that we have studied, they are mostly orthogonal, there is not much of a problem, there is no inter carrier interference which is available, but then when we go to this generalized frequency division multiplexing; this does not come under the category of orthogonal waveforms, it has some kind of non-orthogonality present in it.

And let us see, now why we should look into it and what are the advantages, what are the disadvantages, what are the structures and what are the gains and benefits finally, it provides.

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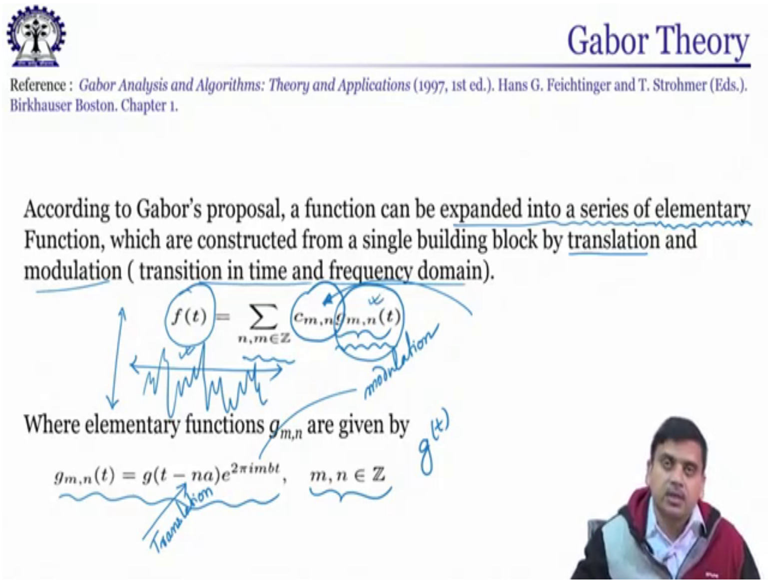

So, to look at that we must briefly look at some of the statements from Gabor theory, which helps us to understand the system in an easier manner. So, according to Gabor's proposal a function can be expanded into a series of elementary functions, which are constructed from a single building block by translation and modulation that means, in this particular expression we are looking at f of t, it can be expanded using g m, n of t whose which are the elementary functions. So, what it says, is that it can be expanded into a series of elementary functions right. So, these are the elementary functions ok, it is expanded into a series.

And of course, you need coefficients we will talk about each, but each of these elementary functions are given by a structure which is this. So, this is more or less self explanatory for people who work in the domain of signals and systems. So, what we see is that there is a translation in time that is what we you see over here, what we also see is that there is a modulation in frequency ok. You can also say, there is a translation in frequency and translation in time, so that is what is happening over here. So, there is a translation in time and frequency domain that is what is happening over here, which can be also seen by translation and modulation terms ok.

So, here g t appears to be the base elementary function, which is translated in time and in frequency which is represented as modulation. Here m and n are integers. So, there is a lattice structure over which this is translated so that means, given a function, so this function could be our arbitrary waveform. And this waveform would exist over a certain duration in time and it would also occupy a certain bandwidth, now what it says is that this waveform can be expanded into a series of certain elementary function. So, elementary function can take a certain shape, you can translate, you can shift in time and frequency and you can make a combination to represent this waveform. These coefficients in that case would represent the information bearing signals or values, which are used along with these basic elementary coefficients.

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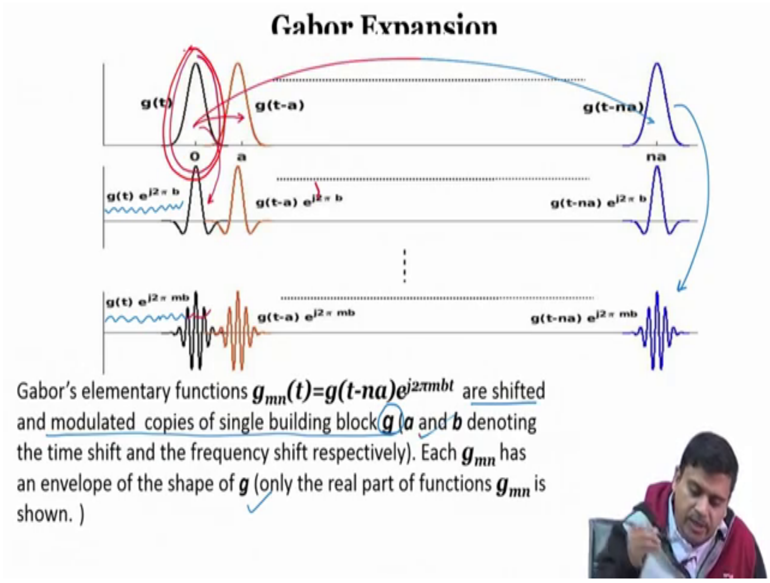

