Evolution of Air Interface towards 5G
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Lecture - 34
Channel Models for Performance Evaluation - Part - 2
(Small Scale Fading)

Welcome to the lectures on Evolution of Air Interface Towards 5G. So, we are discussing propagation model. In the previous lecture we have discussed the large scale propagation model where, we have primarily seen the path loss as well as the shadow fading parameter.

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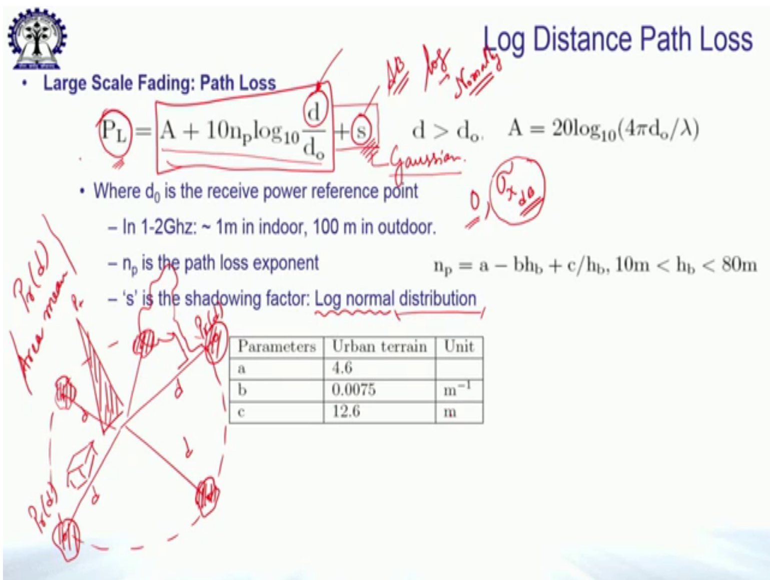

And we have also given you expressions or some samples of the path loss including shadowing expressions which are typically used for evaluation. So, briefly we have discussed about the average received signal strength in the area which is predicted as a function of separation distance between transmitter and receiver d which is predicted only by path loss is enhanced with a shadow fading parameter s. In the dB scale we have declared that it is this Gaussian distributed and with a 0 mean and a standard deviation sigma sub chi or x dB; it is given in dB because this entire equation is in dB. So, that is also given in dB and then we have expanded this thing and we have said that one can look at the typical profiles that are mentioned; as example in this particular one which is about IMT advanced where it talks about the path loss exponent and as well as sigma parameters.

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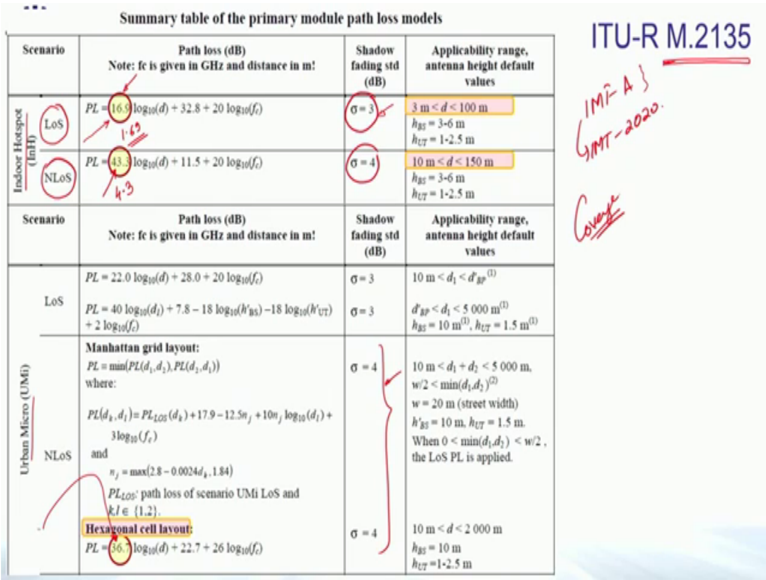

Summary table of the primary module path loss models

Scenario		Path loss (dB) Note: fc is given in GHz and distance in m!	Shadow fading std (dB)	Applicability range, antenna height default values
Indoor Hotspot (InH)	LoS	$PL = 16.9 \log_{10}(d) + 32.8 + 20 \log_{10}(f_c)$	$\sigma = 3$	$3\text{ m} < d < 100\text{ m}$ $h_{BS} = 3\text{-}6\text{ m}$ $h_{UT} = 1\text{-}2.5\text{ m}$
	NLoS	$PL = 43.3 \log_{10}(d) + 11.5 + 20 \log_{10}(f_c)$	$\sigma = 4$	$10\text{ m} < d < 150\text{ m}$ $h_{BS} = 3\text{-}6\text{ m}$ $h_{UT} = 1\text{-}2.5\text{ m}$
Scenario		**Path loss (dB) Note: fc is given in GHz and distance in m!**	**Shadow fading std (dB)**	**Applicability range, antenna height default values**
Urban Micro (UMi)	LoS	$PL = 22.0 \log_{10}(d) + 28.0 + 20 \log_{10}(f_c)$	$\sigma = 3$	$10\text{ m} < d_1 < d'_{BP}$ (1)
		$PL = 40 \log_{10}(d_1) + 7.8 - 18 \log_{10}(h'_{BS}) - 18 \log_{10}(h'_{UT}) + 2 \log_{10}(f_c)$	$\sigma = 3$	$d'_{BP} < d_1 < 5\,000\text{ m}$ (1) $h_{BS} = 10\text{ m}$ (1), $h_{UT} = 1.5\text{ m}$ (1)
	NLoS	**Manhattan grid layout:** $PL = \min(PL(d_1,d_2), PL(d_2,d_1))$ where: $PL(d_k,d_l) = PL_{LOS}(d_k) + 17.9 - 12.5n_j + 10n_j \log_{10}(d_l) + 3\log_{10}(f_c)$ and $n_j = \max(2.8 - 0.0024d_k, 1.84)$ PL_{LOS}: path loss of scenario UMi LoS and $k,l \in \{1,2\}$.	$\sigma = 4$	$10\text{ m} < d_1 + d_2 < 5\,000\text{ m}$, $w/2 < \min(d_1,d_2)$ (2) $w = 20\text{ m}$ (street width) $h'_{BS} = 10\text{ m}$, $h_{UT} = 1.5\text{ m}$. When $0 < \min(d_1,d_2) < w/2$, the LoS PL is applied.
		Hexagonal cell layout: $PL = 36.7 \log_{10}(d) + 22.7 + 26 \log_{10}(f_c)$	$\sigma = 4$	$10\text{ m} < d < 2\,000\text{ m}$ $h_{BS} = 10\text{ m}$ $h_{UT} = 1\text{-}2.5\text{ m}$

So, that is what we have identified in the previous lecture.

