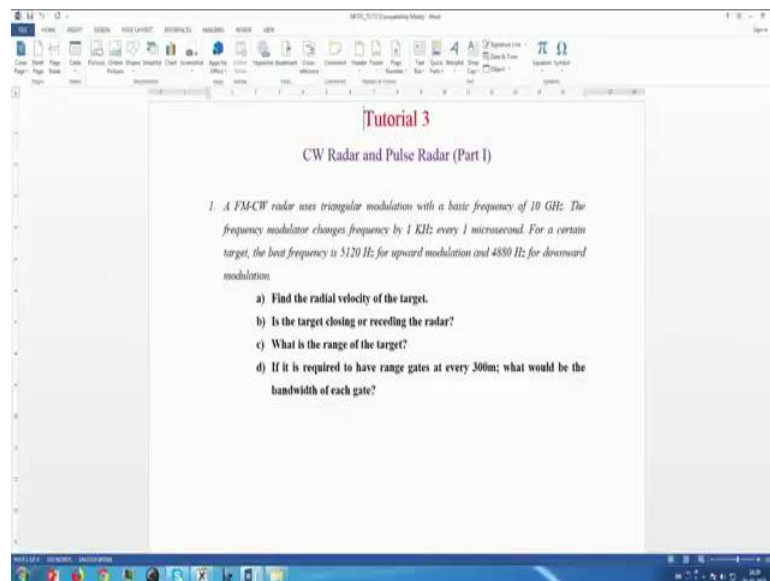


**Principles and Techniques of Modern Radar Systems**  
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**Lecture - 21**  
**Tutorial Problems on CW and Pulsed Radar (Part I)**

**Key Concepts:** Tutorial-3

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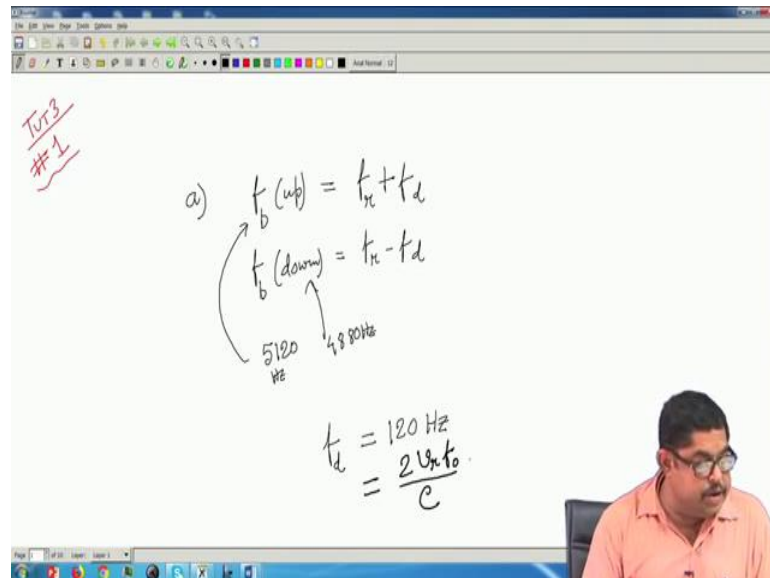
Welcome to this NPTEL lecture on Techniques and Principles of Modern Radar Systems. We were we have seen the CW radar and Pulse radar. Now we will see the today, we will see some problems applying those concepts that we have learned in the previous classes. So, the first problem this is actually tutorial 3. So, the first problem is on an FM-CW radar

So, the problem goes like this; a FM-CW radar uses triangular modulation with a basic frequency of 10 giga Hertz. The frequency modulator changes frequency by 1 kilo Hertz every 1 microsecond. As you know that FM-CW means it changes the frequency in a linear fashion. So, for a certain target the beat frequency is 5120 Hertz for upward modulation and 4880 Hertz for downward modulation.

