Evolution of Air Interface towards 5G
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Lecture - 37
Mimo Signal Processing (Transmit Diversity)

Welcome to the lectures on Evolution of Air Interface towards 5G.

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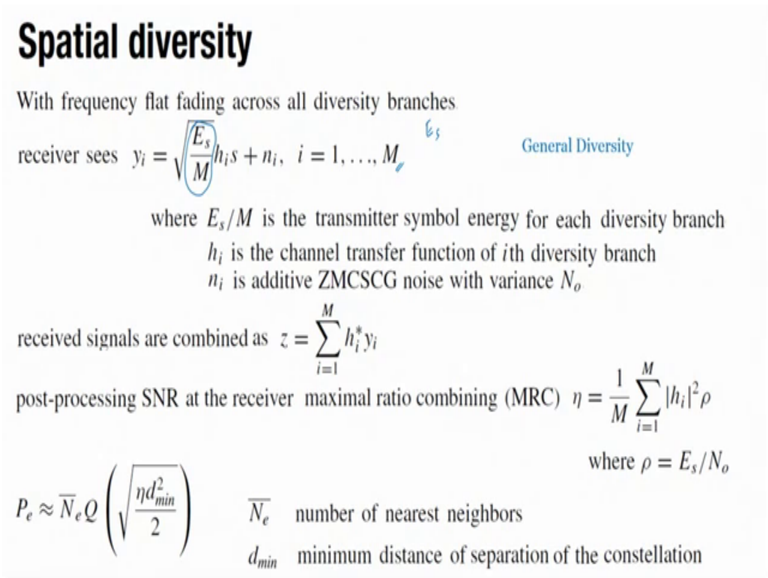

So, we have started discussing about multi antenna signal processing and in the previous lecture we have discussed about the diversity mechanism. We started off with the discussion on general diversity, we will just briefly look at that and then conclude with the other mechanisms.

So, in the discussion on the general order of diversity we did mention that we study our system with the understanding that each of the diversity branch has a power of E s by M. So, overall with M branches there is a power of E s.

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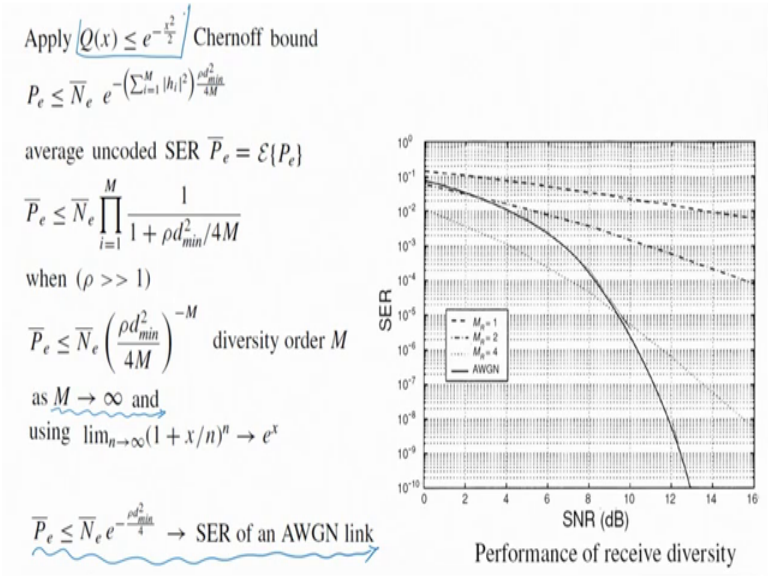

And in that procedure we also studied about the way we should calculate the error probability and finally, we concluded that as the order of diversity extends towards infinity, the probability of error gets asymptotically closer and closer towards an AWGN link that is the best possible link one can imagine.

So; that means, by simply diversity one can make the error probability be as best as possible. This was the one of the first conclusions and for all of these we have used the Chernoff bound as well as the MGF of the Frobenius norm squared of h.

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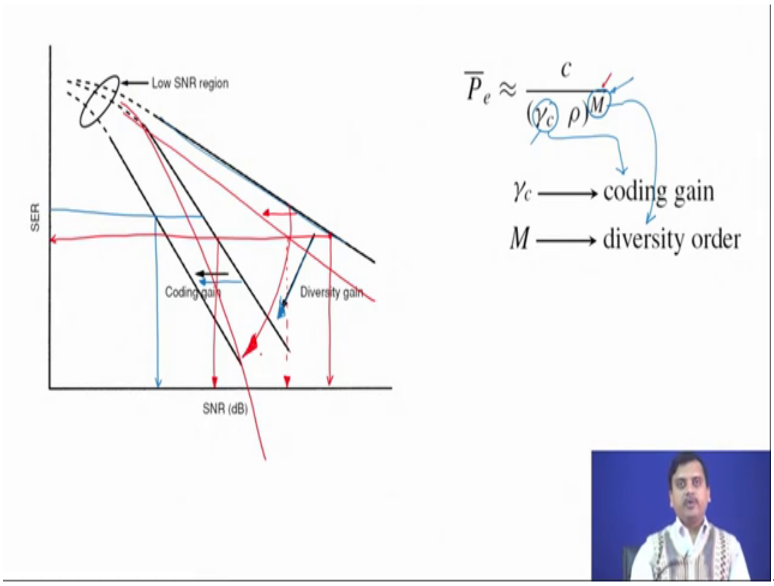

We also described that in the previous expression on probability of error we have the exponent of SNR which implies the diversity gain and the multiplicative factor implies the coding gain and they have the consequence as M increases; that means, as M increases the slope of the error probability curve becomes steeper and steeper and as the coding gain increases the curve shifts more to the left indicating that at lower SNR one can achieve a better probability of error.