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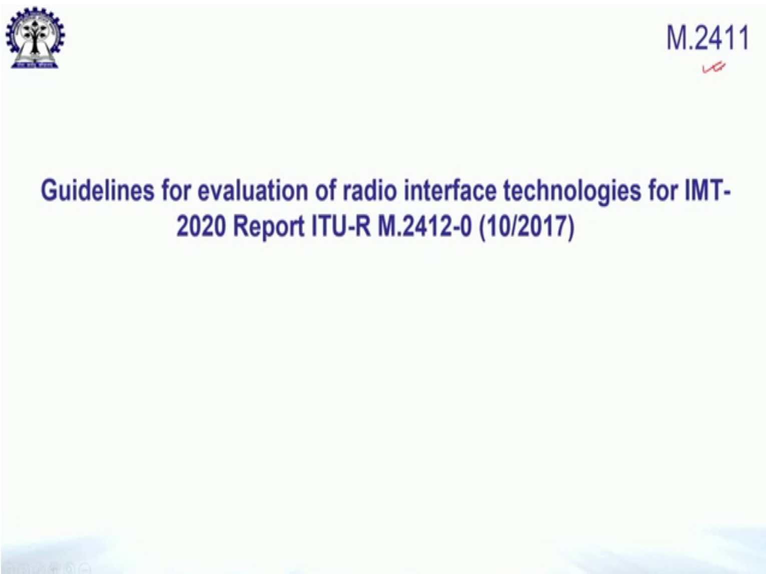

So, using these models one can find coverage probability which will shortly see and we have also been talking about this IMT-2020 which is nothing but the 5G.

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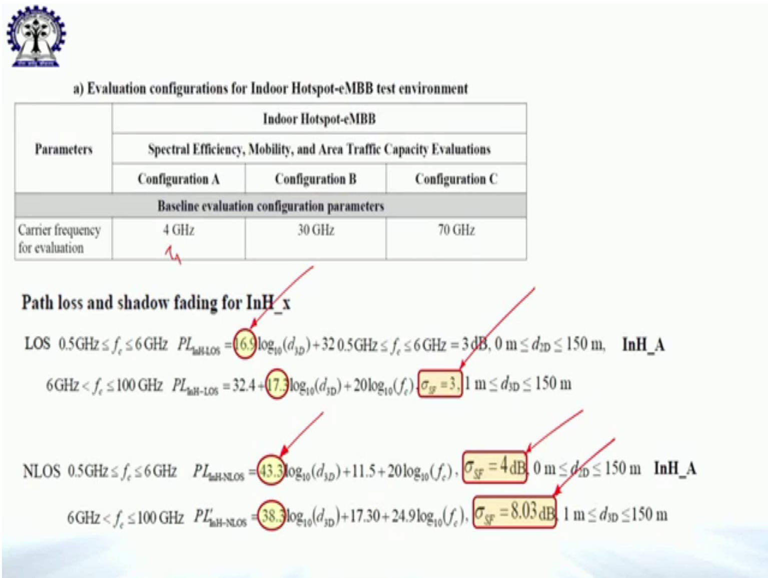

So, there also we have been looking at the different path loss exponent and they have not changed over significantly for the fourth generation evaluation models to the ones used in the fifth generation model and those are path loss exponent these are the sigma dB that is what we were talking about. So, using these one can generate various realizations of the channels and one can find various performance aspects of it. So, that is something which one should be aware and we will briefly talk about one particular way of calculating the coverage probability so, let us look at that.

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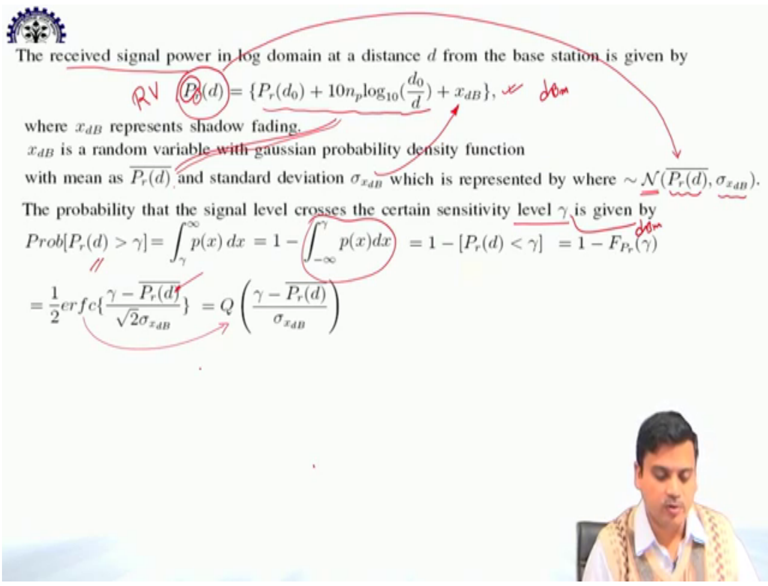

So, what we are interested using these models because these are talking about large scale propagation models about the coverage probability in a particular area. So, again we will talk about the methods that are used. There are various methods we will just talk about by the first principle beyond this there are several advanced techniques which have been developed, but it is probably difficult to put everything into one platform. But, of course there are relevant papers which one can follow using whatever we are discussing here.

So, the received signal strength in the log domain at a distance d from the base station is given by this particular expression which we have explained so long and where x dB represents the shadow fading parameter; it is a random variable with Gaussian probability function because, it is in dB and with a mean of P r d. So, this received signal power in log domain has a mean value of this which is nothing, but this part and a standard deviation of sigma which is for this particular thing ok. And therefore, it is represented as; that means, this is distributed you will find that this is distributed as normal in the dB because, everything is in dB with a mean and the corresponding sigma that is how that is represented alright.

So, the probability now since this is a random variable so, this is a random variable. So, the probability that is the received signal strength, the received signal strength that is how we would call it process a particular sensitivity level gamma right. So, this is in dB m right. So, this is the received signal strength is in dB m crosses a certain sensitivity level which can also be given in dB m is given by the probability, that the received signal strength is above that threshold that is it, that is what we want to calculate.

So, that is simply integrate from gamma to infinity the PDF; Probability Density Function now with the variable s yeah that is where we are back ok. So, this can be expanded as 1 minus integrate minus infinity to gamma and you can clearly recognize this is the CDF. So, which is 1 minus it is less than the threshold gamma which can be expressed as 1 minus the CDF and since it is Gaussian distributed we now know what is it is going to be half complementary error function gamma minus the mean value divided by root 2 sigma.

So, we see that we can calculate the probability of coverage at a particular distance given the description of the path loss model and as one increases the difference one can find the coverage probability at that particular distance. So, this of course, can be translated to Q function where you can see that half and root 2 has been absorbed that is the standard translation.