So, the questions that you will have to solve is; what is the radial velocity of the target, is the target closing or receding the radar, what is the range of the target. If it is required to

have range gates at every 300 meter what would be the bandwidth of each gate? So, we see that I think you remember that FM-CW radar that actually CW radar cannot measure range, because it does not have any marker, but frequency modulation puts that marker. So, now, that is why in the part c what is the range of the target. So, it will be able to measure that range. So, let us solve this problem.

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So, this is tutorial 3, the problem is first problem. So, now radial velocity of the target, we know that this up beat and down beat by proper manipulation, we can get both the range and Doppler frequency from this. So, I think you remember this expression that beat frequency in the up part that is given by the change due to range, because the modulator is changing frequency. So, due to the range only there is a change in frequency that is  $f_r$  and due to the velocity of the target, there is a  $f_d$ . Similarly,  $f_b$  down that we know is  $f_r$  minus  $f_d$ .

So, we are asked to find what is the radial velocity; that means, we will have to find the  $f_d$ . So, in your case in the given case this  $f_b$  up is 5120,  $f_b$  up and this one is 4880 Hertz. So, just by subtracting we can get that  $f_d$  will be 120 Hertz, 120 Hertz and also, we know that what is  $f_d$ ;  $f_d$  is 2 the radial velocity  $f_0$  by  $c$ , we know if that  $f_d$  is related to the radial velocity by this relation. So, from here we can solve that  $v_r$  is  $f_d$ . So, if you put  $f_d$  we already got  $f_0$  is given as 10 kilo Hertz and  $c$  etcetera. So, that will give us 1.8 meter per second ok. So, we got the radial velocity.

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The screenshot shows a whiteboard with the following content:

$$v_r = \frac{f_u}{f_o} \frac{c}{2} = 1.8 \text{ m/s}$$

b)  $f_{b \text{ up}} > f_b \text{ (down)} \Rightarrow \text{target is receding}$

c)  $f_r = \frac{2R}{c} f_o$   
 $R = \frac{f_r}{f_o} \times \frac{c}{2}$

A person is visible in the bottom right corner of the whiteboard frame.

Now for part b, now as given that since we can see  $f_b$  up is greater than  $f_b$  down. So, that implies that the target is receding, because in the if you remember in the theory class, we have seen that if the target is closing then,  $f_b$  up will be less than  $f_b$  down etcetera. Now what is the range of the target? So, range we know that that is nothing, but it will come from this knowledge of  $f_r$  and  $f_r$  is  $2r$  by  $C$ ,  $f$  naught its derivative  $f$  naught dot. So,  $R$  is  $f_r$  by  $f$  naught dash into  $C$  by  $2$ . Now  $f_r$  you see we could have solved from here if we just add we get  $f_r$ .

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The screenshot shows a whiteboard with the following content:

c)  $f_r = \frac{2R}{c} f_o$   
 $R = \frac{f_r}{f_o} \times \frac{c}{2}$

$$f_r = 5000 \text{ Hz}$$
$$R = \frac{5 \times 10^3}{10^{-6}} \times \frac{3 \times 10^8}{2}$$
$$= 750 \text{ m}$$