So, simply this probability of error is achieved at a lower SNR because of coding gain compared to the normal link and at any given SNR at any given SER if this is the probability of error that we are talking about requires a certain SNR without any diversity gain, because of diversity gain what we will find that the SNR required is significantly lower, coding gain adds on top of it.

So, had the curve been like this then we would have said that there is a coding gain and hence the required SNR would have been here. So, with diversity gain as we are increasing the slope; as we are increasing the slope with higher and higher SNR the gains are more and more and more. So, that is what is to be remembered whereas, with coding gain with SNR it is the same gain that appears.

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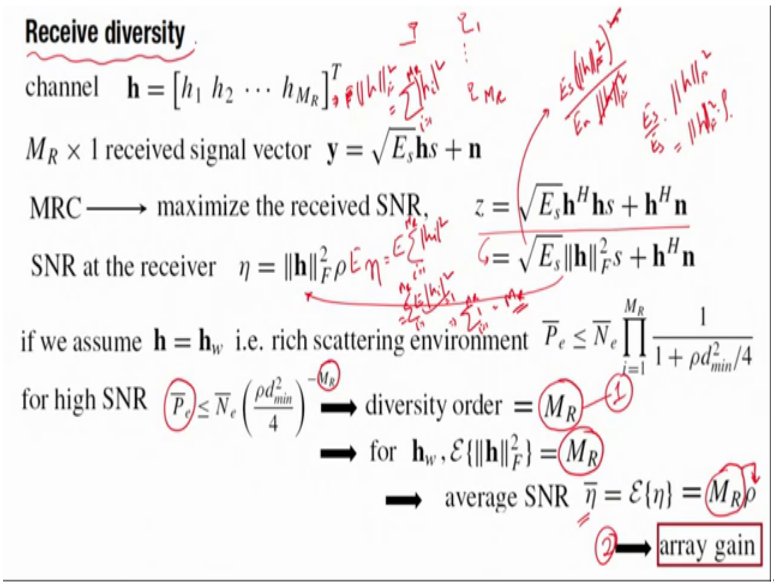

We did talk about a receive diversity; that means, the scenario where there was one transmit antenna and multiple receive antennas and number of receive antennas indicated by M R. What we figured out is that from the error probability expression diversity order is M R because the average probability of error expression had M R in the exponent.

And when you calculate the SNR from this expression I mean this is the received signal expression which can be used to calculate the SNR. So, basically from this when we go there you are going to calculate the SNR. To calculate the SNR you will be getting E s from this term, E s is already there mod of s squared is equal to 1, mod of h F squared and in the denominator you are going to again get E n I mean mod of h F squared whole squared and you are going to get mod of h F squared.

So, these terms will cancel out, you want to get E s by E n into h F squared which would mean that you have h F square times rho that is what we have over here and h F squared from this if you apply h F squared on this; this would turn out to be sum of h i squared, i equals 1 to M R.

So; that means, if we now apply the expectation on eta we are going to get expectation of sum over h i squared i equals 1 to M R which would mean expectation which would mean the summation would go outside. The expectation of h i squared. For h equals to h w we said this term is equal to 1 i equals 1 to M R. So, what we have is summation 1, i equals 1 to M R which would simply mean M R.

So, that is what we have over here. So that means, the average SNR has an M R factor on top of the AWGN SNR which means that there is an array gain. So, we have 2 important things; 1 is the diversity gain and 2 the array gain; when we are talking about pure receive diversity.

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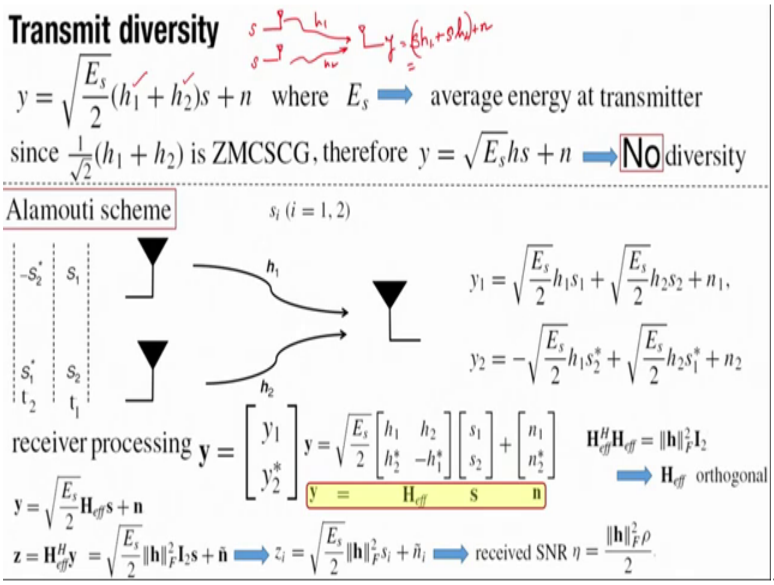

So, then we move on to look at some other mechanism; that means, can we shift the diversity to the transmitter. So, the basic setup that we have is let there be two transmit antennas.

