

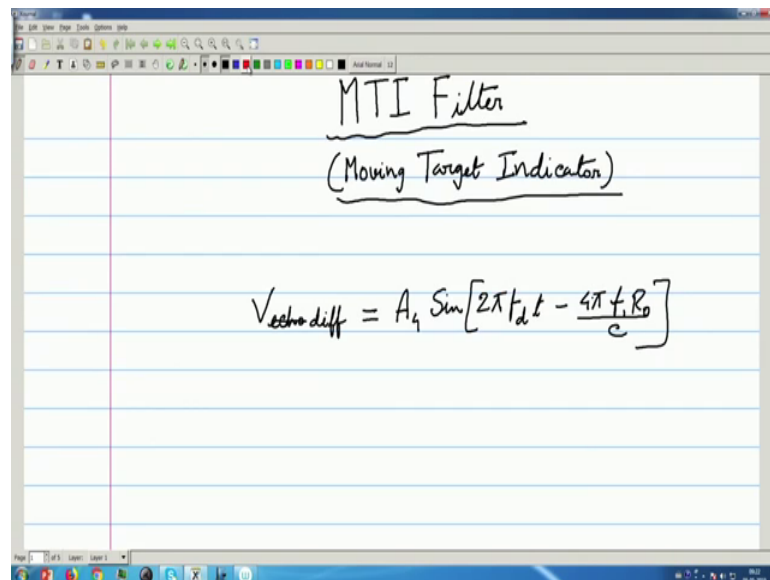
**Principles And Techniques of Modern Radar Systems**  
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**Lecture – 15**  
**MTI Filter**

**Key Concepts:** MTI implementation with single DLC, Transfer function and block diagram of single DLC, Concept of blind speeds

Welcome to this NPTEL lecture on Techniques and Principles of Modern Radar. In the last class we have discussed the block diagram of the pulse radar. Now, there we have seen that in the receiver chain, there is an MTI filter. And we discussed that pulse radar, also have a Doppler measurement facility and that is needed for clutter removal. So, that portion is called MTI filter and today we will see, what is there in MTI filter. Because this is a new thing for radar, this filter is not used in other places and you have not come across this. So, let us today discuss that MTI Filter.

(Refer Slide Time: 01:19)



MTI Filter  
(Moving Target Indicator)

$$V_{\text{echo diff}} = A_1 \sin \left[ 2\pi f_d t - \frac{4\pi f_i R_0}{c} \right]$$

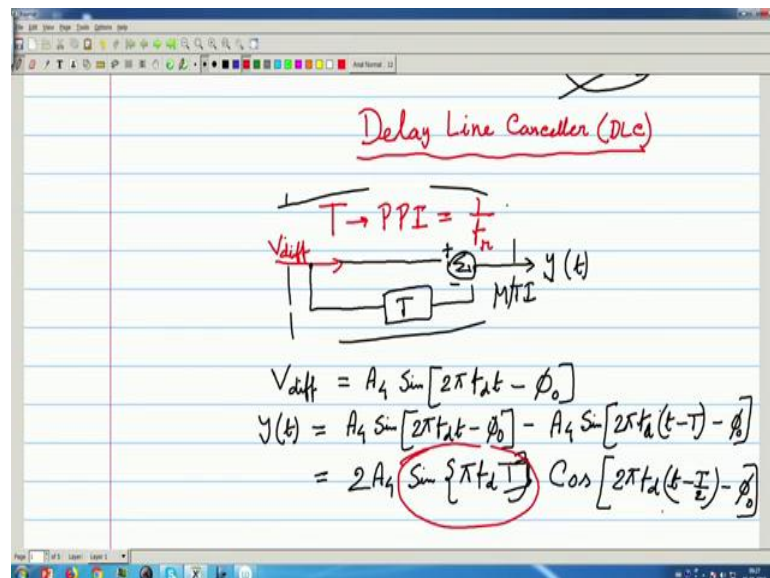
Now, at the end we have seen that a pulsed radar in the receiver, when the echo comes; that echo's mathematical expression we derived that was the  $V_{\text{echo}}$  if it is the; sorry not  $V_{\text{echo}}$  it is the difference between the reference and the target echo.

So, we can say I do not remember what we named, but you see probably something like this V diff phase detector. Sorry V phase detector output that will be something like A, what I said that time A 4 or A 3 some magnitude,  $\sin 2\pi f_d t - 4\pi f_1 R_0$  naught by c.

$$V_{\text{echo diff}} = A_4 \sin \left[ 2\pi f_d t - \frac{4\pi f_1 R_0}{c} \right]$$

So, due to the two way travel, space phase got detect added. So, this is the shape. Now, today we will start that we will see later various implementation of MTI filter.