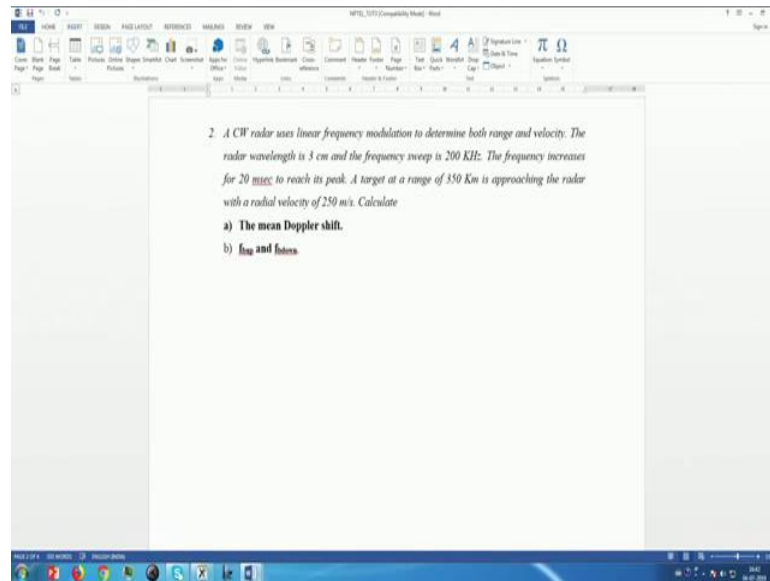
So, from here we can say that fr value will be, fr value is 5000 Hertz. Just you add divide by 2 from that equation. So, fr is 5000 Hertz. So, you can solve for R; R will be fr is 5 into 10 to the power 3. By this modulation we know that it frequency sweep is changed by 1 kilo Hertz for every 1 microsecond; that means, 10 to the power 3 by 10 to the power minus 6 and into c by 2; that is 3 into 10 to the power 8 by 2. So, this will give you R is 750 meter.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, the equation  $R = \frac{10^3}{10^{-6}} \times 2$  is written, which simplifies to  $R = 750 \text{ m}$ . Below this, a calculation for the range gate bandwidth  $\Delta f_n$  is shown. It starts with the formula  $\Delta f_n = \frac{2 \Delta R}{c} \dot{f}_0$ . The values are substituted as  $\Delta R = 300$  and  $\dot{f}_0 = 10^3 \times 10^{-6}$ . The calculation proceeds as follows:  $\Delta f_n = \frac{2 \times 300}{3 \times 10^8} \times 10^3 \times 10^{-6}$ , which simplifies to  $\Delta f_n = 2 \text{ KHz}$ .

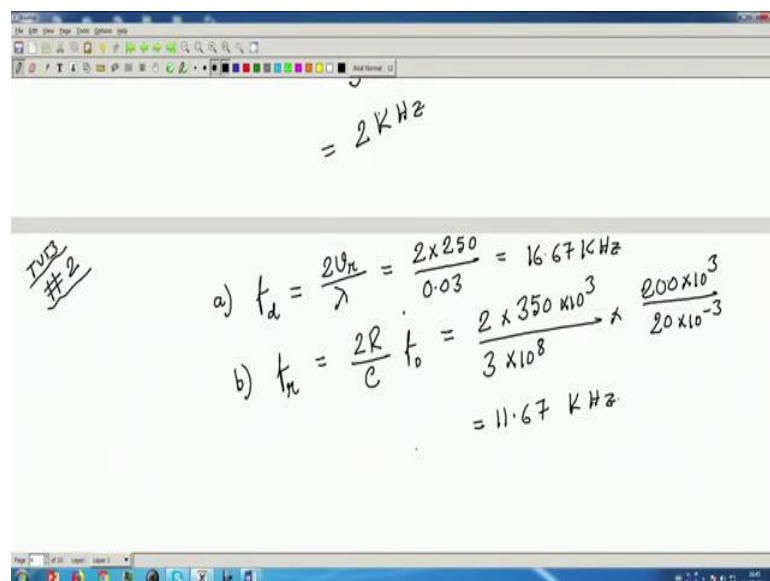
So, the range of the target is 750 meter and now the question is if it is required to have range gates at every 300 meter. So, what would be the bandwidth of each gate? Now you see this same fr this expression, this will give us because every 300 meter if I have range gate. So, what is the bandwidth? So, we can just differentiate that delta fr is 2 delta R by c f naught dot. So, from here delta R is said that range gates at every 300; so, this delta fr this will give us the bandwidth so it is 2 into 300 by 3 into 10 to the power 8 into fr f naught dash is that 10 to the power 3 by 10 to the power minus 6. So, that will give us 2 kilo Hertz. So, the bandwidth of every range gate that we will have to keep as 2 kilo Hertz; so, simple problem.