And let there be one receive antenna and let this link be h 1 and let this thing be h 2. So, what we have is h 1 as signal transmitted from h 1 as the channel gained from one antenna to the receiver, the other antenna to the receiver and from both of this if we sub send s what we are going to receive y is equal to s times h 1 plus s times h 2 plus noise.

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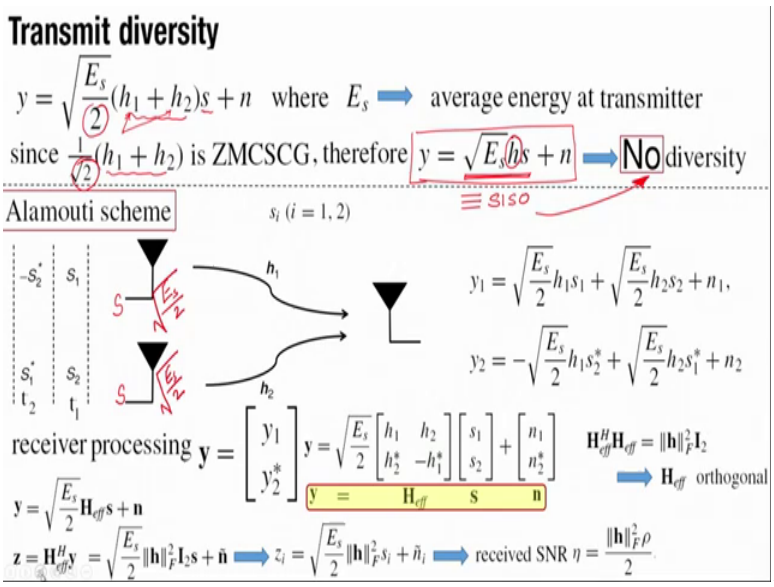

So, which you could write it as of course, there is the scaling of energy so, what you can write it as root over E s by 2 which is the energy term taken outside and h 1 plus h 2 with a common term of s. So, now, h 1 and h 2 are both 0 mean circular symmetry Gaussian random variables and therefore, you could replace these and there is this 1 by root 2 term by a term which is this.

Now, this 2 term comes because we had said that E s by 2 and E s by 2 will be the power equally divided amongst the two antenna internal branches and a square root of that is the amplitude factor. So, what we have is equally divided power. Now when we add them together at the receiver what we find is that just carefully look at this. So, the sum is a random variable with now each both of them are iid in that case you are going to get an equivalent h with the sigma root 2 times that of the previous one which cancels out and if you now focus on this expression this is exactly similar to a SISO equation and hence there is no diversity gain with this mode of transmission if you are sending signal s from both antennas.

So, this mechanism is not a good mechanism and hence we would have to go for a better mechanism which is this celebrated Alamouti scheme. So, the Alamouti scheme is a very very famous scheme and will briefly outline the scheme which is very well established.

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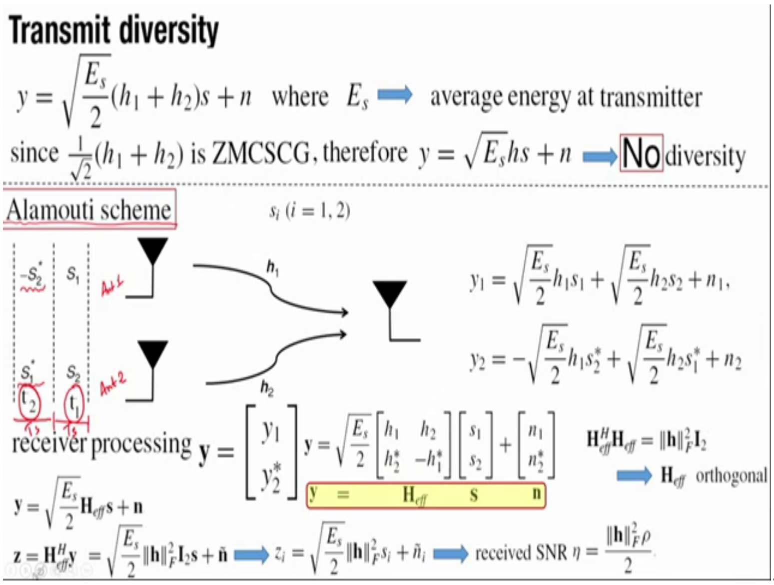

So, it uses two time intervals in the first time interval you send signal s 1 from antenna 1 and signal s 2 from antenna 2. In the second symbol duration, so, this duration one can take it as T s. In the second symbol duration from the first antenna one would say send minus s 2 conjugate and s 1 conjugate from the other antenna. So, these are the transmission mechanism.