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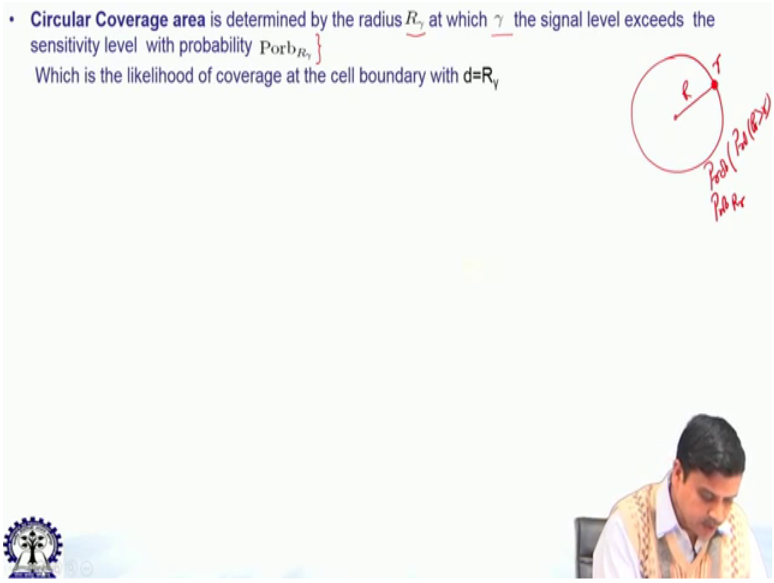

So, what we are interested in calculating is the circular coverage area which is determined by the radius R sub gamma, that is R sub gamma is the radius at which the signal level crosses the threshold of gamma right and what is the probability of doing that. So, you define a coverage area as a coverage area is maybe a circular coverage area with a particular radius wherein, the at the boundary the signal crosses the threshold of gamma with a probability that P probability that received signal strength is greater than gamma which is defined as prob R gamma right that is it that is the probability value.

So, that is how you define the coverage probability that at the boundary with this radius what percentage of time one is covered. So, which is the likelihood of coverage at the cell boundary; let me clear of all the ink on this particular page yeah.

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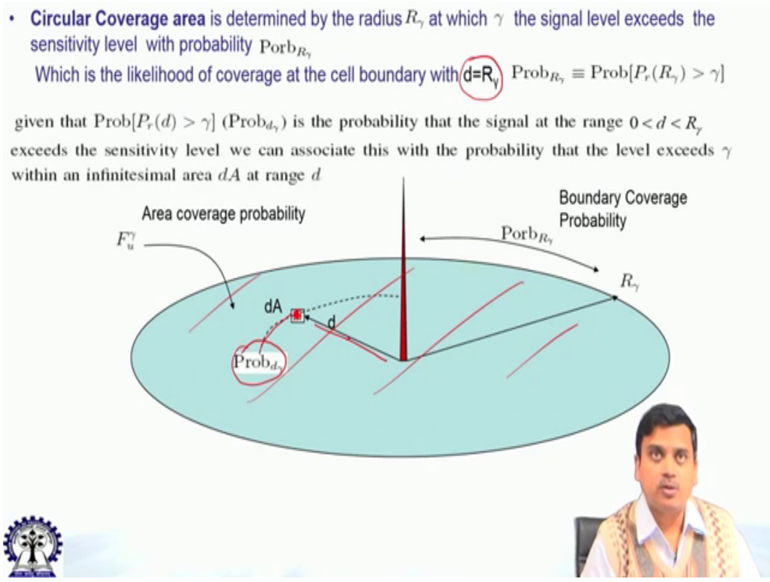

So, the likelihood of coverage at the cell boundary with d will now be set equal to R gamma; that means, coverage at the cell edge. So, the way to do it is whatever probability of coverage we have been discussing can be associated with an infinite small area at a particular distance d from the center. And, then we can simply integrate or average out this probability over the whole area that is the whole idea, that is what we want to do in order to find the coverage probability alright.

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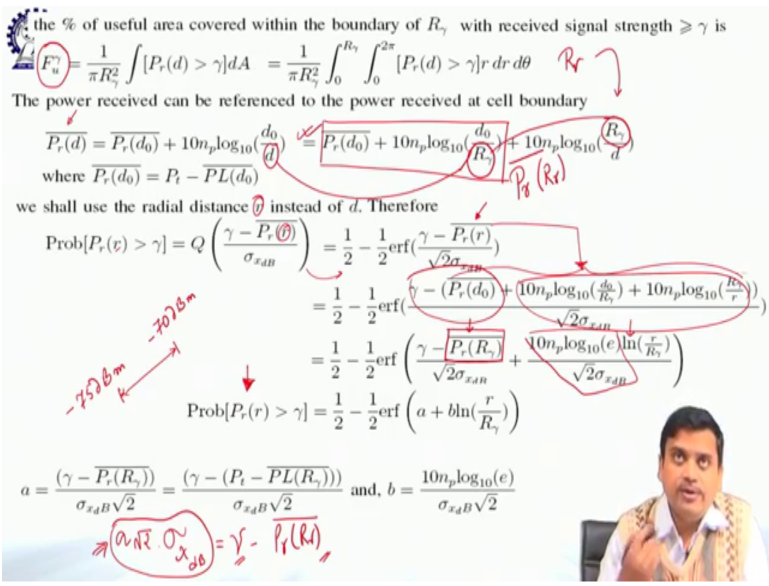

So, moving ahead now; so, the percentage useful area is simply the area averaged probability of coverage right that one can calculate. And, to do that it is sometimes essential to translate the expressions in terms of received signal strength at the cell boundary or the cell edge, then things becomes easier. Because, through path loss model one can easily calculate the average received signal strength at the cell edge and from that one can do all the calculations so, things are better. So, we know that the average received signal strength at a distance t is by this expression. We have been talking about this and now you have to translate this expression to the one in terms of R gamma.

So, if we look at the expression between this and this there is no such big change because this d is now replaced by R gamma and it is again cancelled by R gamma. So, the equation remains the same, only thing is that we have introduced the parameter R gamma. The advantage is when we look at the entire expression; that means, if we look at this particular block then one will easily recognize this is P r at R gamma bar; that means, the average received signal strength at cell edge. So, we can now reference things with respect to this and P r d 0 bar is nothing, but the average received signal strength at d 0 we have defined what is d 0.

In the next few steps you replace d with r because, that is the cell radius or the radial distance and therefore, you can use the probability of coverage at a distance r from the center is the same expression where, the d is replaced by r and the same expression; it was complementary error function which have expanded in terms of error function. And, then this P gamma that is what you have over here P average received signal strength; you have expanded over here which is again not new. And, then what you can find is again this at the denominator and at the numerator cancels out. So, it is the whole expression is nothing, but the received signal strength at the distance right alright.

