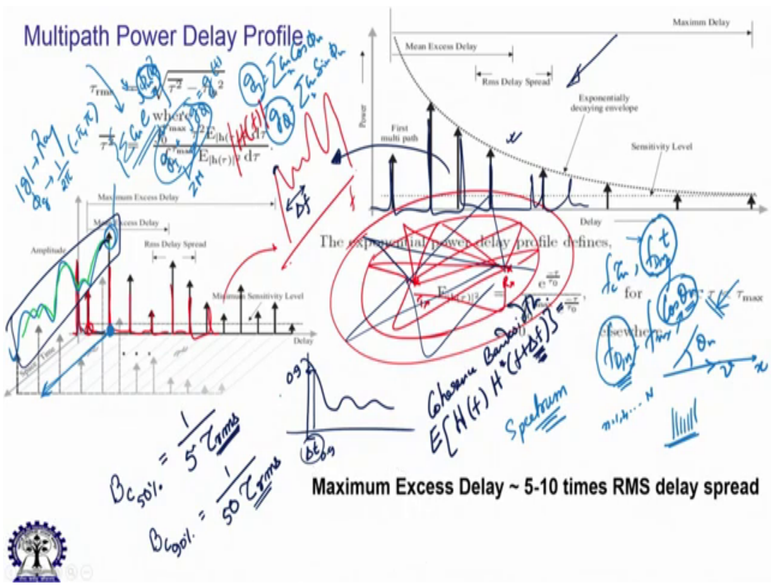
Evolution of Air Interface towards 5G
Prof. Suvra Sekhar Das
G. S. Sanyal School of Telecommunications
Indian Institute of Technology, Kharagpur

Lecture - 35
Channel Models for Performance Evaluation - Part – 3
(Spatial Channel Model)

Welcome to the lectures on Evolution of Air Interface towards 5G. So, we are looking at the propagation characteristics, we have looked at the large scale propagation models, we have started looking into the small scale models. In that we have looked at the flat fading condition, as well as the frequency selective fading conditions. So, we are now ready to move forward towards studying the MIMO channel, but before we proceed there are a few more minor things, which we one should one should look at.

(Refer Slide Time: 00:41)



So, here we take a brief look at it. So, we have discussed the frequency selectivity in the previous lecture and what we need to just take a further look at some of the additional things, when we combine the different aspects together for the different channels that we have. So, we will get a profile which is better described in this particular image. So, what we have essentially is that we have been talking about a situation, where an impulse is launched and we get echoes.

So, we will choose a different color, we will get echoes at different delays which gives rise to if you plot in the frequency domain frequency selective characteristics. So, on this axis there will be H of f ok so, you know that frequency selective characteristics. Now, if we look at any one delay, so this is the impulse that has been launched and these are the echoes that come in. So, if we look at any one echo effectively, this is all about the transmitter and receiver located at the two focal points of an ellipse, which contains the different scatterers reflectors right, this is what we have said.

The second delay is again for a second tier of reflectors or scatterers, this is also what we have discussed earlier. So, what it means essentially is that each of the delays and the magnitude over here is due to summation over several of the components. And we have accepted this particular model and we studied this under the flat fading system. So, if we look at any one particular tap, we find that is the summation of several such coefficients and this can be broken down into two parts g I and a complex because of this particular complex part g Q, where g I one can write it as summation of C n cos phi n and g Q can be easily written as summation of C n sin phi n.

So, because we have a large number of summations over here, each of these individually can be modeled as Gaussian random variables. And hence when we have g of t which is can be represented as g I plus j g Q in the complex form, each having normal random distribution, mod of g will follow Rayleigh distribution. Under the assumption that this is 0 mean, as well as this is 0 mean. And they be in quadrature we will get that the modulus is Rayleigh distributed and the phase of g you will find it as uniformly distributed in the range of minus pi to pi. This is a standard result; we are not going to derive it in this course, details are there in the other NPTEL course on MIMO communications.

So, if we focus on any one particular tap or any one particular delay and its time evolution, we will find that the signal changes with time. And this is well captured within this model, through the development of phi n which is a function of time, the model we have seen before. So, the phi n which is a function of time there are two parameters one is f c tau n that is related to the delay and there is f Dn times t. So, this is the term which allows the entire thing to grow with time.

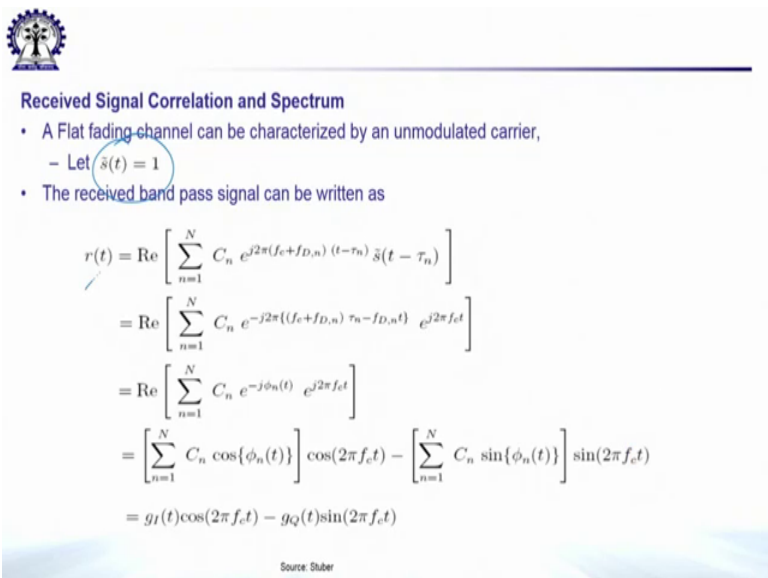
And what we have is several such different components because of different values of n and just to remind you f Dn is equal to f max times cos theta n and cos theta n is due to the angle of propagation that means, v is propagating along this direction that means, the object is propagating along positive x axis and the waveforms are coming at an angle making an angle theta n with the particular receiver.

So, under this consideration what we find is that the Doppler frequency is present in the phase term, which allows it to grow. And you have different Doppler frequencies coming from different directions. So, if f m is the max Doppler frequency because of cos theta n term there is an effective different value of Doppler frequency. So, each tap experiences several such Doppler frequencies added together to get the cumulative effect that we see over here.