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So, we have, so the next problem is that a CW radar uses linear frequency modulation to determine both range and velocity like the previous problem. The radar wavelength is 3 centimeter and the frequency sweep is 200 kilo Hertz. The frequency increases for 20 milliseconds to reach its peak a target at a range of 350 kilometer is approaching the radar with a radial velocity of 250 meter per second. Calculate the mean Doppler shift, fb up and fb down. So, in previous problem it was said now you have to do what will be the fp up and fb down

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Problem 2, tutorial 3; so, the first part says that the mean Doppler shift, now what is Doppler shift?  $f_d$  is equal to  $2 v_r$  by  $\lambda$ . So,  $v_r$  which has been said that radial velocity 250. So,  $2$  into  $250$  by  $\lambda$  is  $3$  centimeter that is  $0.03$ . So, that gives us  $16.67$  kilo Hertz is the  $f_d$  and  $f_b$  up so,  $f_d$  is known. Now  $f_r$ , what will be  $f_r$ ?  $f_r$  the same formula we will use  $2 R$  by  $c$ . So, these values as said, the frequency target is at a range of  $350$  kilometer.

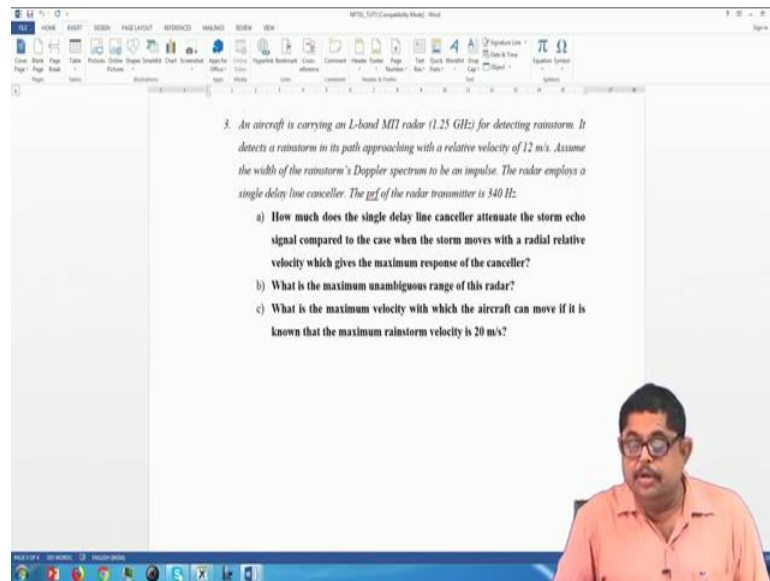
So,  $350$  into  $10$  to the power  $3$  by  $c$  is  $3$  into  $10$  to the power  $8$  and the modulator slope, frequency slope is  $200$  kilo Hertz in  $20$  millisecond I think yes  $20$  millisecond. So,  $20$  into  $10$  to the power minus  $3$ . If you do that it comes to  $11.67$  kilo Hertz and so, we can find out here now, it is said that the target is closing target is closing. This information is given. So, you can easily also we can see that  $f_d$ , let me show that  $f_d$   $16.67$  kilo Hertz  $f_r$  is  $11.67$ ; that means,  $f_d$  is greater than  $f_r$ . So, this in theory we have seen that if this is satisfied then our equation will be like this;  $f_b$  up that will be  $f_d$  minus  $f_r$ . So, that will be  $5$  kilo Hertz and  $f_b$  down is  $f_d$  plus  $f_r$  that is  $28.34$  kilo Hertz that is all; so, simple ok.