Let us look at this, so what we have over here is the Gabor's elementary function g mn of t, which is basically g t minus n a what we saw over here right and e to the power of j 2 pi m b t are shifted and modulated copies of the single building block g, in this what we said, and a and b denote the time and frequency shift respectively. Each g mn has the envelope of the shape of g and only the real party shown in this particular picture. So, this is your g of t, this when shifted in time is g of t minus a and g of t minus n a right.

And each of them can be shifted in frequency or modulated in frequency in a manner g t e to the power of j 2 pi b, t of course is there. And here g t e to the power of j 2 pi m b t right. So, this can also be shifted in frequency, so that means by using this base elementary function, this is the base elementary function one can actually cover the entire space all right.

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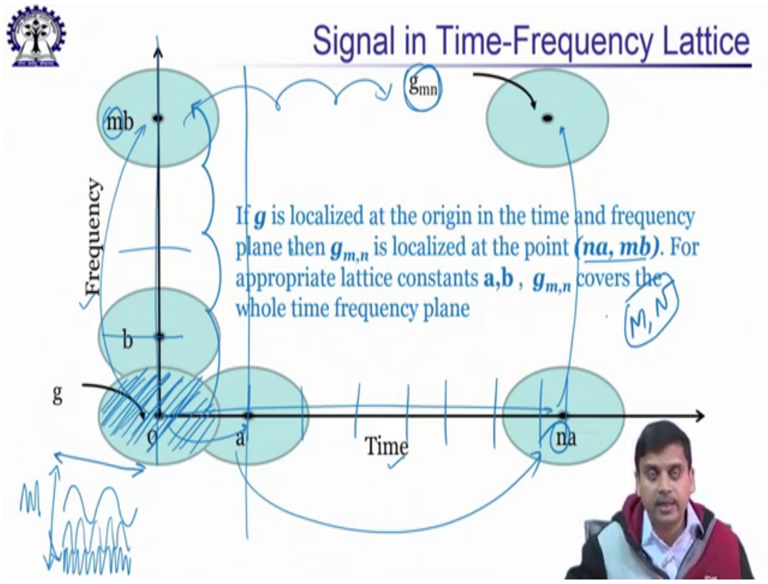

And one represents such a thing pictorially in this manner. So, if g is localized in the origin, so if this is your g which is localized in the origin in the time frequency plane. Then g mn that is this one, which is of our concern is localized at the point coordinate system point na, mb. So, basically you have divided it into such lattice structure and a is the basic unit of shift. So, this is the na unit of shift on this side and if this is the b unit of shift, then you have mb unit of shift.

So, basically this entire Gabor atom if you call it, gets translated in time, then in frequency to be given as g mn which are basically indexes of m and n over here respectively, so that is how you can span a time frequency grid and any particular signal can be represented in this manner. So, just for example if you look at OFDM, then sorry, one of the carrier signal is like this, the next carrier signal would be a higher frequency carrier signal, and then another one which would be here would be basically even higher frequency signal.

So, so basically it covers a certain space in frequency and a certain space in time. So, what this particular thing tells us that this Gabor atom can be used to cover the entire time frequency grid. In case of OFDM, there is only one time slot; so there is only shifting in frequency that keeps on happening. Now, if you think in terms of block OFDM, then one can think of also translating in time as well.