So, had there been only one Doppler shift we would have got a single tone corresponding to that Doppler shift, but here since we have different values of n up to a very large number, you are going to get different such cos theta n and hence different f D n that means, you are going to get several such frequency components thereby giving rise to the Doppler spectrum and not just the Doppler shift this is something important to consider.

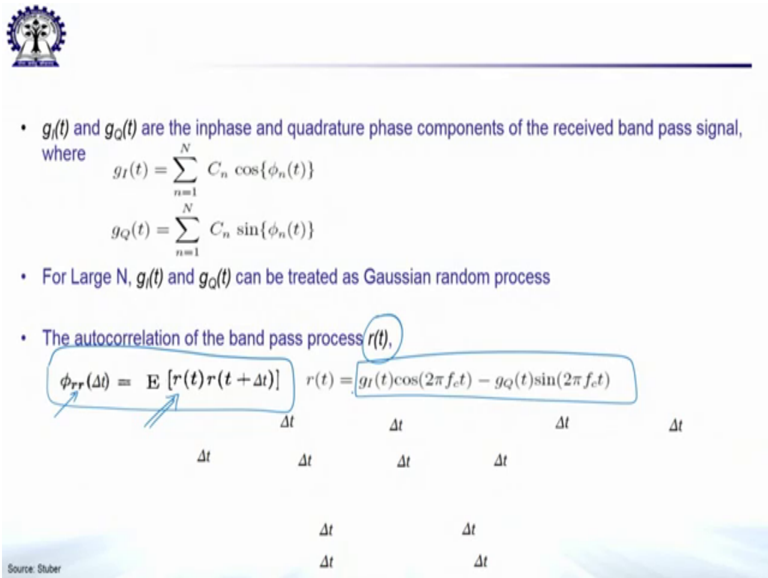
So, then if we have such a situation, let us look at what this would result in when we study that we look into the correlation analysis of the signal that means, we are generally interested to study the received signal correlation.

(Refer Slide Time: 06:14)



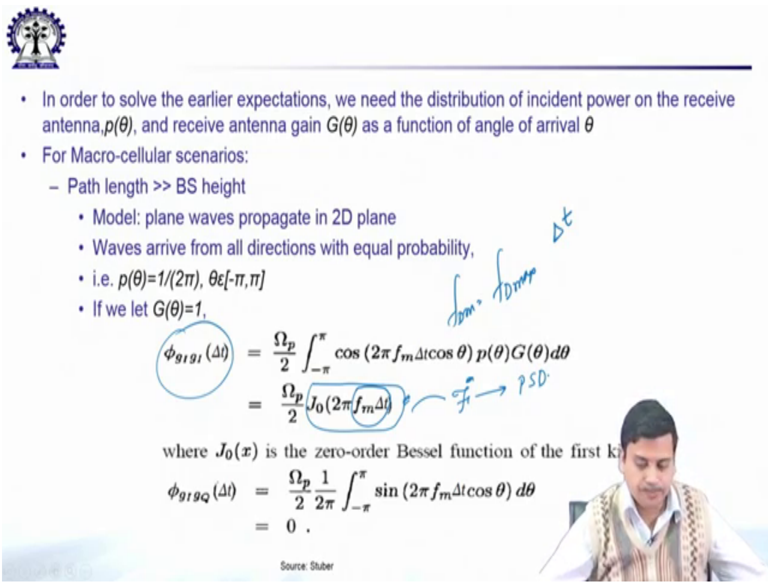
So, we usually let that S tilde t in our model to be equal to 1. So, if you get back to the model and we study the correlation analysis. So, if we look at the correlation analysis, what we do is we would like to take r t that is the received signal.

(Refer Slide Time: 06:35)



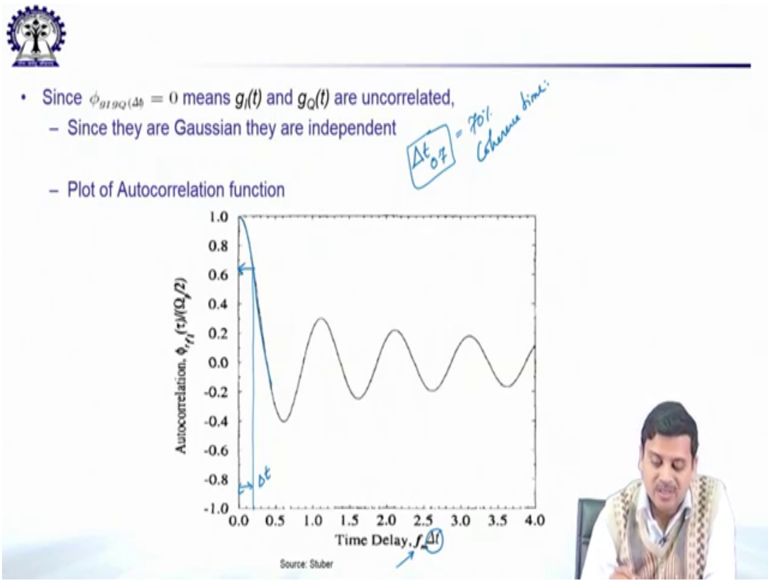
And we would have the correlation of the process r t which is defined by phi rr of delta t, which is given by this particular expression. So, if we analyze this particular expression now, because that is written in these terms there is a sequence of steps which one can follow, one would end up in a situation we just like to show you the result.