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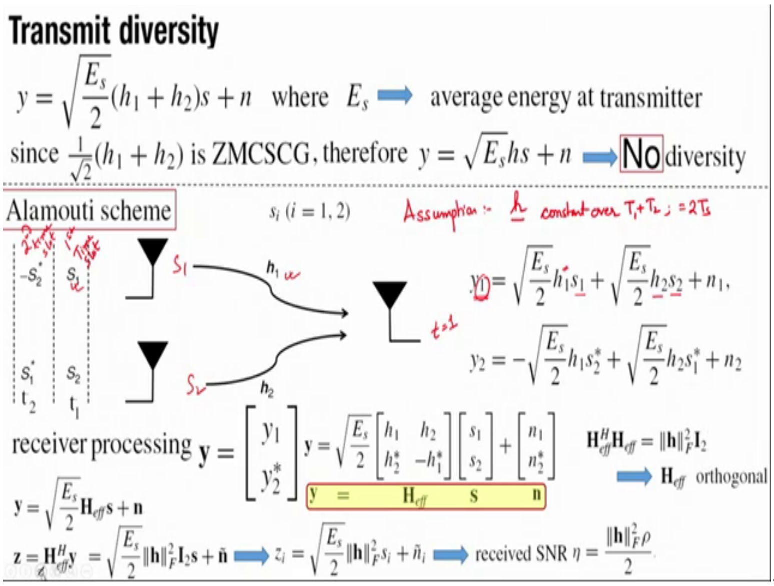

So that means, the first time interval this is the signal and the second time interval 2nd time slot right and we can write that as the 1st time slot so; that means, there is two time duration which is required for processing. And one of the strong assumptions is that the h vector remains constant over time over T 1 plus T 2 right or which is equal to over 2 T s. So, this is one assumption in this whole set of things.

So, the signal received in the first antenna is s 1 that is s 1 multiplied by h 1 that is over here plus s 2 multiplied by h 2. So, look at this so, the first time interval this is at t equals to 1 you have s 1 through this, s 2 through this all right. So, then in the second interval what we find is that you have s 2 conjugate with a minus and s 1 conjugate hence the time interval 2 the signal received is minus s 2 conjugate through channel 1.

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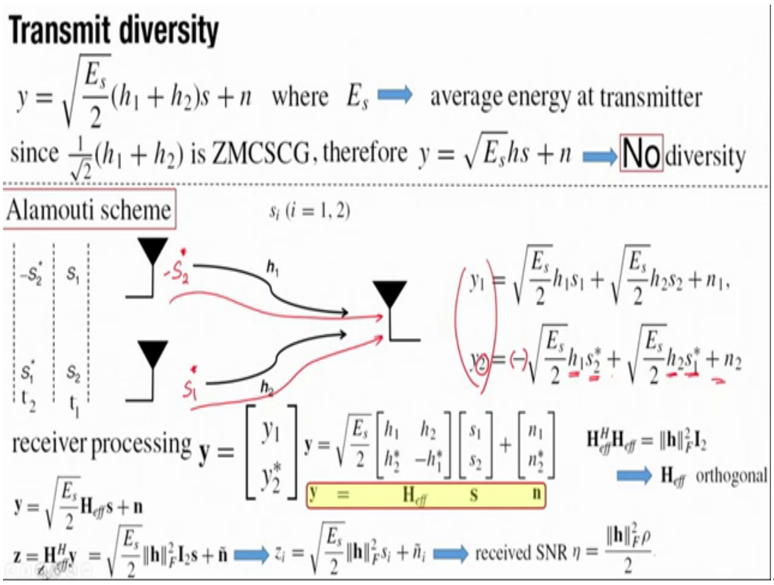

If you follow this path; plus s 1 conjugate multiplied by h 2, s 1 conjugate multiplied by s 2 plus noise. So, that is how you have both the signals.

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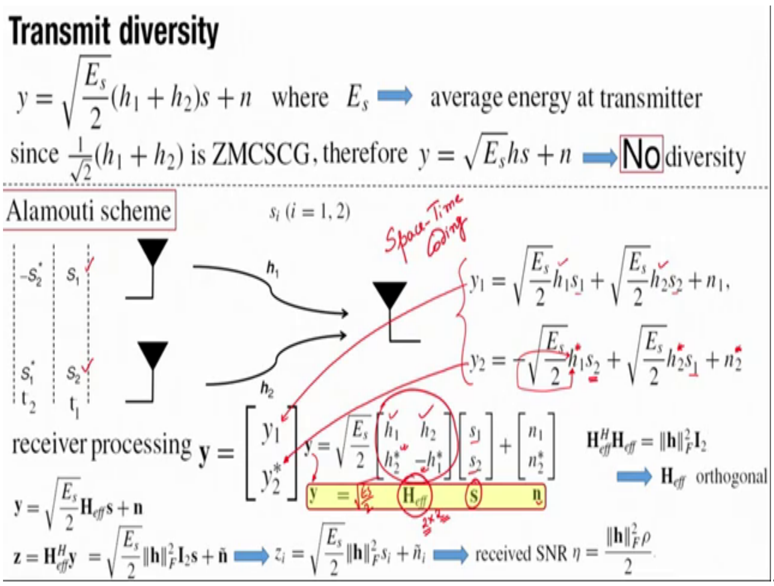

And therefore now you have to process these two signals at the receiver to for processing it appropriately you keep y 1 unchanged, but you allow y 2's conjugate to be taken in the processing. So, because the conjugation would mean that h 1 would become conjugate and you are not going to get anything over here. So, this thing is going to go away and this thing is also going to go away. And you are going to get a conjugate of h 2 and n 2 conjugate now since n 2 conjugate and n 2 would not have any difference in the distribution because of circular symmetricity. So, now, what we have is y 2 conjugate we collect the minus sign and associate it with h 1.