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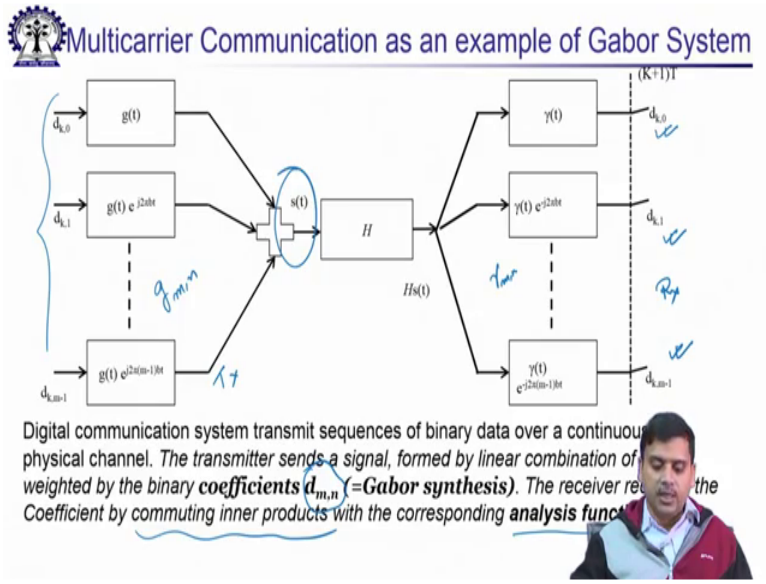

So, moving further digital communication system transmits a sequence of binary data over a continuous physical channel that is of course, what we see. So, this is the transmitter side of things and this is the receiver side of things. And the transmitter sends a signal formed by linear combination of g mn weighted by binary coefficients d. So, this d m, n written over here is the can be thought of as the same thing as c that we had described over here and that time we had said this is the coefficients which carry information right.

So, they are the Gabor synthesis coefficient you can say that through this you are synthesizing the function that you are sending over the channel. And then the receiver recovers these coefficients by taking inner product with the analysis function gamma mn. So, what we have is g m, n on this side and we have gamma mn on this side and there are different combinations different ways you can create them. So, it is not necessary that they are the same, but you can also get sets which would also which would be the same, but you can also go beyond them, which are different set of functions which can create, help you recover the original coefficients that one sends the signal with.

So, this a overall framework on which the system is built and then we move on to the generalized frequency division multiplexing architecture.

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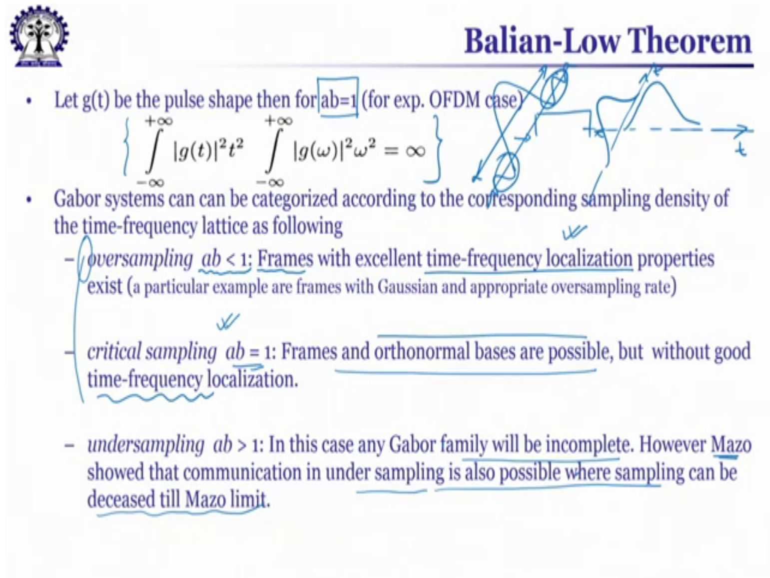

So, before we proceed there a few more things we need to look at is if g is the pulse shape, then for the product ab equals to 1; a and b we had said are the tiles basic coefficients for ab equals to 1, which is the scenario for OFDM. We find that the Balian-low theorem states that the signal cannot be well contained in time and frequency, what it effectively means that time frequency localization is not possible, if you have the coefficient set as ab equals to 1; that means, your lattice if it is structure in this manner, then it is not possible to have well localized time frequency signals.

Now, we have been saying that OFDM has the problem that since it is rectangular in time, it is absolutely well localized in time, but if you look at in frequency it is a sync structure. So, a sync structure means it is actually spans to infinity and there is a huge amount of signal in the out of band region, which we would like to reduce. And what we find is that by a fundamental theorem, OFDM does not allow you to do that and hence you must come up with different mechanisms.

So, within the Gabor framework, it is said that if you let ab product of ab to be less than 1, in that case it is an over sampling systems and they come under the category of frames, we will not discuss it here. You can actually localize the waveform in time and frequency, you can form well-localized pulses that means, which would be localized in time, as well as it will be well localized in frequency.