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Handwritten notes on a whiteboard showing Doppler shift calculations for a closing target. The text includes:

- $f_d = 11.67 \text{ kHz}$
- Target is closing
- $f_d > f_r$
- $f_{b \text{ up}} = f_d - f_r = 5 \text{ kHz}$
- $f_{b \text{ down}} = f_d + f_r = 28.34 \text{ kHz}$

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So, this is an interesting problem an aircraft is carrying an L band MTI radar for detecting rainstorm. It detects rainstorm in its path approaching with a relative velocity of 12 meter per second. So, you know that CW radars there used for detecting rainstorm. It is having an MTI facility.

So, we approaching and rainstorm is coming with 12 meter per second. Assume the width of the rainstorm's Doppler spectrum to be an impulse, because the Doppler, it is it has a finite width but let us assume to be an impulse. The radar employs a single delay line canceller. The prf of the radar transmitter is given. So, and so, so the question is how much does the single delay line canceller attenuate that storm echo compared to the case when the storm moves with a radial relative velocity which gives the maximum response of the canceller?

We know that a delay line canceller makes a sinusoidal attenuation of the various echoes. So, the question is this particular velocity rainstorm how much it will be attenuated compared to the maximum response of the canceller case; so, very interesting question. So, let us see how we can solve it. So, this is our tutorial 3.

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TUT 3  
#3

$$a) \lambda = \frac{0.3}{1.25} = 0.24 \text{ m}$$
$$f_r = 340 \text{ Hz}$$
$$f_d = \frac{2 v_r f_r}{c} = \frac{2 \times 12 \times 1.25 \times 10^9}{3 \times 10^8} = 100 \text{ Hz}$$
$$\text{SDLC} \rightarrow \text{Sin} \left\{ \frac{\pi f_d}{f_r} \right\} = \text{Sin} \left\{ \frac{\pi \times 100}{340} \right\} = 0.8$$

Attenuation ratio = 0.8

Tutorial 3 problem number 3. So, first it is a L band radar. L band you know 1 to 2 giga Hertz. So, this is given as 1.25 giga Hertz. So, that is lambda will be then 0.3 by 1.25 that will be 0 point; that means, 24 centimeter is the lambda. Okay also it is said that relative velocity the prf of the radar transmitter is 340 Hertz. So, prf is 340 Hertz and radial velocity given we can calculate fd. So, fd will be 2 vr f naught by C.

So, vr is 12 meter per second. So, 2 into 12 and f naught f naught is 1.25 giga Hertz so, 1.25 into 10 to the power 9 by c. So, this will give us fd is 100 Hertz. So, you see that the 100 Hertz is the fd. Now, we know that single delay line canceller, single delay line canceller has an amplitude multiplying factor of sin pi fd by fr. So, that will be sin pi fd is 100 and fr is 340. So, this will give us a 0.8; that means, it will attenuate the echo by a factor of 0.8 whereas; any sin function the maximum is 1. So, attenuation ratio we can say; that means, this storm by the maximum so, that if we call attenuation ratio will be 0.8 that is all. So, first part is over.



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Handwritten calculations on a whiteboard:

Altitude ratio = 0.8

b)  $R_{\text{unamb}} = \frac{c}{2f_r} = \frac{3 \times 10^8}{2 \times 340} = 441.2 \text{ Km}$

c) first blind speed  
 $v_1 = \frac{\lambda f_r}{2} = \frac{0.24 \times 340}{2}$   
 $= 40.8 \text{ m/s}$