So, we will erase some of the ink to reduce the clutter ok; proceeding further what we now do is we club these two terms over here and the rest of the terms in the next step and we have a meaning associated with it. So, we say that let a denote this thing so, this is a and we also have this thing denoted as b right that is what is given below. So, if you have these two replacements you can have probability of coverage in terms of error function as written in the expression above. So, now one can use the different path loss models that have been described in order to calculate the coverage probability at the cell edge.

Once you calculate coverage probability at any distance or at cell edge then you can go back in calculating the percentage useful area of the cell that is possible. Now, one quick interpretation on this is that, if you concentrate on the term a what we have I mean, if we look at this a multiplied by root 2 multiplied by sigma x dB is equal to gamma minus P r R gamma bar; indicating that this term is capturing the difference between the sensitivity level and the received signal strength right. So, let us say sensitivity level is minus 75 dB m and the received signal strength is minus 70 dB m.

So, this term is able to capture the margin that is present between the sensitivity level and the average received signal strength and that influences the coverage probability. So, you can now see in terms of margin and that is because you have over here the average received signal strength at this cell edge. And, that is why the earlier introduced concept of translating things to cell radius and receive signals power at cell edge is very very important right.

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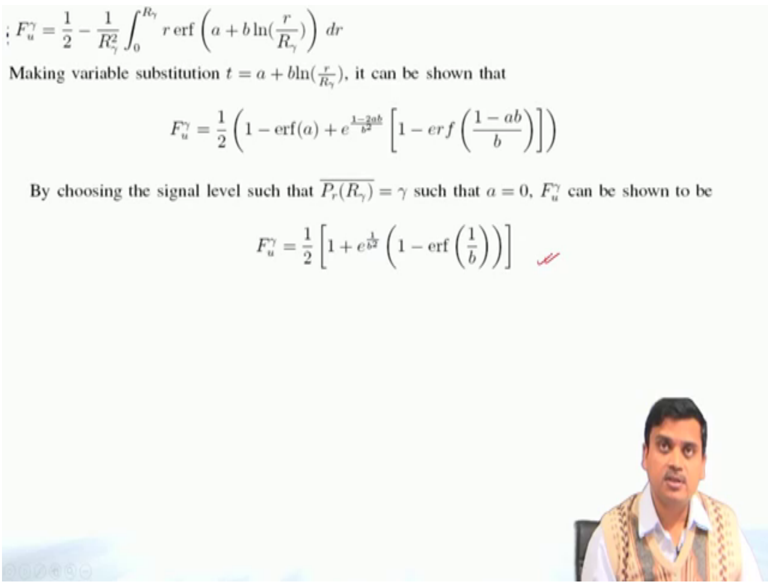

Moving ahead then as we said one can calculate a few and there are different ways of doing it, you get certain expressions. Well, this is for available for reference, but that is it.

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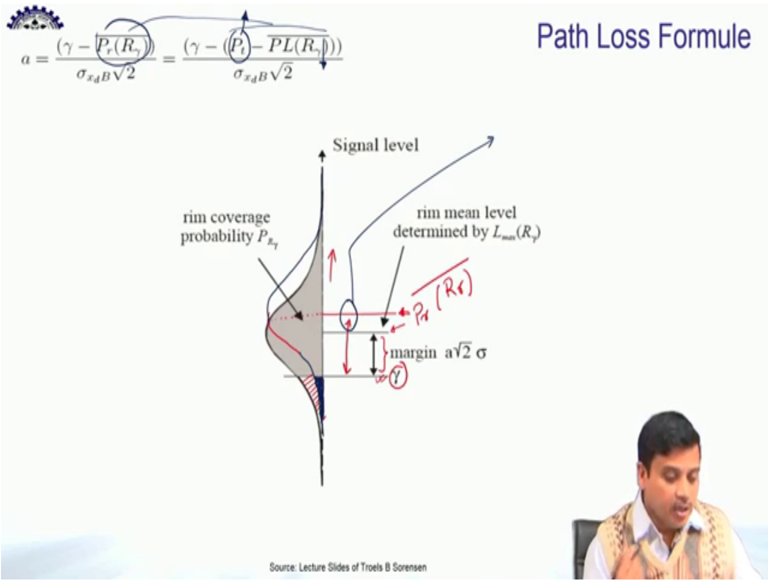

So, we have discussed about the margin and that is a graphical representation of what we have just discussed. So, there is a threshold and this is basically P r received it is P r of R gamma bar. So, that is and this particular thing is the margin right. So, if we are actually calculating we are actually calculating this particular area which is kind of outage probability. So, clearly if this is fixed and we want a higher or higher coverage or lower outage probability then we should shift this entire thing upwards.

In order to shift the entire thing upward all you can do is to increase the margin, if you increase the margin your average received signal strength will now shift there and, hence your mean point is supposed to shift over there. So, you are going to get your curve which will look like this right and hence, your area under coverage will be this. So therefore, which is a much smaller area and you have reduced the outage probability or you have increase the coverage probability. So, to shift this upwards you can do two possibilities, if you look back. So, what you have is a P t term over here. So, basically you have to increase this term, to increase this term either I can increase this P t term that is increase the transmit power or I can decrease this term.

So; that means, the path loss value would be smaller and hence this overall expression has to increase which is nothing, but this expression correct. So, two ways to do it and that is very logical if you reduce the cell coverage, if you reduce the cell radius then you have increase the coverage probability. Or, if you increase the transmit power you have increase the coverage probability, but the problem is interference comes in two plane you have to handle it in a different manner.

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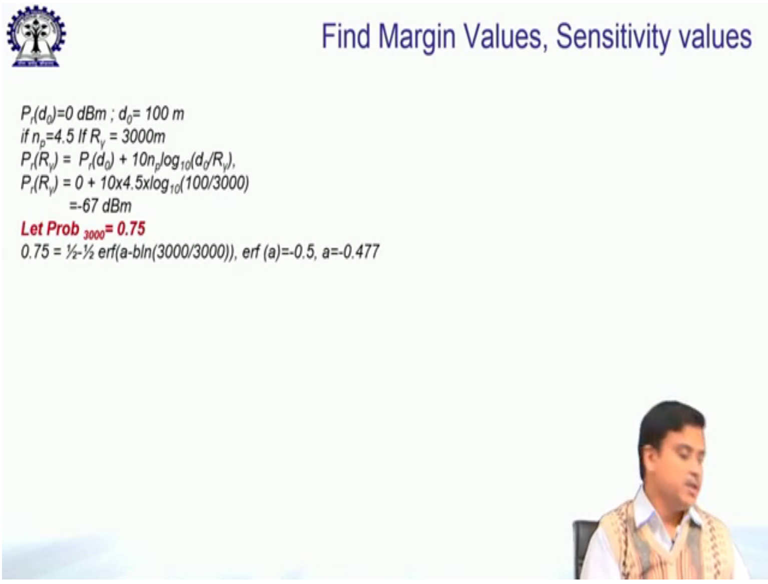

So, there are certain examples assignments which will be made available which you can take advantage of and work on using these things. So, with this we move on to the next set of discussions especially on the channel structures which is primarily about the small-scale fluctuations. So, that is very critical.