And then what we have is h 1 over here is h 1, h 2 is h 2 multiplied by s 1 s 2, s 1 s 2 in the second equation also we have s 1 s 2 coefficient of s 1 is h 2 conjugate. So, we have h 2 conjugate, coefficient of s 2 is minus h 1 conjugate so, we have minus h 1 conjugate and of course, this equation is fine. So, the vectorial notation we have y vector is equal to of course, this root over E s by 2 is there, this matrix you would take it as a effective channel matrix because now you are receiving over two transmit intervals and hence this can be thought of as being received at another instant of channel.

So, together it forms the effective channel matrix which is now a 2 cross 2 matrix 2 indicating the time index and this 2 indicating the two space index. So, we have space time code; this is the most elementary coding in this mechanism and s 1 and s 2 is the signal vector that is what we are required to send and n is the noise. So, in a very crisp notation we have the expression as given by this.

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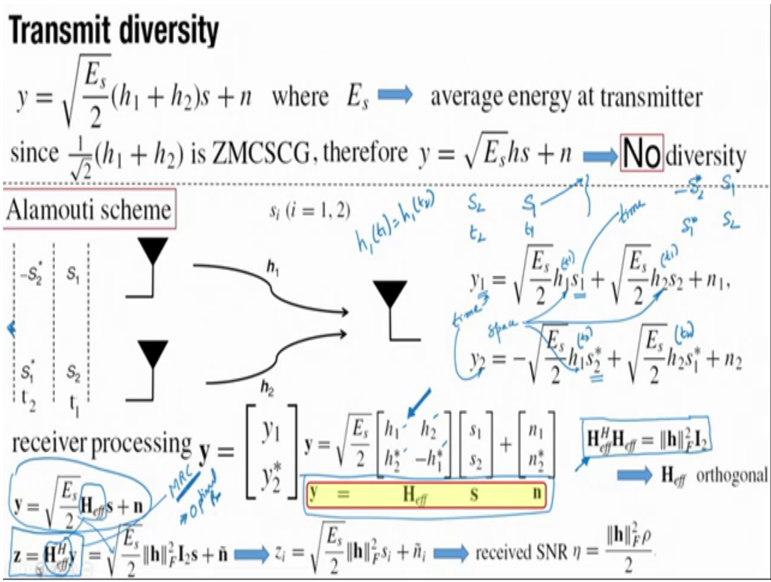

And once it is given in a linear equation then all the things that we have been doing can follow directly. So, we will write the received signal as this y equals to E s by root 2 H effective s plus n. And the processing at the receiver is given by z equals to H effective Hermitian times y which is the MRC that is what we have been doing all the while which maximizes the output SNR and that means, E s by root 2 remains as it is over here H effective times H effective Hermitian this gets multiplied by H effective this whole term gets multiplied over here.

What we have over here is, H effective times H effective Hermitian gives us the Frobenius norm squared of F times I under the set of assumptions that we have already taken and that if you look into this matrix it is an orthogonal matrix. And one of the vital reasons for this is that h at time instant 1 h 2 at time instant 1 and h 1 at time instant 2, h 1 at time instant 2. So, in this notation something has been carefully introduced. This is the signal which is the symbol 1 this is what is the symbol 2;this sub index is for time whereas, this sub index is for space. So, these things have been mixed in a way so that it is kind of a bit deceptive. So, to be very clear one should write it the appropriate indices in the appropriate rotation.

And this is again time at the position which is before this because we are having s 1 symbol coming in time 1 and s 2 symbol coming in time 2 then this s 1 and s 2 together are transmitted in a manner that s 1 s 2 and then you have minus s 2 conjugate, s 1 conjugate. So, one must be careful with these few notations that we are using over here.

So, because of the set of assumptions like h is static; that means, h 1 t 1 is equal to h 1 t 2 which we have already said, we get this matrix where these t terms are not present in any of this and they have remained constant. So, we get by this; by the structuring of the problem we get it as an orthogonal channel.

Now, because of orthogonality the matched filtering or the MRC operation that is happening over here gives us the optimal receiver; we do not have to worry about it.

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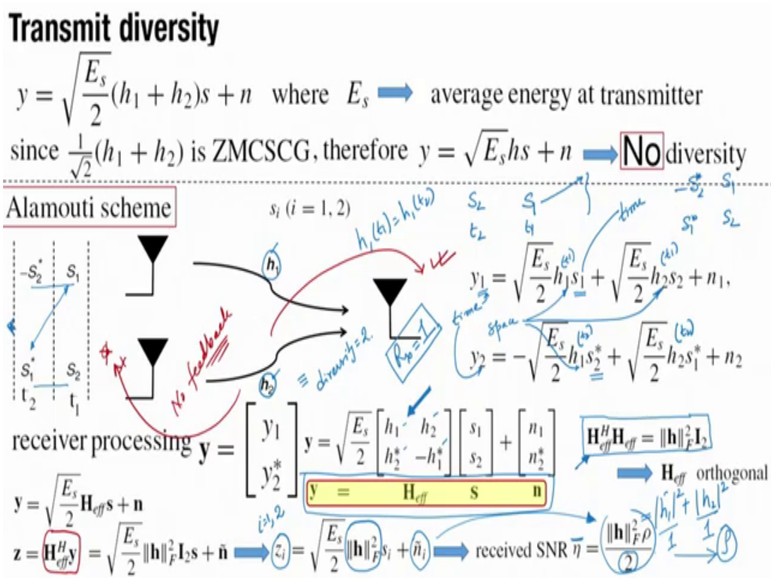

So, what we get because of the processing that we have here yeah because of the processing that we have over here what we see is, that from this each of the received receiver processed things; that means, i is equal to 1 and i equals to 2, we have each i is getting h F squared. That means, each symbol is going through the channel h 1 as well as the channel s h 2 simply because you can see that the first antenna is getting to send the symbols s 1 and s 2, the second antenna is also sending the symbols s 1 and s 2 and this is being sent over two intervals of time.