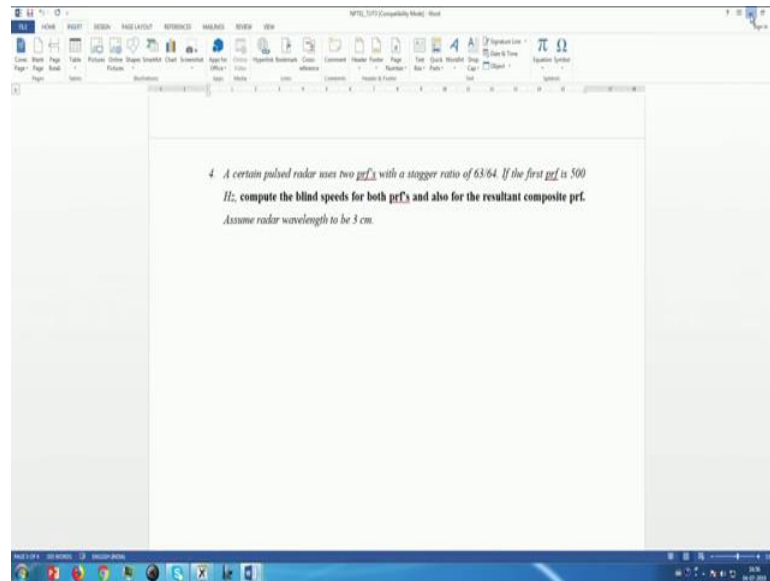
Max = wind of the plane =  $40.8 - 20 = 20.8 \text{ m/s}$

So, let us we can see the question, then part b is what is the maximum unambiguous range of this radar. So, that will be our next step. The maximum unambiguous ranges are unambiguous that is  $c$  by  $2 f_r$  so, that we can put the value 2 into 340. So, that will give us 441.2 kilometer; that means, our unambiguous range is 441 kilometer and what was the part c, what is the maximum velocity with which the aircraft can move if it is known that the maximum rainstorm velocity is 20 meter per second?

Good question so, maximum velocity with what we can move that will be; that means, it should be less than the blind speed. So, let us first find what is the first blind speed. So, first blind speed  $v_1$  is  $\lambda f_r$  by 2. So, that is 0.24 into 340 by 2. So, that is 40.8 meter per second. So, that is the first blind speed. So, if the relative velocity of the storm with respect to the plane is this 40.8 meter per second the storm will be not detected.

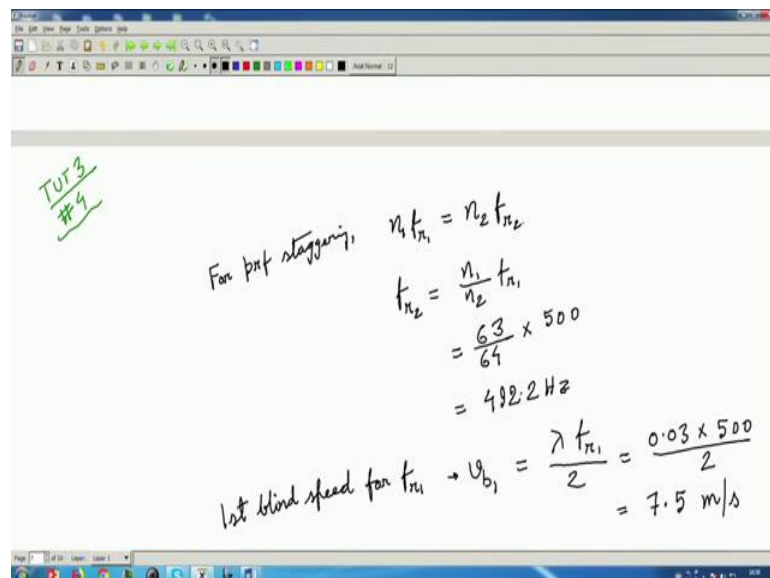
So, so that means, that is the maximum so; that means, the maximum velocity of the plane that will be this 40.8 minus 20, because we know maximum a can be twenty rainstorm. So, that is 20.8 meter per second so; that means, if the plane moves with the this velocity 20.8 meter per second no problem it will not suffer from blind speed, it will be able to detect the rainstorm. You see that from the basic concept you can answer such practical questions; so, third problem gone.

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Then this problem, a certain pulse radar uses two PRFs with a stagger ratio. So, PRF staggering we have discussed it is a question on that with a stagger ratio of 63/64. If the first PRF is 500 Hertz, compute the blind speeds for both PRFs and also for the resultant composite PRF. Assume radar wavelength to be 3 centimeter. So, let us see the solution. So, stagger PRF so, we will have to find blind speed for each PRF and also for the composite PRF when you have staggering.