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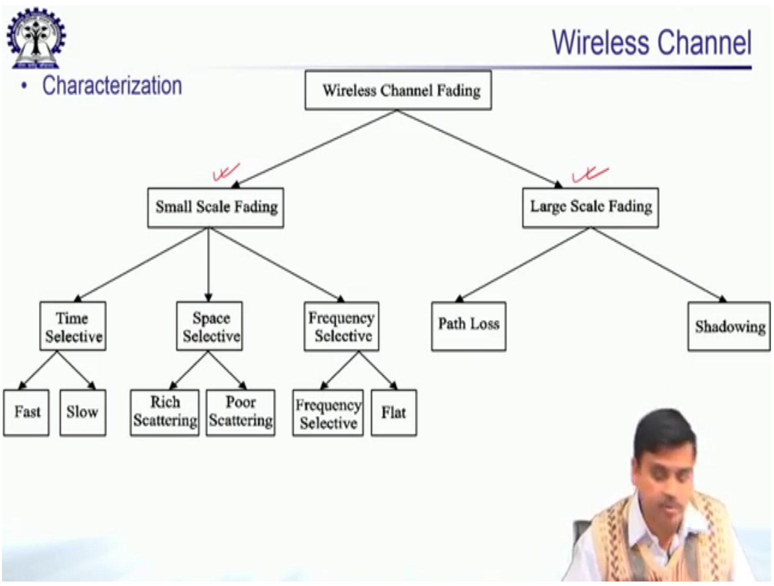

So, we have talked about the large scale fluctuations and so, we are done with the large scale fluctuations. We will take an overview of small scale fluctuation so, that we understand what happens. So, that we can quickly look into the effects of how does mine work and what is the advantage and you can probably these device better schemes even beyond what exists today.

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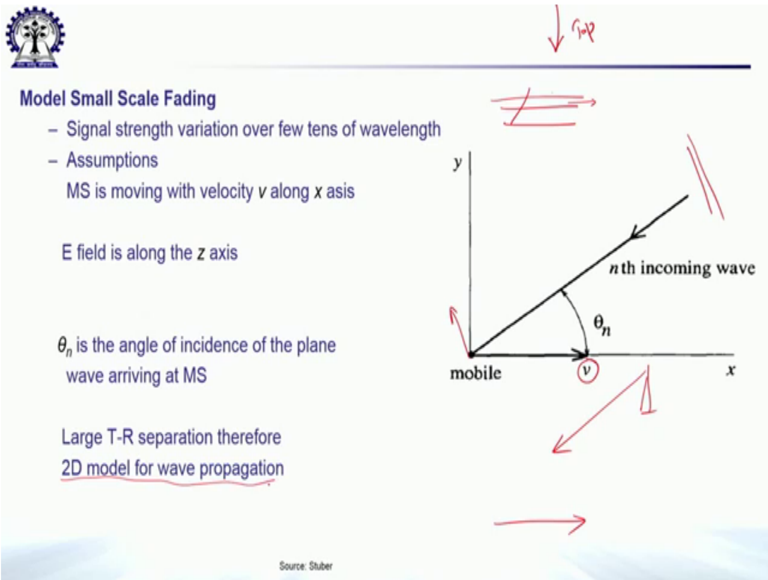

So, in the in the small scale fading we have actually described the small scale fading. So, we will briefly take a look at some of the fundamental models. So, here what we assume that you are taking a top view of this plane. So, basically you are looking from top and there is this floor area or there is a road on which the vehicles are moving. So, you are seeing basically from top that is what is happening. So, the vehicle is moving along the positive x axis with a velocity v and there is a incoming a plane wave from a particular direction indicating that your mobility is in this side. And, some base station is here and the signal is coming there or it might come by a reflection also from that particular direction ok.

So, E field is along the z axis; that means, it is kind of in this direction and theta n is the angle at the MS. So, certain assumptions are that transmitter receiver separation distance is very large and one can use 2D model for a wave propagation. So, that is what we have already discussed.

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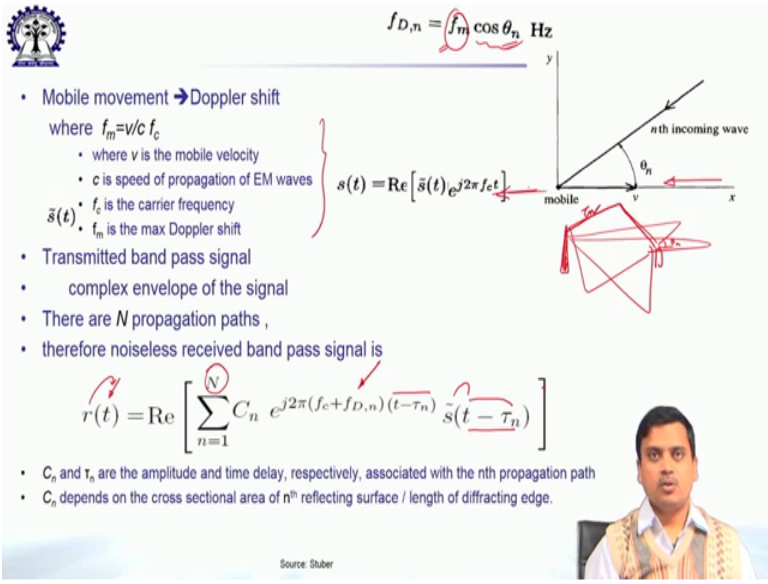

So, it is the same issue now that what we are talking about. So, the Doppler due to mobility is given as f D n is equal to f m cos theta n, where you have the where you have f m denoted as v c; v by c times f c which is the maximum Doppler shift. So, you can clearly see that if cos theta equals to 0; that means, if it is coming from this direction then the Doppler shift is maximum. If theta is from the opposite direction; that means, if it is in the reverse direction it is negative of that value. So, the least possible value in that case or the negative of the amplitude or the maximum value.

And any other direction you can replace with the cos theta, this is a component of Doppler along the direction of mobility and the rest of the terms are defined over here. So, the transmitted band pass signal one would usually write as the real part of the baseband complex envelope s tilde t e to the power of j 2 pi f c t, that is that is a standard model. And, s tilde t is the complex envelope of the signal, there are n propagation paths and therefore, the received signal r t. So that means, if you recall the diagram there is this base station, there is this user device. If the signal comes via multiple paths forming different angles and each of these angles are theta n.

So, the received signal is a summation of the signals that has come via the different paths with coefficients C n indicating the attenuation or amplitude factor of the signal coming along the n th path. And, each of the path has an associated Doppler frequency with it so, f c plus f D n and what we will see is that the signal, that is received at time instant t.