(Refer Slide Time: 02:42)



So, one of that implementation is known as a delay line canceller. So, one of the possible MTI filter is this delay line canceller. We later again discuss, but just to introduce the idea. So, what a delay line canceller is that these signal you have these signal, you also delay it by one time period. One time period is the time; delay time is the pulse repetition interval; that means, the interval between two pulses that are transmitted. Basically, that is T capital T that we call PRI, it is  $1/f_r$ , the  $f_r$  is the p r f.

So, what we have; that means, you have the signal this V difference signal coming and you take from here, a signal, delay it by T and then you put a summer. So, this plus this; not plus you can put a minus here, minus and this is the output. So, this is one possibility

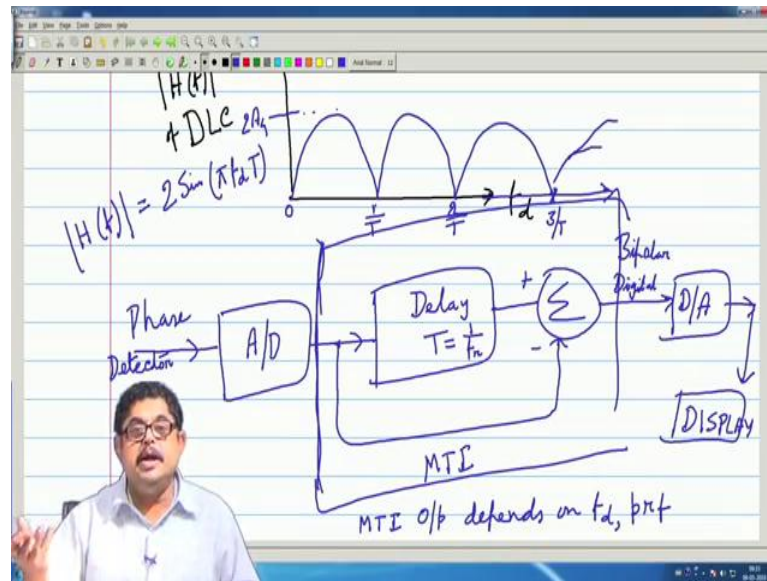
of an MTI filter. One implementation, there are other implementation also that we will discuss.

So, let us now understand if I call this  $y$ , this output how it will vary with time I will now derive that. So, I can write that  $V$  difference is here. So,  $V$  difference, again I am writing I am just will change something, that  $A_4 \sin 2\pi f_d t$  minus this whole thing, I am now calling a phase space phase let us say  $\phi_{naught}$ , because I have got  $R_{naught}$  is the range. So, this is the signal. So, what will be this delayed signal so, or I can directly write the  $y(t)$  expression.  $y(t)$  will be this signal  $A_4 \sin 2\pi f_d t$  minus  $\phi_{naught}$  minus  $A_4 \sin 2\pi f_d (t - T)$  minus  $\phi_{naught}$ . So, this if you do it is a simple  $\sin$  by  $\sin$ . So, we can get that it will be some  $2A_4$ , then  $\sin$  of  $\pi f_d T$ ; sorry, into  $\cos 2\pi f_d t$  minus  $T$  by 2 minus  $\phi_{naught}$  ok.

$$\begin{aligned}
 V_{diff} &= A_4 \sin[2\pi f_d t - \phi_0] \\
 y(t) &= A_4 \sin[2\pi f_d t - \phi_0] - A_4 \sin[2\pi f_d (t - T) - \phi_0] \\
 &= 2A_4 \sin\left\{\pi f_d T\right\} \cos\left[2\pi f_d \left(t - \frac{T}{2}\right) - \phi_0\right]
 \end{aligned}$$

So, you see that we are interest; we are interested that actually what is this MTI filter output? You see it is the original signal the phase detector output; obviously, instead of  $\sin$  that has become  $\cos$ ; that means, a phase change has taken place. But frequencies same, but the amplitude portion, this portion is multiplied  $\pi f_d T \sin$  of  $\pi f_d T$ . So, this is the job of the MTI filter. So, I can write, that what will be the transfer function of this MTI filter. You see it is not doing anything it has a phase, but in magnitude it is changing the amplitude.