(Refer Slide Time: 07:06)



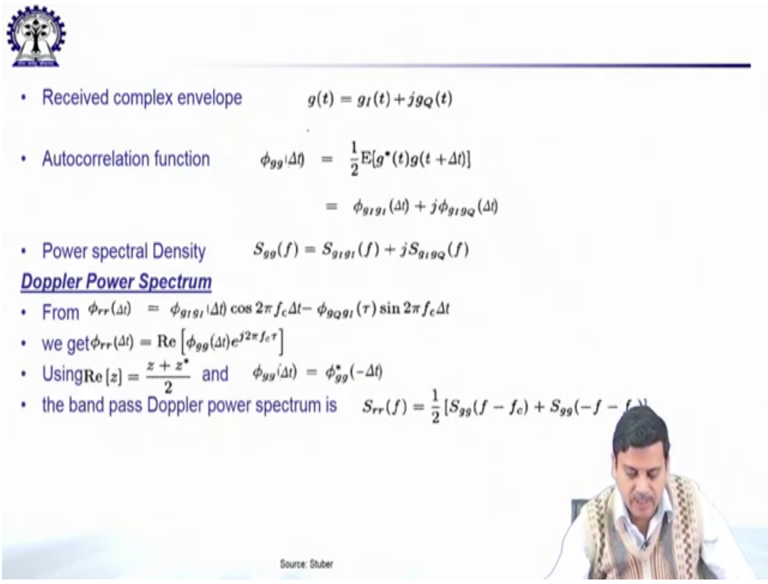
Where the n result of this would appear in a form as given here that the correlation coefficient of the baseband equivalent component appears as zeroth order Bessel function of the first kind, parameterized by f m delta t where f m is basically f D max and delta t is the lag that is the correlation. So, with the correlation we can study the time evolution and if we take the inverse Fourier transform of this, sorry if we take the Fourier transform of this, we will get the power spectral density of the Doppler spectrum that we are talking about it.

(Refer Slide Time: 07:48)



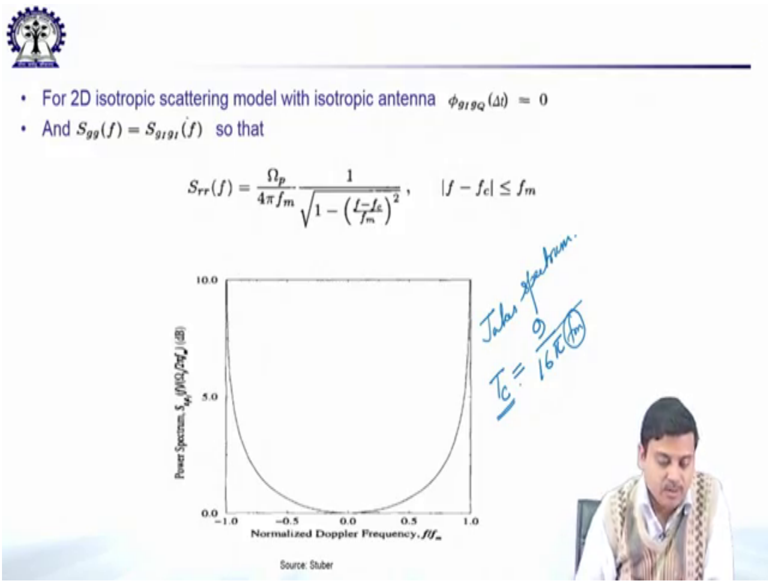
So, if we proceed further what we get, what the picture that we see over here is the autocorrelation function. So, in the autocorrelation function we find that the correlation function drops with increase in delta t for a fixed value of f m, effectively meaning that at a particular offset of delta t for a given f m, there is a certain correlation value. And this correlation value, let us say it is 0 point; in this case it is if it is 0.7. So, this delta t value at 0.7 is the 70 percent coherence time.

(Refer Slide Time: 08:36)



For this particular situation that we have been analyzing, if we proceed further and look at the Fourier transform of the same.

(Refer Slide Time: 08:41)



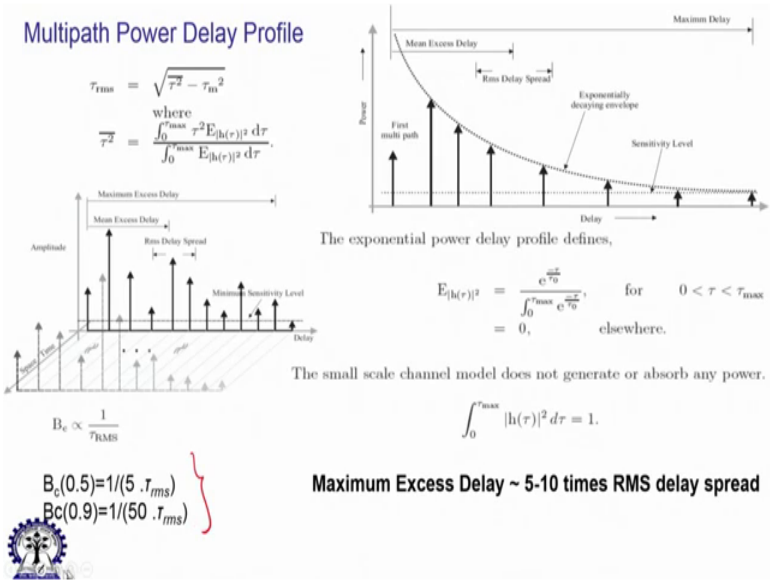
Where we lead is a spectrum characteristics which is a very famous Jakes spectrum. And under these conditions if one has to find the coherence time that can be calculated as 9 by 16 pi fm, sorry it cannot be written like that 9 by 16 pi fm. So, if I know the value of f m, I can roughly calculate the time duration over which the receive signal is coherent with itself; so that means, when we go back to our earlier description that means, when we are here in this particular model. So, this particular extension in time that means, this fluctuation in time that we had drawn earlier, so this is a time evolution.

So, if we take the correlation of this time evolution, we will end up in a pattern as shown in the previous graph. And we will be able to read off the coherence time corresponding to the value of coherence over here, so then in this case it is 0.9 t. So, this will tell us over how much duration of time is the channel coherent with itself that is it does not change significantly. So, this is capturing the time domain fluctuations along with this because of this power delay profile that means, because the channel is having delays, resolvable delays and if you take the Fourier transform you are going to get frequency selectivity.

In a similar manner, one would like to find the bandwidth or the set of range of frequencies over which the channel is relatively flat and this description is given by the term coherence bandwidth and it can be calculated as E of if this is the Fourier transform f H conjugate f plus delta f, and then one would find the value of this separation delta f for which this gets to a particular value.

So, what one can find is that the coherence bandwidth with 50 percent correlation can be roughly calculated as 1 by 5 tau rms, where tau rms is the rms delay spread of this particular power delay profile. If 1 is interested in calculating the 90 percent coherence bandwidth, 1 is going to use the description 1 by 50 tau rms. So, tau r m s can be calculated from the power delay profile kind of description which is given over here.