And since the energy is split to E s by 2 E s by 2 over two interval of time, but again at the receiver you are combining them, you are not losing out on the energy front, but each signal is travelling through two antennas, physically if each of the symbol travels through two antennas you are getting notionally a diversity order of 2 and that we will see how it appears numerically also.

So, n i is the processed noise n i tilde process noise of the ith symbol and hence the received SNR from this expression one should be able to calculate the received SNR as we have done earlier as h F squared by 2 times rho. So, one may be wondering about this 2 factor, but one may note that h F squared contains mod of h 1 squared plus mod of h 2 squared. Now on an average if this produces a value of 1 and on an average if this produces a value of 1; this entire thing on an average is going to produce an SNR which is rho.

So, average SNR has not increased, because on the receiver side you have only 1 antenna, if the number of receive branches are more then you get an array gain. So, there is no increase in array gain, but if you at the probability of error using the methods we have calculated.

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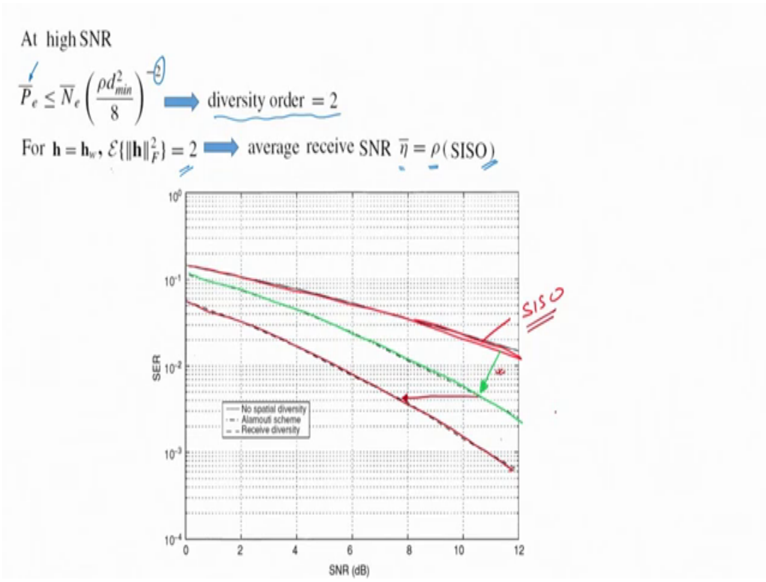

We have used earlier; you will find the exponent getting a factor of 2 we are of course, calculating the average probability of error. And hence the diversity order is 2 and as we have said the expected value of h F squared being 2and that is what we apply over here. So, you are taking the expected value of eta if you take the expected value of eta that is what is over here you get the value of rho which you have just shown that is for SISO.

So, if you compare the different performance this line which I am trying to sketch is the one for SISO link and this the second line, that I am trying to sketch is the one for the Alamouti scheme that we have described. So, there is an increase in the slope whereas, if you are using pure receive diversity; that means, you are having an extra antenna at the receiver then this is the new line that is what we are getting. So, what you can clearly see that both of them have the same slope, but there is an additional shift due to the array gain which is present in the system.

So, what we conclude is that because of one received branch you have only the diversity gain and, but it is better compared to a SISO scheme and it is useful in the sense that instead of doing all processing at receiver you can transfer the processing to the transmitter side. There are various other advanced space time codes which have been developed over years.

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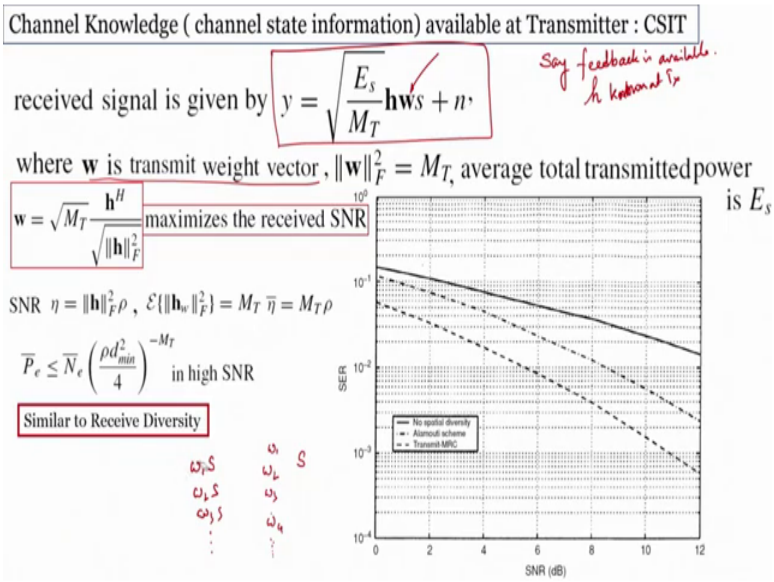

The next scheme that we would like to discuss is the one where channel knowledge is available at the transmitter. Now what we have discussed over here is that we are not using this h information over here. It is not being made available, but h information is being made available at the receiver because you are seeing that you are doing H effective multiplied by y at the receiver side, but the transmitter side you are not doing any particular such processing. So, this is very very advantageous because the advantage is simply because you know feedback is required. It is a very very simple mechanism, very very powerful mechanism but of course, is had its limitation.