So, if this is your time axis and this is your frequency axis so, overall you will find that the signal is well contained in time and frequency grid, it does not spread beyond a certain small region and time and frequency. For critical sampling, which we have already said that frames and orthonormal bases are possible. So, like OFDM you have orthonormal bases, but time frequency localization is not possible, if ab equals to 1. And the under sampling system that means ab greater than 1, in this case the family will be incomplete at however it was shown that by Mazo, which we are going to shortly see that communication under in under sampling system is possible, where sampling can be decreased until the Mazo limit.

So, essentially we would primarily like to be in this region in order to get well localized time frequency pulses, but the problem is in that case your spectral efficiency would be reduced. So, again you pay something and you get something whereas, here your spectral efficiency in terms of number of bits that you are sending is good, but there is significant amount of adjacent channel interference, which would like to reduce. So, one would like to come up with systems which are close to critical sampling in some manner and has good out of band emissions, so that one can squeeze in wherever you find empty spaces.

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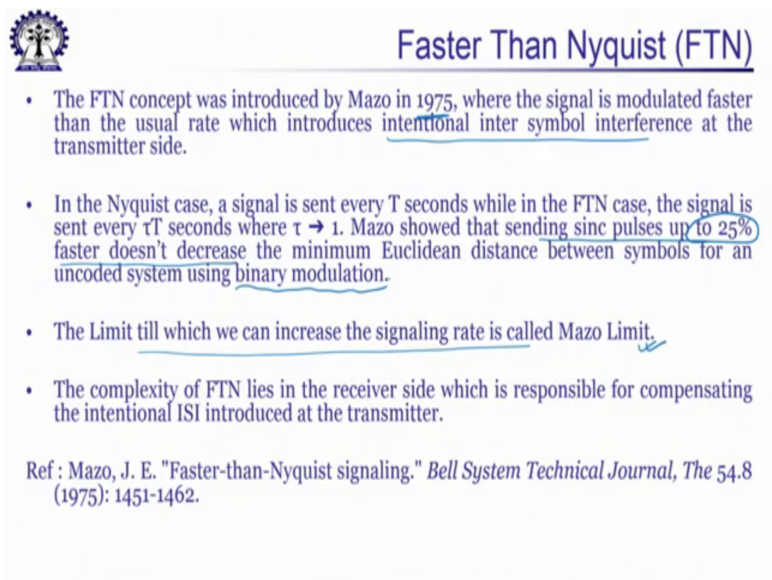

So, as we said earlier about this under sampling system. So, will just put one slide on this, will not spend more time on this. There is something known as the faster than Nyquist system, which is introduced by Mazo in 1975, where the signal is modulated faster than the usual rate which introduces inter symbol interference. Naturally, we know that if you are using sync pulses and you have a bandwidth of w, then you can send the highest rate of signaling as 2 w symbols per second without inter symbol interference that is what is the primary statement.

So, here what a Mazo had actually shown that you can actually go beyond that and yet you can recover the signals up to a certain limit. So, the limit till which you can increase the signaling rate is known as the Mazo limit and for sinc pulses it is shown that up to 25 percent faster does not decrease the minimum Euclidean distance between the symbols right, so that means using a binary modulation of course.

So, when you go with higher modulation then things would be different as it is expected. So, this is another domain which is to be investigated further in quite great details and whether this actually brings us the desired benefit in the overall complicated framework, which we already have needs to be explored further. But for us it is important to know that there exists something, which is beyond the ISI free signaling that we have been discussing.

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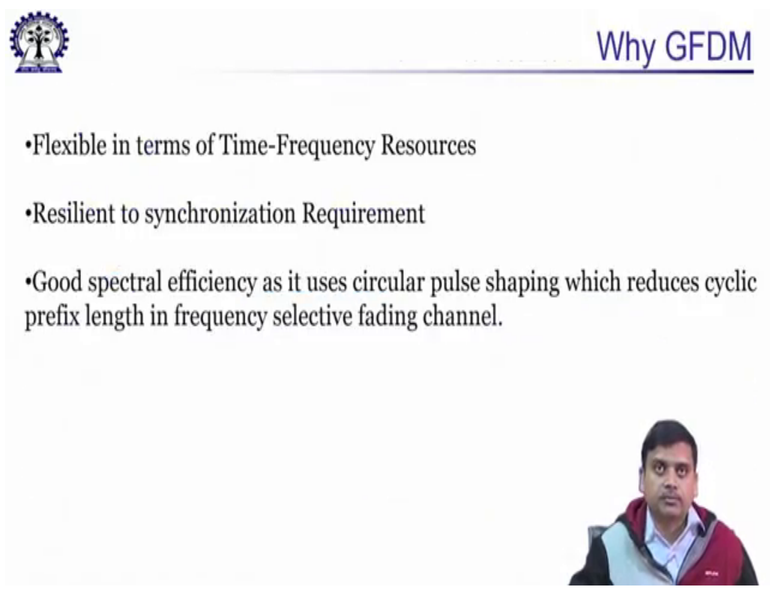