Signal that is received at time instant t is the original source symbol, but it must have started at a time tau units before the present time. If it starts at time tau tau n units before; that means, it has taken a propagation time of tau n corresponding to the delay associated with that particular path. And, hence we have t minus tau n replacing t in all the equations.

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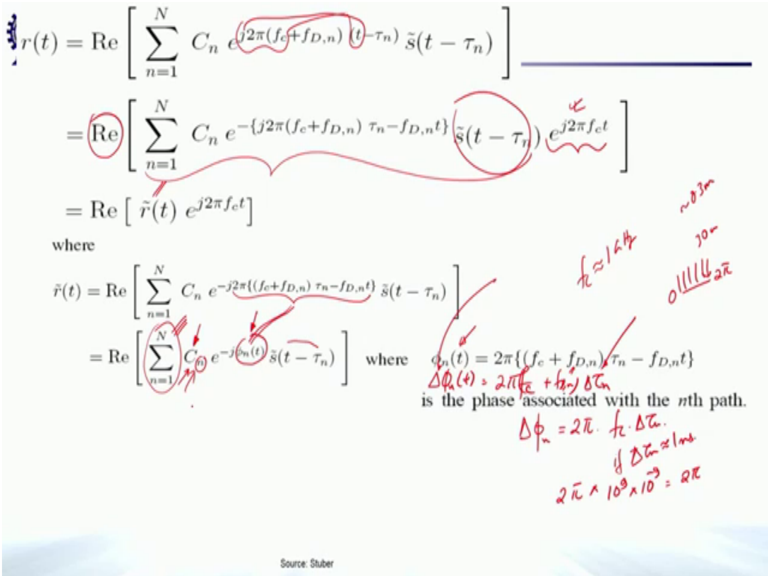

So, we proceed with this so, that is that is basically the signal structure. So, this is the received signal which we had seen in the previous page. And, if we expand the equation what we will do is we will just see e to the power of j 2 pi f c t is the term which we have collected over here and rest of the terms we have kept together. And, that is there is a specific reason for this because, e to the power of j 2 pi f c t is the situation what we would like to handle separately because, this is the pass band signal and with the with the real part of it.

So, what we can clearly realize is that this is the equivalent received baseband signal when this has been transmitted and that is what exactly is written over here. So, this entire thing is equal to r t under t right . So, that is that is what has been expected. And, then in the next expression what we have is this entire term is we are collecting together into the term phi n of t indicating that each path C n each n th path is having an associated amplitude and an associated phase with it. This is very very important and the signals by each path are coming with a delay; that means, different signals are coming by a different paths right.

So, the signals which have been generated at different instants of time corresponding to the path delays are coming and getting added together at the receiver. So, this is causing ISI and this is well known. So, what we look at is the is a phase component associated with each of the paths and one can easily calculate the phase difference because, of path difference. And so, if you are taking the delta at the same instant of time; that means, if two paths differed by delta tau n, two different paths then we can compute the resultant difference in phase.

So, simply since let us take f c is almost equal to 1 giga Hertz, let us neglect f D with respect to 1 giga Hertz. So, what we have essentially is the path difference between let us say 2 is 2 pi times f c times delta tau n. Now, if you let delta tau n is approximately equal to 1 nano second which means that two path lengths are different from each other by 1 nano second. What you will find is that the phase difference is 2 pi times 10 to the power of 9 1 giga Hertz multiplied by 10 to the power of minus 9 that is ; 1 nano second. So, together it is giving you around 2 pi phase rotation; so, if two paths are different by 1 nano second.

So, if you take the speed of life it will approximately turn out to be 0.3 metres; that means, 30 centimeters. If two paths are different by the 30 centimeters there will be a 2 pi phase difference and you are adding up several of them. So, even if paths are different even less than 30 centimeter difference; so, you are going to get the phases which are spanning 0 to 2 pi. And, many of them are coming together and we are adding them together. So, what you can see is that there is a mixture of different phases along with different amplitude factors associated with the coefficients of reflection.

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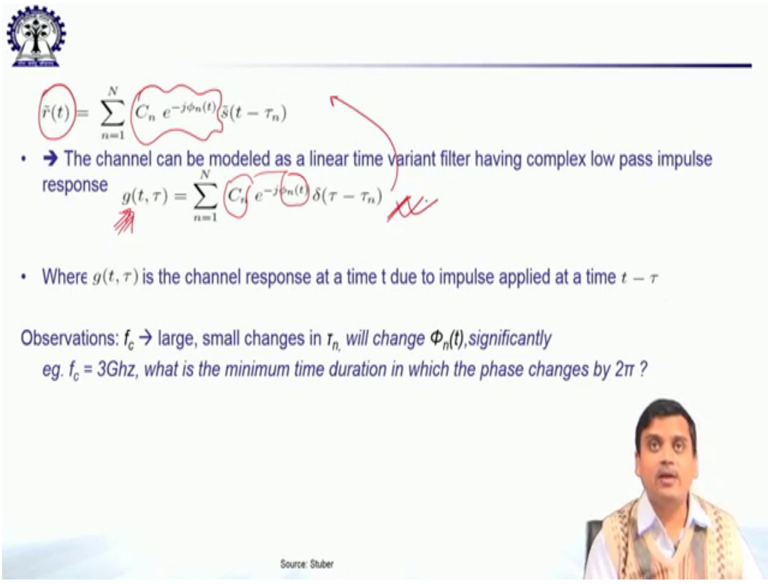

So, this particular channel representation of the received signal that we see over here can be modeled as a linear time variant filter having a complex low pass impulse response given by g t t comma tau as given in this particular expression. So, if you compare these two you will easily figure out that these are coefficients of the channel impulse response, where the channel impulse response is given by this particular expression. So, where each tap has a gain factor and a corresponding phase factor. So, then we are interested to look at the channel impulse response characteristics at any instant of time. So, what we will find is we have actually discussed that part.