Now instead of that if we change the situation and say suppose feedback is available; that means, say feedback is available and in the modern communication systems that what we are talking about feedback is one of the fundamental mechanisms to provide a higher spectral efficiency. So, feedback is provided either in the time division duplexing mode and they use the reciprocity of the channel or it is sent back through the frequency division duplexing in the reverse channel.

So, in all cases we will assume that h is known at the transmitter. The received signal in that case would be given by the expression as outlined in the box that I am drawing, where we have introduced this w which is an additional vector which is the transmit weight vector. So, what we have is if you are sending s 1 from first antenna s 2 from second antenna s 3 from the third antenna. So, then you would multiply each of them by sorry I mean we are talking about the single stream.

So, we will not have this; you can simply have w 1 w 2 w 3 and so on. You can have this particular situation w 4 right and you can have such situations. So, when you have such a situation then effectively what you get is the signal that means, I am going to have w 1 s, w 2 s, w 3 s and so on and so forth.

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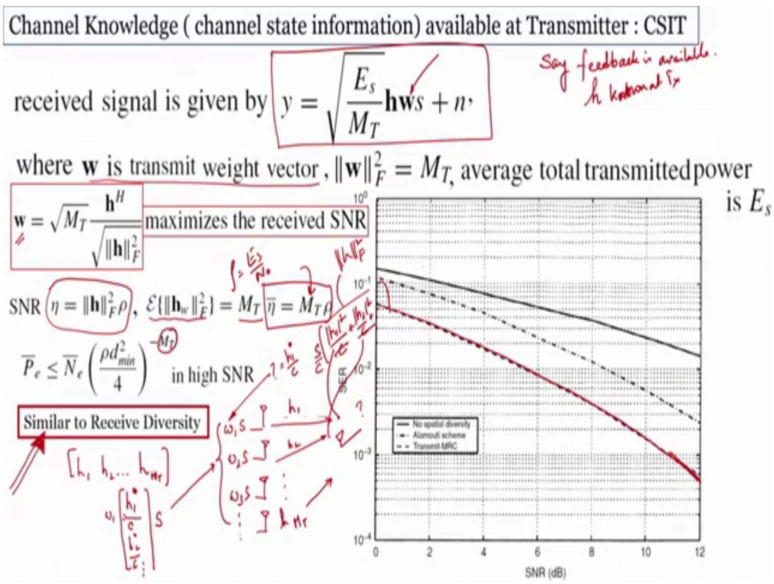

So, when we are transmitting; let us go to the transmitter structure. So, when you are transmitting from the transmit antenna side so, now these are coming back for some reason. So, you have w 1 multiplied by s, w 2 multiplied by s, w 3 multiplied by s and so on and so forth, and they are going through the channel h 1, h 2 up to let us say h M T to the receiver. If the receiver has only one receive antenna the situation is what possible choices of w can be made and what is the processing gain that is available at the receiver are the questions that need to be answered at this point of time.

So, there are various mechanisms for this and one of the mechanism that we have over here is the MRT or Maximal Ratio Transmission; that means, we choose this weight in the form h vector Hermitian divided or it is normalized. So, we have seen that h in this case is h 1, h 2 up to h M T let us say. So, when you take the Hermitian and normalize it is h 1 conjugate of course, by the normalization factor h 2 conjugate by the normalization factor and so on and so forth. This gets multiplied by s so each of these terms are the w terms which now lead to the situation that we have just described.

So, now you can clearly see that if w 1 is equal to h 1 conjugate by some normalizing factor when it goes through the channel the signal at this point is h 1 mod squared by c from normalizing factor. The signal coming from the second antenna would be added h 2 mod squared upon c and so on and so forth and all of them are going to have s as the common term so; that means, if we take s divided by c that means c we take out common what is left inside is the Frobenius norm squared of the channel again.

So, what we see is that the SNR at the receiver so, if we choose w in this particular case in this particular way the SNR at the receiver can be computed to be h F squared which is same as the situation what we have done for the received diversity thing. So, again the expected value of h F squared would be M T in a similar manner and the average SNR would be M T times E s by N naught rho is E s by N naught we should not forget this particular thing and the average probability of error if we calculate exactly following the same mechanism we will find M T coming to the exponent indicating that the order of diversity is M T.

And hence what we have is the performance which should not be any different from the receive diversity case. So, if full CSI is used and maximal ratio combining is used then we are going to get the transmit MRC's performance which will be same as that of the received MRC performance without changing; without providing any extra transmit power. That means, you are transferring the complexity of this processing to the transmit side yet getting all the advantage that are possible.