So, now we go on to the generalized frequency division multiplexing structures. So, why is GFDM the primary reason is that it is flexible in terms of time and frequency resources, which we will see. And it is resilient to synchronization requirements will also see that, it is good spectral efficiency as it uses circular pulse shaping like the one which

OFDM uses similar, but it is different which reduces cyclic prefix length in frequency selective fading channel. So, these are some of the benefits of GFDM.

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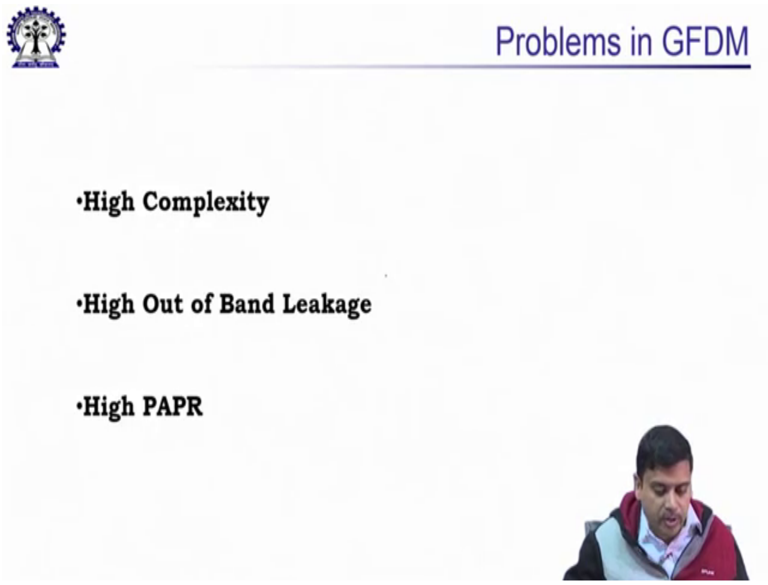

But on the other hand, GFDM has high complexity we will see there. It also has out of band leakage, which is high; one of the reason for it being high is that it is a uses circular pulse shape amongst other things ok. So, one has to find methods to take advantage of these particular things and see if something can be done in order to reduce, in order to reduce the out of band.

It also has high peak to average power ratio, because when you are shifting away from the rectangular pulse shape your PAPR is going to increase. So, again these are some of the areas, where GFDM needs to be improved whereas, these are some of the areas which are its advantage.

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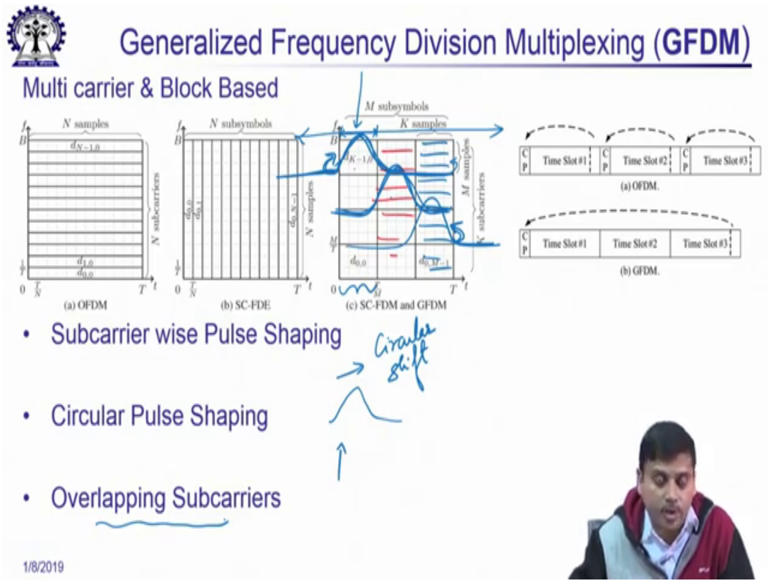

So, let us look into the generalized frequency division multiplexing framework. So, what we see is that it is a multi carrier framework that means, it has spectral efficiency advantage. It is also a block based system that means, that one can actually transmit in blocks of data. For example, if one would have OFDM all go in parallel in frequency, this is well understood by all of us ok.