(Refer Slide Time: 07:57)



So, if I plot the transfer function magnitude. So, if I plot that, transfer function magnitude of MTI filter or delay line canceller sorry, because there are others also so I will say that of DLC. So, versus  $f d$  if I plot it the plot will be something like this, it is a sin graph and plotting the magnitude part. So, it will be something like this and at so this is our zero, this is the magnitude that is that  $2 A 4$  something like that and this where it will cut, it will cut at  $1$  by  $T$ ; that means, the nulls will and  $2$  by  $T$ ,  $3$  by  $T$  and this graph will continue.

So; that means, the problem is that the output that is becoming a 0 at certain points. So, I can draw the block diagram. So, actually you can say that this DLC the full block diagram is we had that phase detector output. Before MTI filter if you see phase detector output is coming, it is a bipolar video we will put and A D convertor here.

Then we will give a delay block in; an digital domain you can put a delay in analogue domain also you know that, if we put a transmission line of appropriate length, then we can put these delay and these delay is equal to  $1$  by  $f r$  and you put a summer block. So, this output, you take and put it here and, here you are getting a bipolar digital signal and again you put to D A convertor and you give it to the display.

So, you see that this voltage that has now dependence on the prf as well as; obviously, the other portion is that sinusoidal variation with  $f d$ . So, we can write that this; so, this is MTI filter sorry, this is the MTI filter part, MTI. So, MTI output, I can write that the

MTI output depends on the  $f_d$ , the Doppler due to the range of the object range or if the moving target; obviously, moving target. So, range and velocity both are producing these things and then it also depends on the PRF of the signal through that  $T$ .

Now, this is a problem. This is a problem, because we can say that the output of the delay line canceller is the wave with same frequency as  $f_d$ , but amplitude modulated, its transfer function is; we have already seen the transfer function. The transfer function magnitude, we can say its expression is  $2 \sin \pi f_d T$ .

$$|H(f)| = 2 \sin(\pi f_d T)$$

And so when it is giving this null even if there is a target actually the MTI filter output is giving 0; that means, as if the Doppler is 0 because Doppler amplitude is getting modulated by this one.

So, this is a problem of MTI filter, that though it can distinguish whether a stationary target and moving target, but it also has suffered ambiguity in the Doppler measurement, because it is blind to certain speed. So, the speeds for which these values are there, at that the radar becomes radar receiver becomes blind. So, they are called blind speed. So, we can say that fall out of all these is that, because there is a sin function.

(Refer Slide Time: 13:55)

Drawback  $\rightarrow$

- Ambiguity in Doppler (blind speed)
- Diff. vel. targets give diff. attenuati.
- Roll off of the filter is not sharp near  $f=0$ .

Blind Speed

$$\rightarrow f_d = \frac{n}{T}, \quad n \in \mathbb{I}$$

$$V_{n, \text{blind}} = \frac{\lambda}{2} f_d$$

$$= \frac{\lambda}{2} \frac{n}{T} = n \frac{\lambda}{2} \frac{f_r}{2}, \quad n=1, 2, 3, \dots$$

1st blind speed  $>$  maximum expected target velocity

So, the drawback we can say that we have now invited to have MTI filtering; that means, to have moving target indicator, we have introduced a ambiguity in the Doppler measurement. So, I can say ambiguity in Doppler, which is called blind speed. And also you see there is another problem with this transfer function, that suppose in these zone also there are echoes, but different echoes are given different amplitude modulation. So, echoes are also depending on their range's, is their velocities different velocity targets there giving different things.

So, some of the targets will miss etcetera so, that is also another problem so; that means, the whatever we are making the  $f_d$  that is not uniform throughout. So, we will have various wrong information about the, their velocities etcetera. So, that is the ambiguity in Doppler so, because ideally, we should have a uniform here and these notches cannot be avoided etcetera.

So, now we will analyze these, so I will say that it is given you the thing, that different velocity targets given different attenuation. Also actually this part you now note, that this transfer function is another thing that near 0, actually this is a pure sin function. So, sin near 0 it is not changing very fast; I mean whatever is required from the practical. So, that is called roll off. So, near 0 frequency the roll off is not high. What is the implication of that, will see later, but now I will write another drawback, that roll off of transfer function is not sharp, near  $f_d$  is equal to 0. Actually I will say now, that we will see later that the clutter is near that; so, from the clutter the actual targets Doppler they are not very sharply distinguished, if you have this property. So, these are the drawbacks of these DLC etcetera, We will see that how to improve that what are the various MTI filter designs etcetera.