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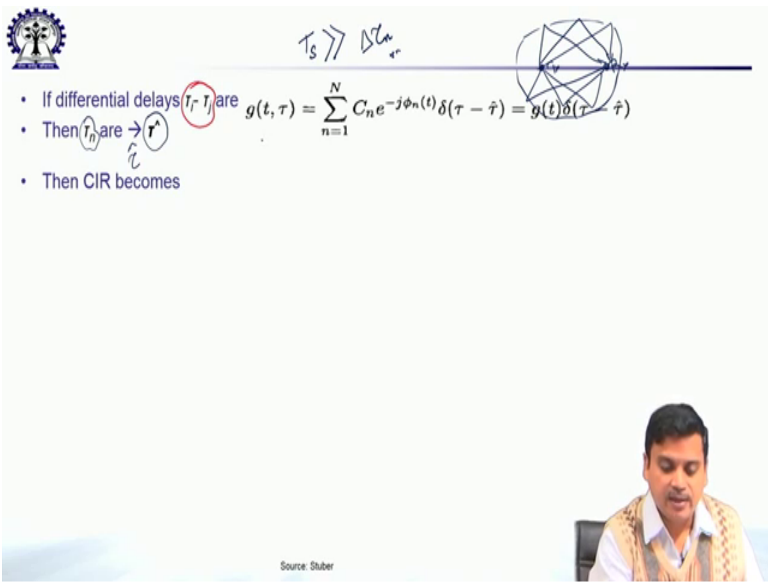

So, what we will now make is a very very important assumption is that the let the delays, the relative delays be very very small relative to the symbol duration. So that means, if we assume that the symbol duration is really large; that means, T s is the duration is much greater than delta tau n, I mean for all n let us say right. That means, this is not the exact notation because we are talking about difference between two delays.

Then we can approximate all the different delays to one particular delay that is tau cap that is an; that is a kind of situation. So, what is that? So, this is the situation when all the path lengths are almost same right. So, if they are on an ellipse, if all the scatterers reflectors are on an ellipse whose two focal points are the transmitter and receiver. So, in that case the trans; the length would be the same and that is a kind of approximation or a scenario that we are looking at.

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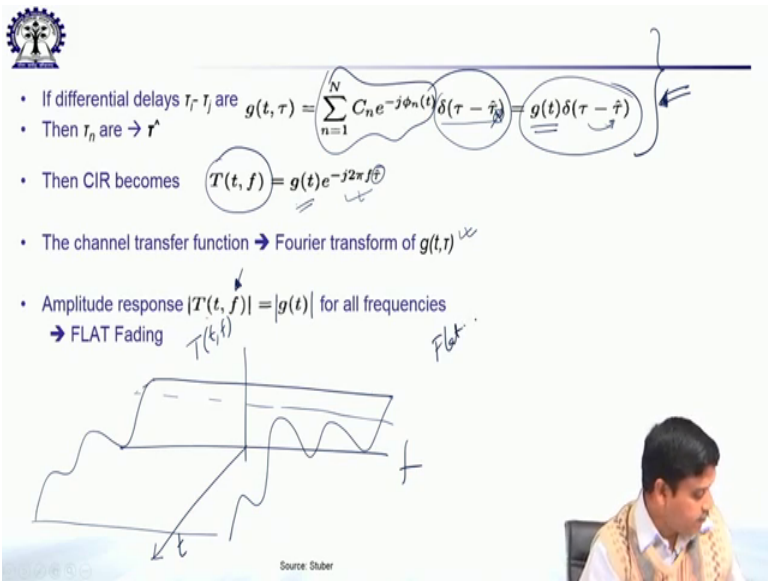

So, under that condition we will find that the impulse response which we had drawn which we had written can be written in this form; that means, you are going to get delta tau minus tau cap instead of tau n. So, you are removing this tau n and hence this is out of the summation, it goes beyond the summation. So, this entire set can be written as g t with the single delay factor right. So, that is what is the channel impulse response and you are interested in the in the Fourier transform of it.

So, if you take the Fourier transform of the channel impulse response. So, you get it as capital T f and you just look at it. There is a delta function, you take a Fourier transform;

if g t with e to the power of j 2 pi f tau cap right and then what we are interested in is the amplitude response. So, if you take the amplitude of g t what you will find is that the modulus of g t modulus of T f would be left with modulus of g t. So that means, it is not a function of frequency anymore and this is primarily because of this assumption set that we have made.

So, under these conditions the amplitude response is not dependent on the frequency; that means, if we have a frequency f and if we write T of t comma f at a particular instant, it will be a constant value across all frequencies. And, if we have time in this axis then at every instant of time we can imagine that this bar to be at different values right, that is what this bar is going to fluctuate like ok. So, that is what gives rise to flat fading; what we have been talking about for a long time and that is one of the components of the study that is what we have been looking at right.

Quickly we can even say that, if that condition is not true anymore; that means, if this condition is not true. If this condition is not true, if this condition is not true, if this condition is not true; that means, we are left with a situation like this then what is going to happen.

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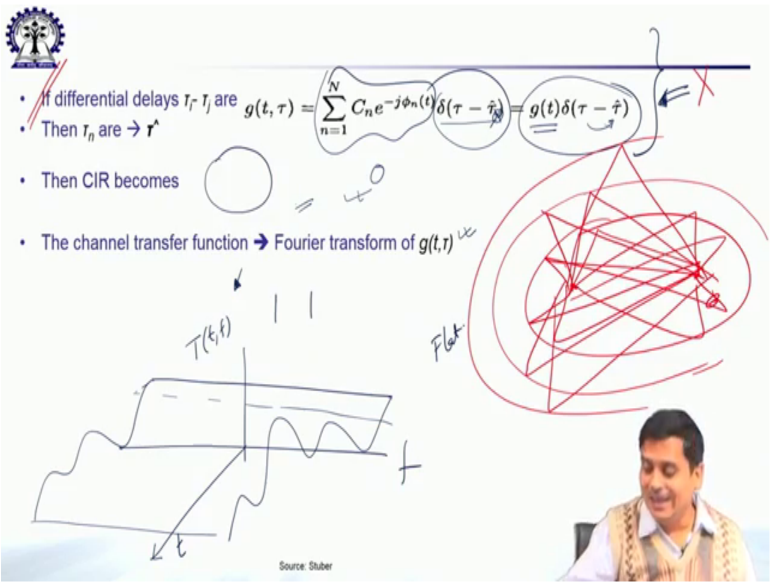

That means we do not have a situation where, the transmitter and receiver are at the focal points of an ellipse and all reflectors and scatterers are coming from the same ellipse, this is the right path I mean like this. What if there is they are the focal points of other ellipse also and the signals keep coming like this. So that means, there are resolvable delays right. So, let us take a quick look at what happens when they are dissolvable delays right.