And all of them happen in one symbol duration ok, if it is a single carrier frequency domain equalization, single carrier means there is only one carrier, but one would send multiple time slots ok. So, this is f frequency division multiplexing, this is time division multiplexing, equivalent one can think in those notions, but here we have both time and frequency at the same time meaning one can equate one time block to that of an OFDM symbol. So, as if there are multiple OFDM symbols like not OFDM, but if one understands OFDM which can be grouped together and it forms one block.

So, I would erase all of these things, so instead of seeing it as like OFDM as we just said, OFDM would be thought of as one block in comparison to this. Whereas, this has multiple such blocks together and one can actually choose the different block combinations, one can choose to use this block, one can choose to use this block, one might choose to use this block or one might choose to use this block. So, this is a flexible structure that is has, it can have various combinations of parameters which is given by m parameter on this side and n parameter on the other side. So, there are two parameters m and n parameters. So, names can be different m means the number of time slots and the number of sub-carriers. So, one can flexibly choose it and can make various combinations.

Now, in OFDM what you would find is that generally for OFDM there is a symbol duration there is a C P, the symbol duration there is a C P, symbol duration in C P, whereas in GFDM it is suggested that you have a block of symbols and a common C P for it, so that effectively means that you process the entire block simultaneously that is another feature. And you can also see directly that because there is a cyclic prefix for every OFDM symbol, the amount of overhead that is present in such a system is much more than in GFDM system.

So, in GFDM system what we find is there is only one cyclic prefix and we have described earlier that the cyclic prefix length is dependent upon the channel constraint. And hence the cyclic prefix length in this case and this case would be the same. So, the percentage overhead is less in GFDM system as compared to OFDM system.

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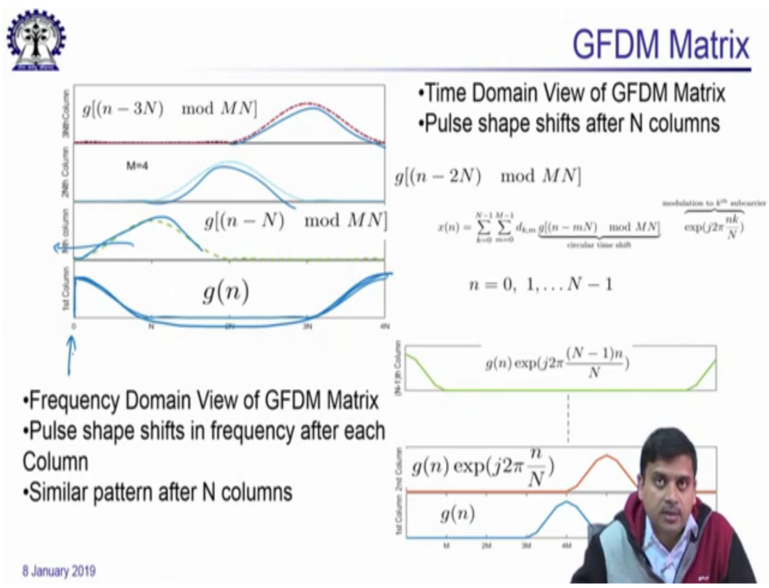

Moving down further, now in GFDM system what is done is there is a pulse shape for every symbol duration. So, for example, we can clear of all of these what happens is that yeah. So, what happens is this symbol duration there is a certain pulse shape ok, the next symbol duration there is another pulse shape and this symbol duration there is another pulse shape. So, this is one way of visualizing the entire thing.

And all of these subcarriers are having the same pulse shape. All of the next set of subcarriers, I mean usually all of them have the same pulse shape. It is one way of viewing it, the but more accurate picture would be that this every symbol there is there is a pulse shape and the pulse shape is not exactly restricted to this time duration and that is that can be understood from the way we have drawn this thing. So, this indicates that the pulse shape is a long pulse shape right. This is the ideal pulse shape for the center one, because it is symmetric.