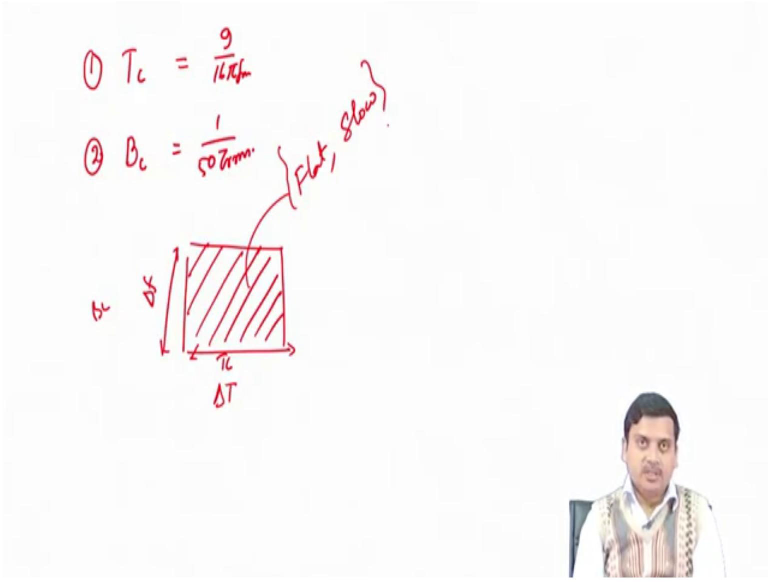
(Refer Slide Time: 11:43)



So, if we accept these things, then so in this what we have is for a particular situation that is if we are taking the exponential power delay profile that means, expected value of h tau squared is given in this form that means, e to the power of minus tau by tau naught, here tau naught is the one which characterizes the r m s delay spread. And in that case, 1 would be able to easily calculate the power delay profile or the RMS delay spread analytically. Otherwise, this is the set of expression one has to use in order to calculate it calculate the tau r m s.

So, what we see over here is tau m is the mean excess delay of the channel and tau squared bar is the weighted delay of the channel that means, you take the tau squared multiplied by the power of the channel at that particular delay, integrate over the entire range of it, normalized by the energy of the channel; so that is how 1 would calculate the tau rms, once 1 calculates the tau rms, then 1 would be able to calculate the coherence bandwidth in this manner. So, once one calculates the coherence bandwidth, then one would be able to get a few things.

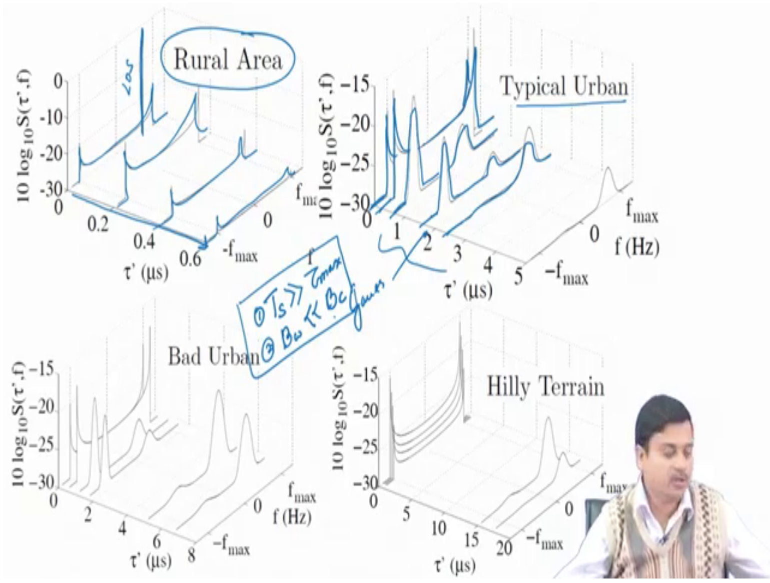
(Refer Slide Time: 13:06)



So that means, first one has coherence time and second one has coherence bandwidth, coherence time is given as 9 by 16 pi fm and this is given by 1 by 50 tau rms. So, this essentially gives us the range of frequencies over which channel is not fluctuating and T c, so this is B c, gives us the delta time over which channel is not fluctuating.

So, if we are taking a time frequency grid which is contained within B c and T c, we are looking at a portion of time frequency which is not fluctuating with time. And this is flat in frequency and slow in time, which is most of the things that we are going to be concerned with.

(Refer Slide Time: 14:08)

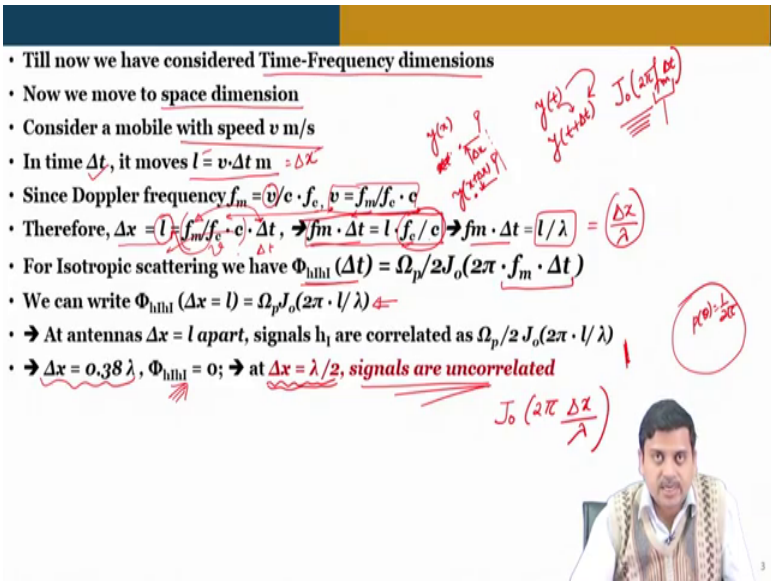


So, moving ahead when we combine everything together; so, the combined picture that we get is depicted in this particular figure. So, let us look at any one particular image that is the rural area. So, if we look at the rural area, we have the delay axis along this and what we find is that along the first delay that Doppler is Jakes spectrum, which is again Jakes spectrum along all delays as shown in this. However, on the first delay there is a strong specular component which is the line of sight component.