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So, that will be, let me tutorial 3 problem 4. So, for prf staggering prf staggering, we know that if we have two prf's then,  $n_1 f_r 1$  is equal to  $n_2 f_r 2$ . So, what will be  $f_r 2$ ;  $n_1$  by  $n_2 f_r 1$ . So, it is said one of the prf is 500. So, other one will be 63 by 64 into 500 so, that will come to be 492.2 Hertz.

Now, we will have to find what is the first blind speed for  $f_r 1$ , what is the first blind speed for  $f_r 2$ . One by one we will calculate. So, first blind speed for  $f_r 1$ , let us call that  $v_b 1$ . So, that is  $\lambda f_r 1$  by 2. So, you can put  $\lambda$  is 3 centimeter. So, 0.03  $f_r 1$  is 500 by 2. So, that will be 7.5 meter per second. So, first blind speed for prf 1 is 7.5 meter per second.

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The image shows a whiteboard with handwritten mathematical derivations. The first derivation calculates the first blind speed for  $f_{r2}$  as  $v_{b2} = \frac{\lambda f_{r2}}{2} = \frac{0.03 \times 492.2}{2} = 7.38 \text{ m/s}$ . The second derivation calculates the average constant prf as  $f_{r_{av}} = \frac{f_{r1} + f_{r2}}{2} = 496.1 \text{ Hz}$ . The third derivation calculates the first blind speed for  $f_{r_{av}}$  as  $v_{b_{av}} = \frac{\lambda f_{r_{av}}}{2} = \frac{0.03 \times 496.1}{2} = 7.44 \text{ m/s}$ .

Similarly, calculate first blind speed for  $f_r 2$ . If we call that  $v_b 2$ ; that is  $\lambda f_r 2$  by 2. So, that is 0.03 into 492.2 by 2. So, that will be 7.38 meter per second. Now composite; that means, when you are staggering what is the average constant prf? So, that we can call  $f_r$  average. So, that is nothing, but  $f_r 1$  plus  $f_r 2$  by 2. So, that will be 496.1 Hertz. Now, first blind speed for that blind speed for  $f_r$  average if we call that  $f_b$  average, that is  $\lambda f_r$  average by 2. So, that is 0.03 into 496.1 by 2 so that is 7.44 meter per second.

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The image shows a screenshot of a whiteboard with handwritten mathematical formulas. The top section is titled "1st blind speed" and shows the calculation: 
$$= \frac{0.03 \times 496.1}{2}$$
$$= 7.44 \text{ m/s}$$
The bottom section is titled "For staggered prf" and shows the formula for the first blind speed: 
$$1^{\text{st}} \text{ blind speed } U_{b \text{ st}} = \frac{n_1 + n_2}{2} U_{b \text{ av.}}$$
Below this, the calculation is shown: 
$$= \frac{63 + 64}{2} \times 7.44$$
$$= 472.54 \text{ m/s}$$

Now whereas, so, this is a average constant prf, but for staggered prf for staggered prf, first blind speed formula we know first blind speed if we call that fb staggered; that is  $n_1$  plus  $n_2$  by 2 by  $v_b$  average. So, that will be 63 plus 64 by 2 into 7.44. So, that will be 472.54 meter per second. So, you see that staggering is qualitatively changing the game.

So, if you have either fr 1, it is 7.4 meter per second then for fr 2 also not much change; that means, if we have a constant prf, constant prf may be average of the two prf's fr 1, fr 2 you are not changing much. The blind speed is almost same, but the moment you do prf staggering with two nearby prf's, you are getting almost a jump of from 7 to almost 80 or more than that 85 times you are getting the blind speed removal. So, that is the advantage of prf staggering. This example also sees that. We will see more such thing in the next tutorial.

Thank you.