For the other cases, it has to wrap around right, for the other cases it has to wrap around. So, this wrap around thing is this circular shift that is present in GFDM system, because if one would not wrap it around in that case the if we think of this pulse shape, this is the central pulse shape. So, when we use it over here, then it will be extending on the left hand side; if we use it here, it will extend on the right hand side.

So, effectively the signal the symbol is stretching over a much longer duration and hence the efficiency would be less. To improve the efficiency, you actually wrap it around you wrap it around on both the ends, and hence you are able to create a much higher spectral efficiency. And the flip side of it is that because you are wrapping it around, you are not able to get a smooth start at the beginning. So, if you are not able to get a smooth start this beginning abrupt start, in that case it is obviously, going to give you a high spectral leakage.

The other thing is that it has overlapping subcarriers like we had seen others so hence spectral efficiency is high, so that is that is not much of a problem. So, let us look deeper into the thing. So, what we have over here is one of the pulse shapes ok, then it is shifted, then it is shifted, then it is shifted and you can clearly see that if you wrap it around that means, when I move it here part of it is here and part of it is here. So, because of this when the signal suddenly starts, then you are going to get a sudden burst of increase in the signal level which causes out of band leakage. So, this needs to be controlled in a generalized frequency division multiplexing system.

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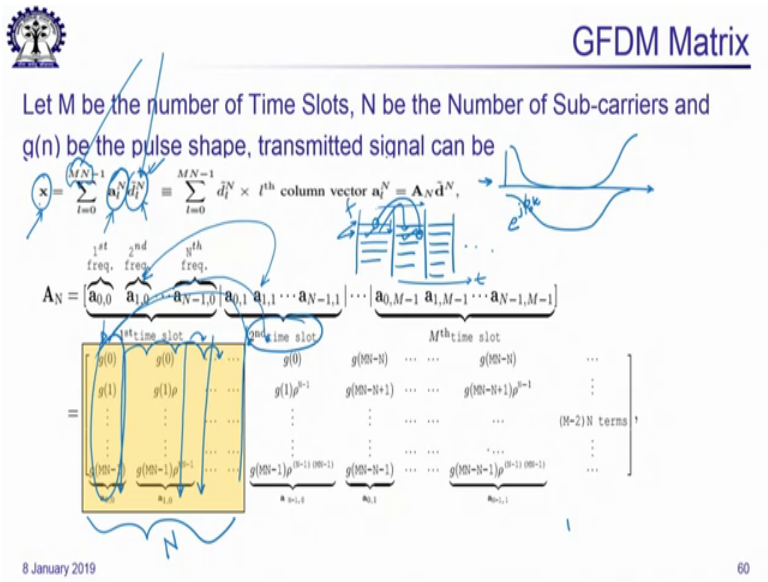

So, let us look deeper into the structure. So, what we have is there is this data symbols and these are the pulse shapes which are of critical nature. So, let us see how these are addressed. So, in the first time slot the g 0 to g MN minus 1 are the samples of the pulse shape of the first time slot.

The second set of signals indicate the frequencies shifted version of the pulse shape, so that means, if there is a certain pulse shape. So, let this be a pulse shape in the first time slot ok, in the first sub-carrier. The second sub-carrier is going to get the same pulse shape, but it will be frequency shifted by e to the power of j phi 0 times k indicating that it is a frequency shift from the first sub-carrier to the next sub-carrier ok.

The next column would be the same pulse shape with additional frequency shift, like that you are going to have N such sub-carriers, all of them are going to have the same time domain pulse shape, but they will each of them will be frequency shifted. The next set of pulse shapes, would be the pulse shape for the second time slot and that is a time shifted.

So, when we are talking about the Gabor version, so this is what we have in the Gabor version. This is the basic Gabor atom, these are frequency modulated Gabor of thus of the primary atom and from here that is the first column over here to the first column of the second slot, it is the time shifted version. And from the first column to this column over here, it is the time shifted and frequency shifted.

Now, if we compare the second frequency over here with this one, these two have the same frequency shift, but there is one unit of time shift difference between them. So, between this and this as has been pointed out there is no difference in the frequency shift that means, they are the same sub-carriers.