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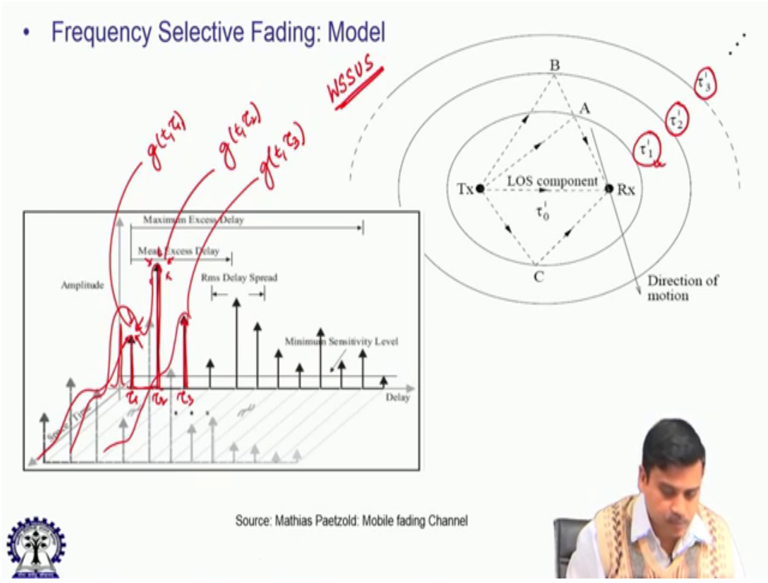

So, that is the kind of picture that we get when they are resolvable delays and there are groups of reflected waves which come at a certain delay, which we have just finished studying. Another group would come at another delay and we have also studied the effects of that, another group would come at another delay which we have also studied. So, what is happening is if you would launch an impulse, an echo is going to come at a certain delay which is let us say tau 1 and here all rays from different directions are going to come. And, they are within the same delay unit that is tau 1. The next group of echoes are going to come together and they will add up and will not be able to recognize them separately.

So, this one would be g of t comma tau 1, this would be g of t comma tau 2 in correspondence to what we have just discussed. So, in this situation what we have is all the properties that we have discussed in the previous set, same and valid for any one particular delay at any particular delay. But, we have this entire series that is valid in the entire thing and each of them fluctuate in time according to their own policies; policies in the sense in a random manner. There are various ways of structuring this, there are various models of doing it. One of the most common models that are used is Wide Sense

Stationarity Uncorrelated Scattering model which is followed in such analysis. So, we will just briefly tell you the effect of such a thing.

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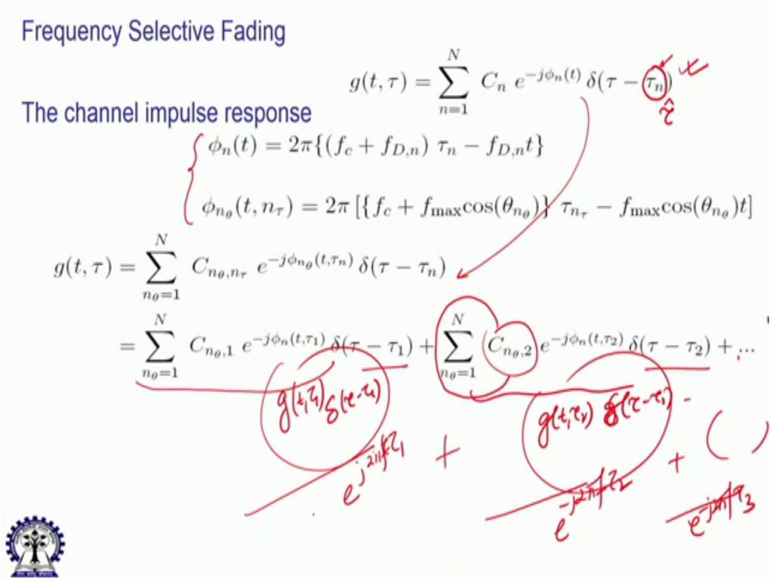

So, what we have is this expression which you can easily recognize from the previous set of equations, that we had been talking about. We had made the assumption that let these different tau n's be replaced by tau cap. Now, we are saying that no, let us go back and keep this original tau n and see what happens. So, simply you are going to expand this expression. So, you can you can decide to omit this because, it is just rewritten over here, we have the same expression over here. So, you would simply have these things as delays, delays and these coefficients. So, we have already explained that at each delay you are going to get adding up of all the different rays that come at the same delay.

So, any one of them, if I look at this would correspond to g of t comma tau 1, this thing would correspond to g of t comma tau 2 and so on and so forth. So, basically this is g of t comma tau 1 multiplied by delta tau minus tau 1 multiplied by delta tau minus tau 2 and so on and so forth. Now, if you take the Fourier transform, Fourier transform of this it is a linear operator plus Fourier transform of this plus Fourier transform of the next and so on and so forth. Each one of them are flat that is for sure, but they come with a certain phase factor, it comes with a certain phase factor right. And so, when they add up together they create a complete different picture.

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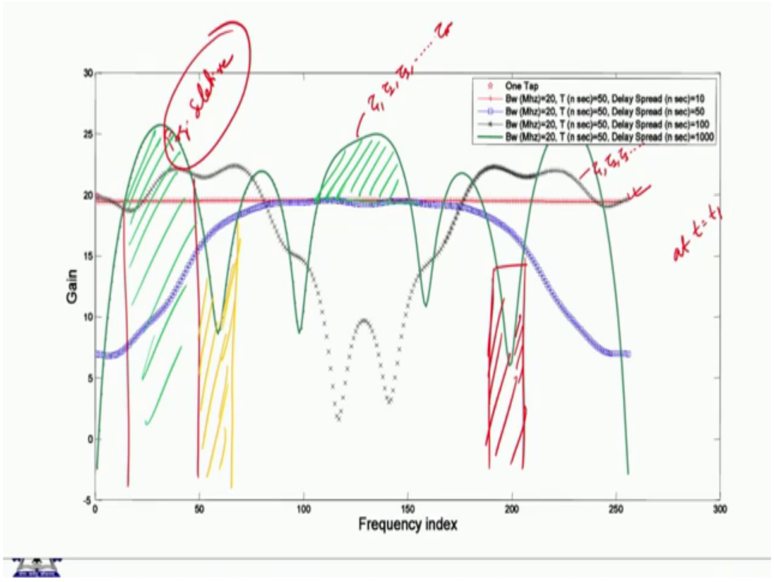

So, what we will see is that if you look at the graphical representation, if there is only one tap only one equivalent delay, this is the frequency response that you want to get. If you have let us say two delays; then the frequency response time snapshot; that means, at any instant of time at t equals to let us say t 1. It is not tau, it is time it will be this; as the number of delays resolvable delays increase; that means, in this case if I say I have tau 1, tau 2, tau 3 and so on the channel becomes more and more frequency selective. Why do we call it frequency selective? The simple reason is, if I look at this last one or maybe if we go further this particular one may have tau 1 tau, 2 tau, 3 and so on up to some tau n.

These set of frequencies we just change the color to match it, are allowed to pass through with a certain amount of gain. If we look at another set of frequencies here, these set of frequencies are kind of relatively attenuated with respect to other frequencies. So, again we choose back the color, these set of frequencies are allowed to pass through with less attenuation and these set of frequencies are kind of subdued or they are more attenuated. So, there is selectivity across the frequency compared to flatness across the frequency between the two conditions. So, this is again a very very vital situation that we have to use while studying the different effects.

So, we stop this particular lecture over here and we will continue with the one more lecture at least to consolidate these issues, before we can get into a study of (Refer Time: 32:13) communications.

Thank you.