Now, since we have introduced the concept of blind speed. So, let us now discuss something about the blind speed. So, we can go here and see that what is a condition of blind speed? Those speeds which are for which Dopplers are integral multiples of  $1/T$ . So, those are the blind speeds. So, we can easily write that what are the blind speed that in those will have  $f_d$  is equal to  $n/T$ . So, the corresponding velocities will be blind speed; what is this  $n$ ?  $n$  is some integer. So,  $n$  belongs to  $I$ ; that means, 1, 2, 3 etcetera.

Blind Speed

$$\rightarrow f_d = \frac{n}{T}, \quad n \in \mathbb{I}$$

So now, we can convert that  $f_d$  corresponds to a speed; so, what is the blind speed I can say  $n$ th blind speed,  $V_n$  blind that will be we know that what is the relation between the Doppler and the  $\lambda$  by 2 into  $f_d$ . So, I can put this  $f_d$  value, that  $\lambda$  by 2 into  $n$  by  $T$ . So, now this  $T$  if I express in terms of  $f_r$  because it is more customary to make it there so,  $n$  sorry I can write  $n \lambda f_r$  by 2; where  $n$  is equal to 1, 2, 3 etcetera;

$$V_{n, \text{blind}} = \frac{\lambda}{2} f_d$$

$$= \frac{\lambda}{2} \frac{n}{T} = n \frac{\lambda f_r}{2}, \quad n = 1, 2, 3..$$

that means, what is the blind speed? So, if the targets they are having these blind speeds, the first blind speed for  $n$  is equal to 1, second blind speed, then the I would not be able to detect that target there will be blind speed.

Now, one way is that when a radar is designed there is an expectation that, what is the maximum speed with which the target will come? Because you know generally that for which targets you are trying to detect? So, if you design it in such a way that the first blind speed is just beyond the maximum velocity expected. So, we can say that, we can put a condition that first blind speed if we can make just greater than maximum expected target velocity, then the problem can be avoided. But this is not so, easily to do. So, because from this expression, if we put  $n$  is equal to 1 here, what does that say that; that means,  $\lambda f_r$  will be 2 into maximum expected target velocity.

(Refer Slide Time: 21:16)

Angular resolution

Unambiguous range

$f_r \rightarrow \text{Large}$

$1100 \text{ Km/h} = \frac{1.11 \times 10^4}{36} \text{ m/s}$

$\frac{1.11 \times 10^4}{36} = \frac{\lambda f_r}{2}$

UHF Radar  $\rightarrow f_c = 300 \text{ MHz}$

$\lambda = \frac{3 \times 10^8}{300 \times 10^6} \text{ m} = 1 \text{ m}$

$f_r = \frac{2.22 \times 10^3}{3.6} \text{ Hz}$

So; that means, target velocity whatever so; that means, we want  $\lambda f_r$  to have this that if you say that I will put blind speed faraway first blind speed, at least greater than the maximum thing then you need this thing should go to a large value.

Now, we have seen that if you make  $f_r$  large, you will have lot of ambiguity in the range. So, to mitigate the ambiguity in velocity you are then falling into the trap of range ambiguity. On the other hand, if you make  $\lambda$  large; that means, you are having low frequency radar; that means, your carrier frequency will be low that has severe problems etcetera, we have seen that then you have resolution problem.

So, actually you see that these two are more importantly this from for velocity measurement these are not chosen. Already  $\lambda$  and  $f_r$  has been chosen for the radar for some other two purposes. We have already seen that when we discussed the basic radar on what we choose generally  $\lambda$ , that is the we have seen that if we choose low frequency radar. The antennas beam width will be larger and so resolution of; the angular resolution that will be worst; that means, angular resolution value will be high. So, that we do not one.

So, usually from angular resolution stand point the  $\lambda$  is chosen and prf that is chosen from the concept of unambiguous range, that we have already seen in basics. So, these are not free to the designer. So, blind speed mitigation by this technique, that I will put it like this. This is generally not feasible in practical radars.



So, let us do some calculations that will give you an idea. That suppose a commercial aircraft these our normal planes. How much is their speed? Typical speed I can say 1000 or let us say 1100 kilometre per hour. Typically, this is the speed actually this is going on for long time that the normal aircrafts they fly around 1000 kilometre per hour, let me take 1100 kilometre per hour so, this is kilometre per hour.