So, in the time frequency diagram when we had if these indicate the first block, if these indicate the second block, these indicate the third block and so on. So, basically this one and this one they have the same frequency shift, but they have two different time shifts. So, this is the time axis this is the frequency axis whereas, this and this they have a frequency shift with respect to each other, but there is no time shift between them, but if I take this one and compare with this one, so there is a time shift as well as frequency shift. So, there is a time shift as well as there is a frequency shift between these two right. So, this is what we should keep in mind. So, this is the primary Gabor atom and then we are shifting it and creating it right.

So, if you look into the equation carefully, you will be able to get how the signal is generated. So, every data symbol is multiplied with a pulse shape and there are M times N number of such pulse shape right. So, if we revisit back to the structure that we have over here; so, this is the basic Gabor structure and if there are M, N which is small m small n in this particular thing and capital M capital N over there. So, you have M.N number of Gabor atoms right, each Gabor atom is multiplied by a coefficient that is what we said, each Gabor atom is multiplied by a coefficient and it is sent out.

So, here correspondingly what we have over here is that each of them are multiplied by a coefficient and they are added together to produce the final signal form. So, because of the time and frequency block domain representation, you have M times N number of such Gabor atoms, and each must get a corresponding coefficient which is indicated by d in this case and the cumulative is the x in this particular slide, which is f in the original slide, where we had discussed about the Gabor structure.

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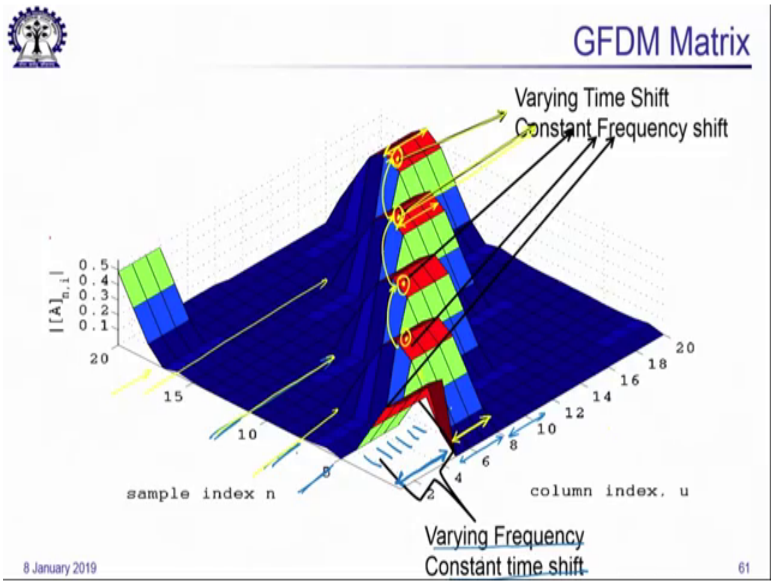

Like that this entire time frequency grid is filled out. So, this particular diagram helps us understand the basic layout that is what we have is on one side if we look at it carefully, so here the first few set of columns this picture is a scaled down version of what we should have. So, this entire equation that you see it is summation over l equals to 0 to MN can be constructed into a matrix equation ok. And it is a matrix multiplication, this is a vector and this is a matrix of size M N cross M N. So, this is a matrix of size M N cross M N and hence you can create this entire operation right.

So, what you see is that the first few columns they have the same time domain pulse, but each of them are frequency shifted version right, so that is what you see that varying frequency, but constant time shift ok, they have the same time domain. Now, what you see in the next set of columns corresponding to what we have here, the next set of columns. So, there is time shift, so from here you can clearly identify, you can clearly identify the time shift in the pulse.

The next set of columns, you can again identify the time shift right. So, we take a different colour yeah, you can identify the time shift, the next set of columns you can identify the time shift and then this one right. So, there are different time shifts for a particular time shift, these are the subcarriers different subcarriers. So, these are the frequency shift. So, now if you as been pointed over here this one and this one, these are two different time shift pulses, but within this structure they have frequency shifts, within this structure these are frequency shift. So, this one and this one they have the same frequency shift and this one has the same frequency shift, but between them there is a time shift. So, this is a pictorial representation of this column structure of what is actually represented in the analytical expression over here right.

So, we stop this particular lecture over here. And in the next lecture we will discuss about, the ways the receiver can operate there are different receiver structures and what is the performance comparison of all the different waveforms, that we have seen till now.

Thank you.