If we look at the typical urban profile, what we will find is that at different delays there are average echoes. And at each delay, there are different kind of Doppler spectrum that is present and these are usually from measurements. And the earlier few delays encounter Jakes spectrum and the later few delays encounter double sided gauss spectrum. So, like this you can characterize the overall channel power delay profile and what we will be concerned is with the situation, when the symbol duration is much much greater than the tau max and the signal bandwidth that means, the bandwidth of the signal is much much less than the coherence bandwidth.

So, if these two conditions are satisfied, then we are situation where the signal is not experiencing fluctuations in the frequency or fluctuations in time that means, within that small region the channel is as if held constant and most of our discussion will be with these set of assumptions, all right. So, with this we have the basic profile of the things that we require and then we move on to discuss some of the additional components that are required to understand the MIMO propagation.

(Refer Slide Time: 16:15)



So, what we have discussed till now is the time frequency analysis. So, thereafter we have to move to the space dimensions. So, from time frequency we have to go to the space dimension, so that means as if we have an antenna over here, we have antenna over here; so what about the signal which is received let us say I call it y at x and y at x plus delta x, which is this separation is given as delta x. What we had studied till now, is if y of t is available can we say anything about y plus t plus delta t and this was achieved through the correlation analysis. And what we found is that the correlation follows the J 0 function because of certain set of assumptions, which are under laid within that analysis.

So, now what we do over here is we consider, so we use that same analysis to the space dimension. So, what we consider is that the mobile is moving with velocity v, which is within our scope. And in time delta t, it moves a distance l which is v times delta t, which you can also write as delta x ok. Now, since the Doppler frequency f m is given by this term therefore, you can write v in terms of the other parameters. And hence this l or delta x you can easily write as, f m upon c multiplied by c, because this is the v term v multiplied by delta t. So, this is the v term that we have over here, so that v term is this term multiplied by delta t.

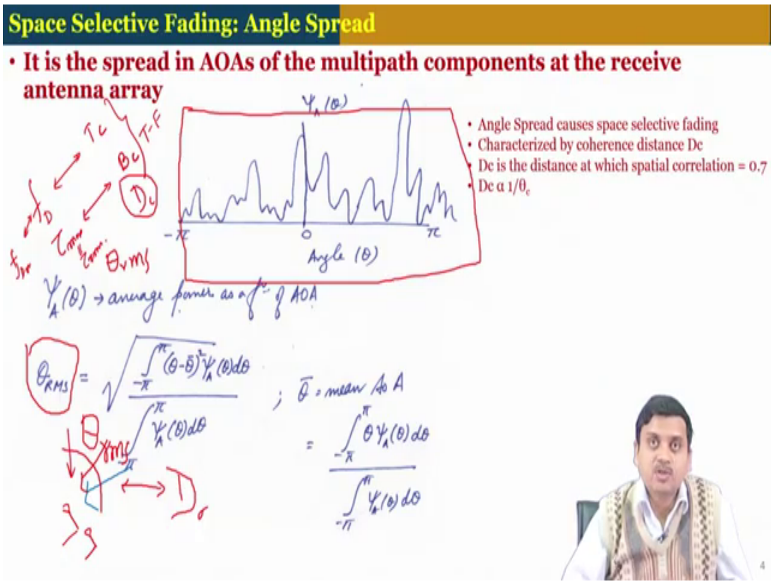
So, now what we have is f m multiplied by delta t right; so, f m multiplied by delta t can be translated to l or delta x multiplied by f c, because f c is in the denominator gets multiplied and c comes to the denominator side. So, we have f m delta t is equal to this and then since you have in the denominator c by f c or f c by c in the numerator. So, f m delta t can be written as l upon lambda or you can also write it as delta x upon lambda.

So, now what we see is that instead of measuring the signal at two different intervals of time, if we say that in this time interval something has moved across this distance delta x, then we can potentially reuse this entire expression that we had got and replace this f m delta x by this particular term. So, what we have in the next few statements is that the correlation which we have designed; which you have derived between the signals with the separation of delta t is this expression within which we are going to replace f m delta t and what we get back is the expression over here.

In other words, we are saying that the correlation of two signals spaced apart is given by spaced apart by delta x is given by this expression, under certain set of assumptions that means, when the signal is coming from all directions with equal probability under this set of assumptions. So, what we conclude from this set of assumptions is that if we set the separation between the two positions or if we in other words if we look at two antenna positions and consider the signal in those two physical locations, and if these two physical locations are separated roughly 0.38 lambda, we will find that the correlation goes to 0; or approximately we can say that when the separation between the spacing is lambda by 2, we get signals which are uncorrelated right.

And if they are 0 mean Gaussian random variables, then we are going to get independent. This is also one of the big assumptions or the setup that we consider in the analysis of MIMO that we are going to describe very shortly.

(Refer Slide Time: 20:17)



So, there are a few more things like we have RMS delay spread. So, what we have discussed is that Doppler leads to coherence time, delay leads to coherence bandwidth right, this is the delay tau max I have written influences so tau max is basically connected to tau r m s and this is connected to actually Doppler spread.

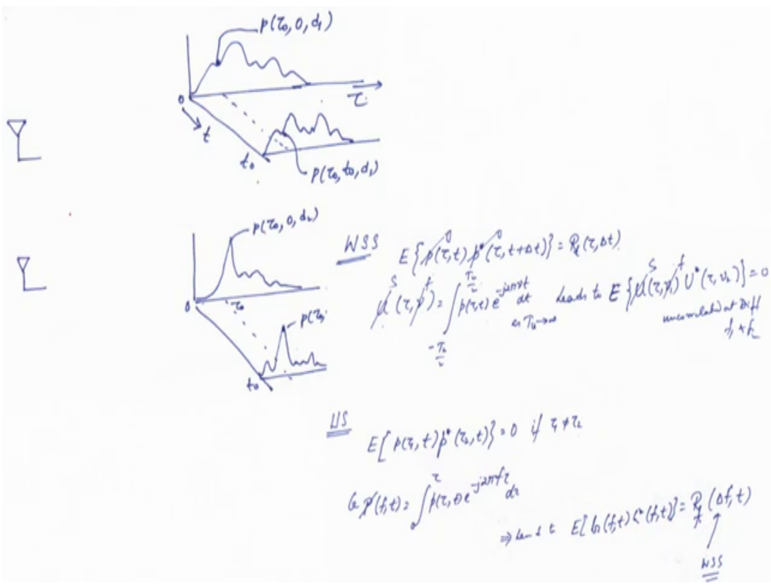
Similarly, what we have over here is angular distribution in case of spatial dimension. So, these were the things which we discussed in the time frequency plane, but when we are going to the spatial dimension, what is happening is that the signals which arrive at the receiver antennas, they can come from different angles. So, these signals they can come from various different angles with a certain spread in this angular dimension, which can be described by theta rms ok.