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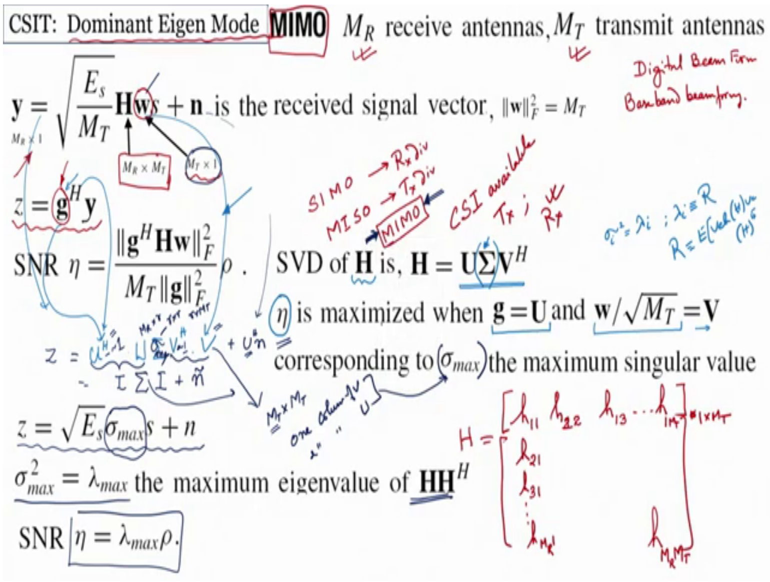

The next important scheme that we look at is known as the Dominant Eigen Mode one can also think of this as the digital beam forming, one can also call it as the baseband beam forming so, let us look at how does it work.

So, we have M R receive antennas we have M T transmit antennas, the received signal y that is there with us is written as ok; so, I should mention that we are talking about the scenario of MIMO; that means, previous cases we have talked about receive diversity, then we have talked about transmit diversity.

So, in receive diversity we had single input multiple output, in transmit diversity we had multiple input and single output and now we have the case which is multiple input multiple output. So, how do we handle this; that means both diversity the transmitter and receiver and we have also assumed that CSI is available at transmitter; at receiver it is always assumed to be available right. So, we are continuing with the CSI available at the transmitter.

So, under this case because you have multiple transmit antennas the H matrix will now be M R cross M T we have discussed how the received matrix would be that is h 1 1 indicating received signal in antenna 1 from transmit antenna 2, h received in signal antenna 1 from transmit antenna 2, signal received in trans in receive antenna 1 from transmit antenna 2 and so on and so forth and this forms the channel vector for the M T cross this is basically 1 cross M T, M T transmit antennas and then we have received in antenna 1 transmit receive antenna 2 transmit antenna 1, receive antenna t 3 transmit antenna 1 and so on up to h M R 1. So, basically you have h 1 M T and then you fill up the entire matrix and the last entry would be h received in M R antenna and transmitted from M T antenna. So, this is your H matrix right.

So, we have M R cross M T, this is an M R cross empty matrix as you can clearly see w is M T cross 1 as we have discussed, but we have to find what w compared to the previous situation and also we have to find the g which is required to be at the receiver right. So, we are kind of doing a same kind of linear processing, but we have to choose the g that we are going to use on the y. So, what we have over here is basically the y.

So, then what we see is that H, which is the channel matrix can be decomposed into the singular value decomposition using the single value decomposition. And sigma contains the singular values and we have already mentioned that the singular value squared would be equal to the lambda i, where lambda i are the Eigen values of R, where R is the expected value of vec H vec H Hermitian.

So; that means, we are actually talking about the channel strengths involved in this; so, this a single value of decomposition. So, what is known is this eta which is the SNR the combined expression would be in this form one could easily derive this without any complexity is maximized, if you set the received vector to U because you are doing this you g Hermitian processing.

And if you would set the transmit vector weight vector to V. So, let us see what would happen, what is going to happen is the received signal. So, if we process it over here H is broken down into U sigma V Hermitian and w would now be V and the g that is at the receiver would be U and this Hermitian would come over here to put; to be put as a Hermitian. So, let me erase this H right this is your Z plus U Hermitian times noise; this is what you are going to get because the noise is over here.

So, now what we see U Hermitian U is an identity. So, you are left with sigma V Hermitian V is again another identity plus processed noise; this is a unitary matrix. So, that some variance is not going to change.

Now, sigma is a diagonal matrix and hence when we do diversity mode or dominant Eigen mode from this we do not choose the entire matrix [vocalized-noise], not the entire matrix U because you have a weight vector which is M T cross 1 and V over here would be of the order of M T cross M T; I mean if you take it as a square matrix then it is; this will become an M R cross r sigma will be r cross r, this would be r cross M T. So, what we have over here is we will have M T such columns. So, instead of that you have to select only one column of V and one column of U. Now which columns would you select; you would select the columns corresponding to the maximum value of sigma which is going to maximize the eta.

So, if we select the maximum; the vectors corresponding to a sigma max what you will end up is in the value of z. So, this diagonal entry would now contain only one value that is sigma max because there is only one vector and there is only one vector over here. So, the received signal would be root over E s sigma max times s plus eta. So, this is the maximum strength of the channel that is what you are exploiting and then you can calculate eta as lambda max times rho where lambda max is equal to sigma max squared and we have already described this Eigen value of HH Hermitian.

So, this way we can extract the maximum order of gain from a MIMO channel if we have multiple antennas at both the transmitter side as well as at the receiver side while there is CSI information available at the transmitter. There is various such mechanisms, we stop our particular lecture over here; we will continue with the discussion in the next class.

Thank you.