So, let us convert it to the meter per second if you do that I think that will be into 10 to the power 4 by I am not exactly calculating it is will use this. So, with this speed then we can find that if this is the first blind speed, we have already found that this speed already the expression is there,  $V_n$  blind. So, this will be our first blind speed; so, first blind speed we have that expression. So, we can write that  $1.11 \times 10^4$  to the power 4 by 36 that should be my  $\lambda f_r$  by 2 I am putting  $n$  is equal to 1  $\lambda f_r$  by 2 ok.

So, from this I can find out now let us say that we are talking of a UHF Radar. So, UHF let us take that our carrier frequency typically, 300 megahertz. So, 300 megahertz radar; so, we know the what will be the value of lambda? Lambda will be  $3 \times 10^8$  to the power 8 by  $3 \times 10^9$  in metre so, that will be 1 metre ok. So, lambda is 1 metre and from here we can putting this lambda 1 metre, the PRF required we can find that PRF will be  $2.22 \times 10^3$  you put in that expression here; so,  $2.22 \times 10^3$  to the power 3 by 3.6 so, this much Hertz ok.

(Refer Slide Time: 26:42)

UHF Radar  $f_c$

$$f_n = \frac{2.22 \times 10^3}{3.6} \text{ Hz}$$

$$R_{unamb} = \frac{c}{2f_n} = 243.2 \text{ km}$$

S band,  $f_c = 3 \text{ GHz}$       $\lambda = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ m}$

$$R_{unamb} = 24.3 \text{ km}$$

X band,  $f_c = 10 \text{ GHz}$ ,  $R_{unamb} = 8.1 \text{ km}$

MTI  $\rightarrow$  unamb in range  
amb in vel

So, if we have this PRF typically, you see something a bit less than 1000 Hertz; 1 kilo Hertz less than that. So, 700, 800 it will be. Now, for that for an UHF radar what will be R unambiguous, this thing we know. So, if this is the PRF required for mitigating the blind speed in the way we have mentioned so, then you have R unambiguous we know, that R unambiguous is determined by the PRF and that expression is  $c$  by  $2 f r$ . So, you put it here that for this value of  $f$  this thing will be 243.2 kilometre. If you do the calculation  $c$  is 3 into 10 to the power 8  $f r$  these value you put it is. So, for UHF radar you may say that ok; the 243 kilometre let me go.

But suppose the same thing we require for S band, and S band radar typically say that  $f_c$  is 3 gigahertz. So, for an  $f_c$  3 gigahertz means, immediately our  $\lambda$  will be that 3 into 10 to the power 8 by 3 into 10 to the power 9 so,  $\lambda$  will be 0.1 metre, 10 centimeter. So, you put it now, that for this speed what will be  $f r$  that you can calculate and that you can put into again this unambiguous range. So, for that it will drastically reduce and it will become 24.3 kilometre.

So, for S band radar, by the technique that you cannot get then if the same thing you do for X band radar; that means,  $f_c$  let us say 10 gigahertz, then your R unambiguous that will become you see 8.1 kilometre. So, these are not realistic values, you cannot have radars; a radar which is supposed to detect the aircraft 24 kilometre or 8 kilometre. So; that means, though at low frequency as we have seen that making  $\lambda$  high means low. So, far UHF radar mitigating in that way may be possible that you make your PRF in such a way choose that this that may be possible, but for S band or X band this type of thing not possible. So, this is a; this blind speed is a serious problem will see that there are techniques by which this blind speed can be detected, but for this low frequency radars like UHF radars.

So, this ambiguity sometimes is tolerated and because if you are not so bothered with the velocity because detection; that means, not a very enemy type of situation, but you want to detect whether the aircraft is there, then velocity measurement you can have ambiguity, but you make the unambiguous range good so, that thing. So, that we have seen that is called MTI radar.

We have not discussed that how it mitigates clutter, that part we will see next, but so, that technique we have said that MTI filter it is unambiguous in range; unambiguous in

range measurement and ambiguous in velocity measurement. Also that time I have told that for the modern radars, modern pulse radars particularly awacs type of thing, this type of ambiguity in velocity measurement is not permissible. So, there you will have to have this that MTI filtering that should not give you the ambiguity.

So, how to mitigate that we will see that there are 3-4 techniques that you can have multiple prf, prf staggering which we discussed in the basic thing, but by that how you can have this thing calculated that, what is the true velocity. So, you can operate with multiple prf, you can operate with higher prf, you can operate with multiple rf frequency etcetera, there are various options. But next actually we will see it, to design a good MTI filter with this clutter removal things we also require what are the how much clutter has been eliminated by this MTI filter. So, we will have to understand clutter, we will have to see various MTI filter implementations their relative merits demerits that we will start from the next class.

Thank you.