So, this theta rms is now connected to something known as the coherence distance. So, we have D c which is called the coherence distance. So, instead of T c, B c, we have D sub c indicating coherence distance which is connected to the term theta rms which is nothing but the angular distribution of the received signal. So, when the signals are coming from various directions, the signals would form a power angular spectrum which is described by the picture which is given over here.

So, in a similar manner like one has calculated the tau rms, one can calculate the theta RMS as is shown over here right. So, if one calculates the theta RMS, then from this one can find out a similar thing the coherence distance. So, coherence distance is the distance over which the signal is correlated to itself.

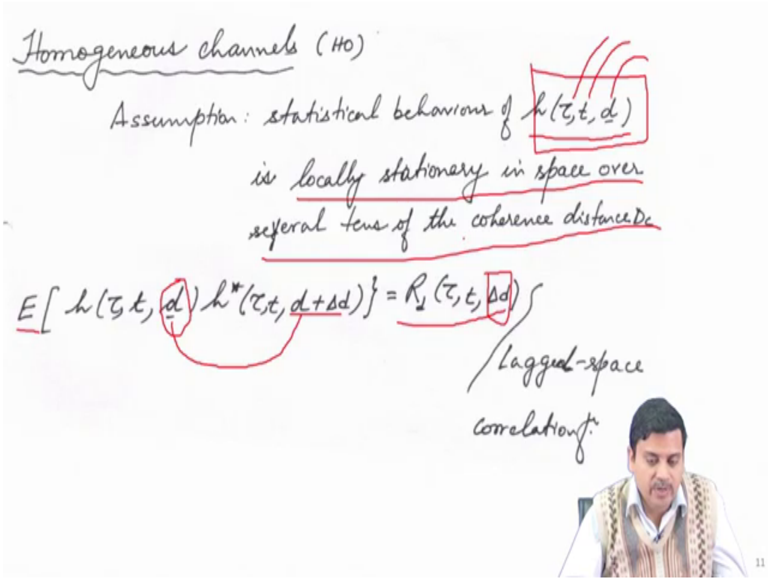
So, if we go back over here under the set of assumptions that the signal is coming from all directions with equal probability, under such assumptions what we have seen is that at a separation of lambda by 2 you get uncorrelated signal. So, if you are within that separation, then you will get highly correlated. Now, unlike in time frequency when we go for MIMO signal analysis, we would generally look at conditions where the received signals in two different antennas would be uncorrelated whereas, in the time frequency we would like to take that grid in time where the signals are highly correlated with each other.

(Refer Slide Time: 23:08)



So, moving ahead we have a certain set of assumptions which we summarized as a channel which contains which is supposed to be wide sense stationary, uncorrelated scattering that means, wide sense stationary means the correlation function is not a function of time that means, it is dependent only on the time shift, wide sense stationary. Uncorrelated scattering means that the signals coming at different delays are not related to each other.

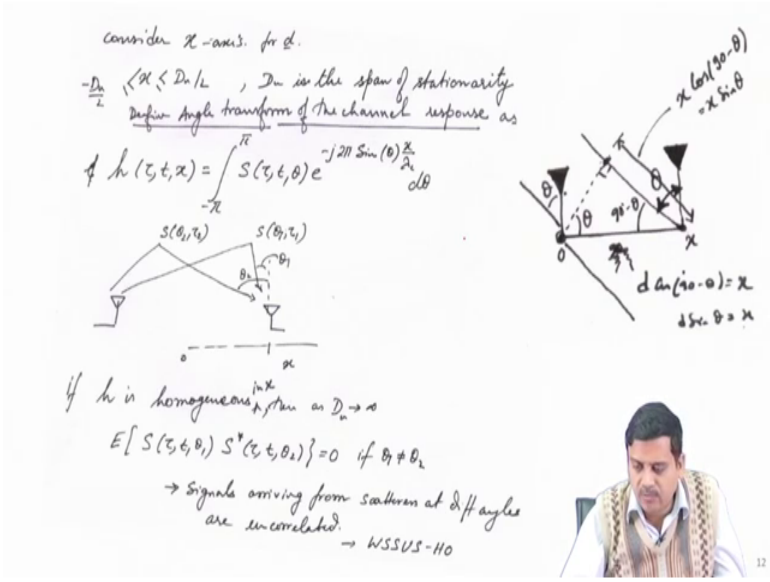
(Refer Slide Time: 23:36)



And along with this, we have something called homogeneous channels. So, with the homogeneous channels what is assumed is that the statistical behavior of the h component, which is given by h tau, t comma d; tau means the delay, t is a function of time because of Doppler and this is the spatial separation is locally stationary in the space over several tens of coherence distance; that means, within a few tens of coherence distance. The distribution of this is not changing or it is not changing over within that spatial distance.

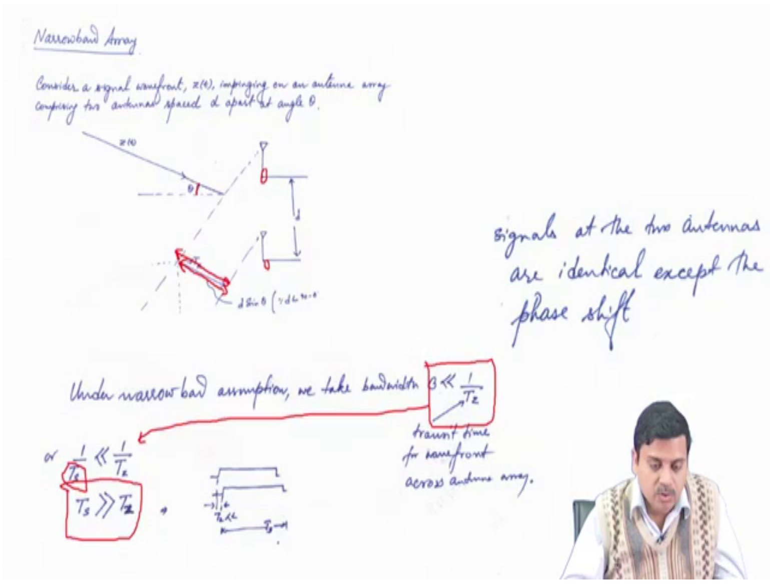
So, under that assumption if we are calculating the correlation at the location d and d plus delta d we would call it, the lag correlation coefficient that means, it is not dependent on d, but it is dependent only on the separation of the antenna elements. So, in other words what we are saying is that the channel if it is wide sense stationarity uncorrelated scattering with homogeneous assumption. We have the frequency domain correlation or the coherence bandwidth is not dependent on the frequency, but only on the separation between the frequencies. Coherence time is not dependent on the time, but only on the lag in the time and coherence in the spatial domain is not dependent on the location, but between the antenna separations.

(Refer Slide Time: 25:05)



So, combine together what we have is a channel which is wide sense stationary uncorrelated state scattering with homogeneousness. So, there is also one more important set of things that we considered, while taking into account the MIMO channels is that there is a narrow band antenna array assumption. The narrow band antenna array assumption means that the signals which are arriving at the first antenna and the last antenna element of the antenna array are not different from each other than a phase term.

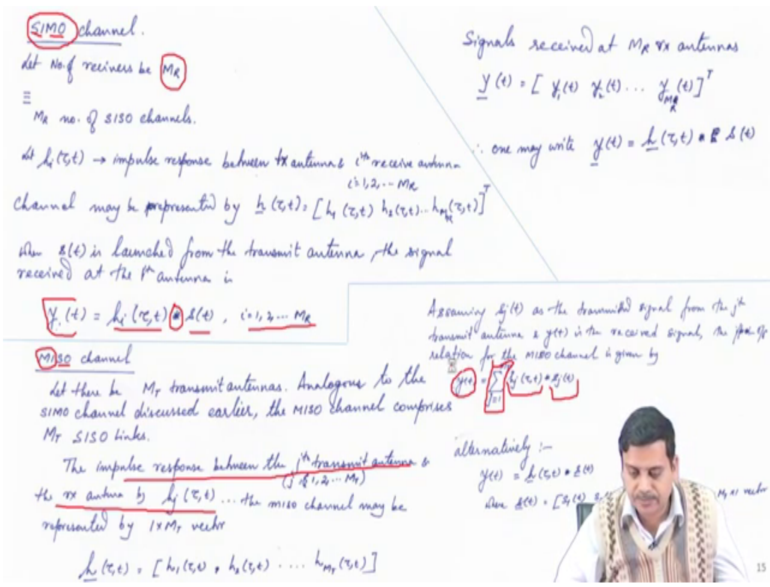
(Refer Slide Time: 25:44)



So, effectively what it means is that effectively what we get to is that the, I mean if you go into the details of it what you finally end up is that, the propagation time between the first antenna element and the second antenna element. So, in this picture we have made the assumption theta that means, the time it takes to propagate from this to this suppose, we mark it as T z.

And if we have T s as the symbol duration, so we say that it is under the narrow band antenna array assumption if the bandwidth of the signal is much much less than 1 by T z. So, if we translate this what we get is 1 by T s, where T s is the symbol duration; this is the symbol duration is much much less than 1 by T z or in other words the symbol duration is much much larger than the propagation time between the two antenna elements right. So, all these conditions have to be taken into account, before we get into the study of MIMO channels.

(Refer Slide Time: 26:45)

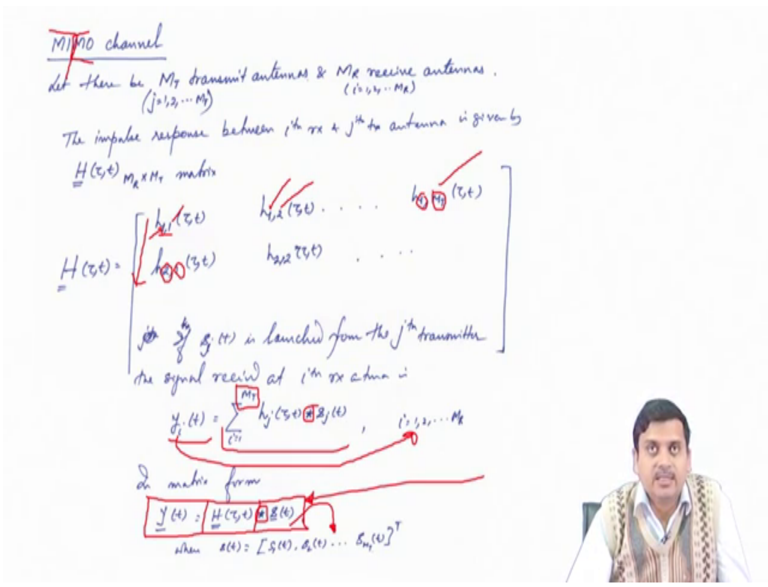


So, a quick discussion about how we model the signal so, in case of SISO links we have one transmit, one receive antenna. The first class of channels is the SIMO channel, where we have M R number of receive antennas. The second class of course, we look at is the MISO case, where we have multiple input and a single output; here we have a single input and a multiple output and finally, we look at a MIMO case.

So, in the SIMO case what we have single input multiple output. The received signal at the i th receive antenna is equal to the h tau comma t. This is the SISO channel coefficient as we have seen, convolved with the signal. And this kind of signal has to be received at the different M R antennas ok.

So, now if we go for a MISO system that means, multiple input single output; what we have over here is that the impulse response between the j th transmit antenna and the receive antenna is given by h j tau,t all right. So, let all the antennas are sending signals at the same time. So, when the signal is received, so what you find is that s j is the signal that is being sent from the j th transmit antenna. And h j is the channel impulse response between the j th transmit antenna and the receive antenna. So, now all these signals add up together and they are combined at the receiver right. So, you can write all these different equations in a matrix vector notation and things will be easier.

(Refer Slide Time: 28:23)

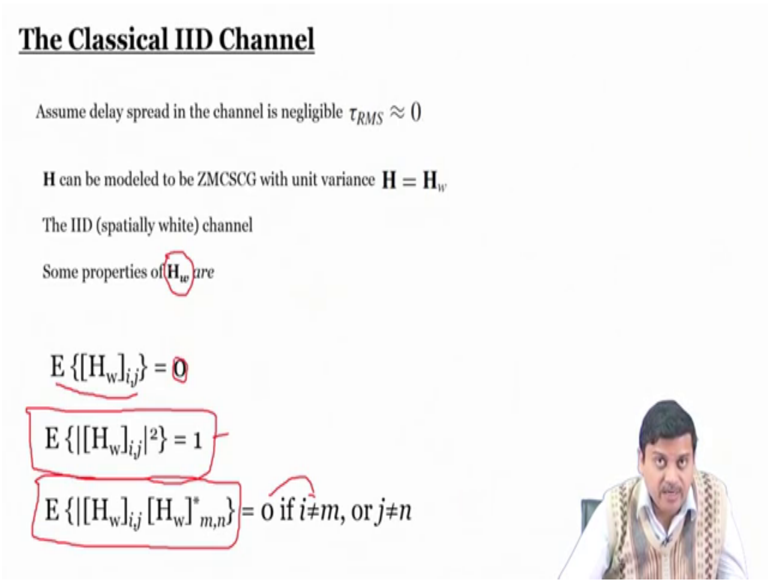


So, then we move on into the situation where there are multiple transmit antennas as well as multiple receive antennas. So, together it forms the MIMO system under that what we have is h 1,1 indicating the channel impulse response between the received antenna element 1 and transmit antenna element 1; this is the channel impulse response at received antenna 1, transmit antenna 2 and so on and so forth. This is the receive channel impulse response at receive antenna 1 and transmit antenna empty.

In a similar manner, if we go down the column, this channel impulse response received in antenna 2, while transmitted from antenna 1. So, if you are able to write down the equations, so for any one receive antenna we have a summation of the signal which is convolved with the corresponding channel impulse response and it is summed over the M T transmit antennas.

And then in the matrix notation you can write, so this y i is for all the different receive M R number of antennas. So, when we write it in a matrix form you can write that the vector y, which is a column vector is this channel impulse response matrix convolved with the transmitted vector s t, where s t is described by this vector which is the signal vector that is being transmitted from the M T transmit antennas. So, once we write in the linear equation form in the matrix notation, we will be able to handle the entire analysis of MIMO using linear algebra.

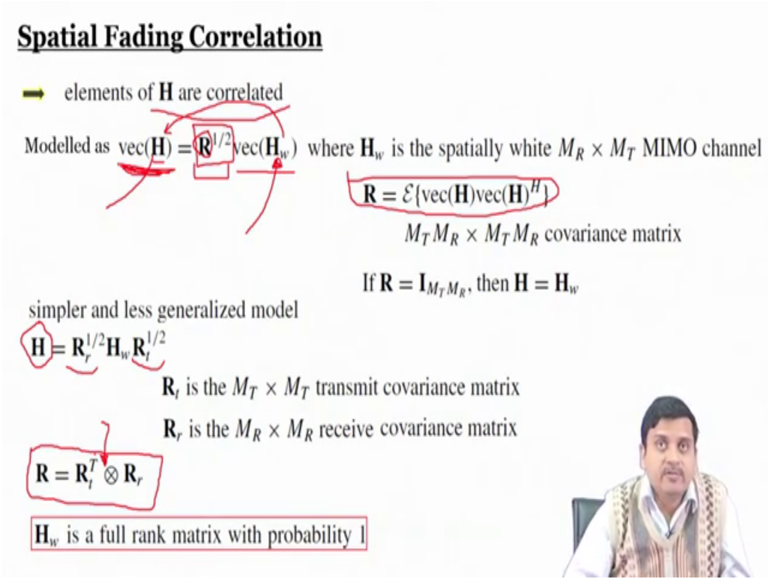
(Refer Slide Time: 30:10)



So, one of the important results of the MIMO channel that we will be looking at is known as the classical IID channel that means, we are characterizing this particular H channel and the classical IID channel would be called the specially wide channel and denoted as H w. So, in this the set of assumptions are that expected value of H W that means, each of the elements is 0 that means, each of these coefficients are on an average 0; so there is 0 mean.

We also have the power of the individual elements are 1, this is matching with the description of large scale and small scale fading and the correlation between two different elements are 0, if they are not the same element and otherwise it is 1. So that means, if you take the covariance matrix of a specially wide channel, you are going to get an identity matrix right; otherwise it will be the covariance matrix. So, in general the elements of H w are such that the expected value of H w, H w hermitian is an identity matrix.

(Refer Slide Time: 31:24)



So, now the other important fact that remains for us to be described is that in case the elements of H are correlated, then how do we capture it? So, first thing what we do is we translate the matrix to a vector using the vec operation, which simply stacks the columns one on top of the other. And we can write that the vec of H that means vectorial form of H, which contains the correlated variables is some R covariance matrix to the power of half that means, square root of that multiplied by the vec H w channel that means, from H w we can generate a correlated MIMO channel matrix.

And the correlation is described through this spatial covariance matrix R, which is a property of a particular propagation area or a particular situation. So, this is the general model. So, here this will be generated using 0 mean Gaussian random variables, while when it is multiplied by R half, you get H where a expected value of H H hermitian is no longer an identity matrix, but that will be R which is the matrix right. So, effectively R is expected value of vec of H vec of H hermitian ok.

So, this model can be relaxed and a simpler model can be used, where this covariance R is split between the transmitter and receiver and you can generate the H coefficient. And this is usually known as the Kronecker model, because the relationship between capital R that we have described earlier and the transmitter receiver correlation can be described through this Kronecker product. And H w is full rank matrix with probability 1.

So, if we have since now we have defined these different matrices, we should be able to discuss the different performance of MIMO schemes with a prior understanding of these descriptions about the channel.

Thank you.