

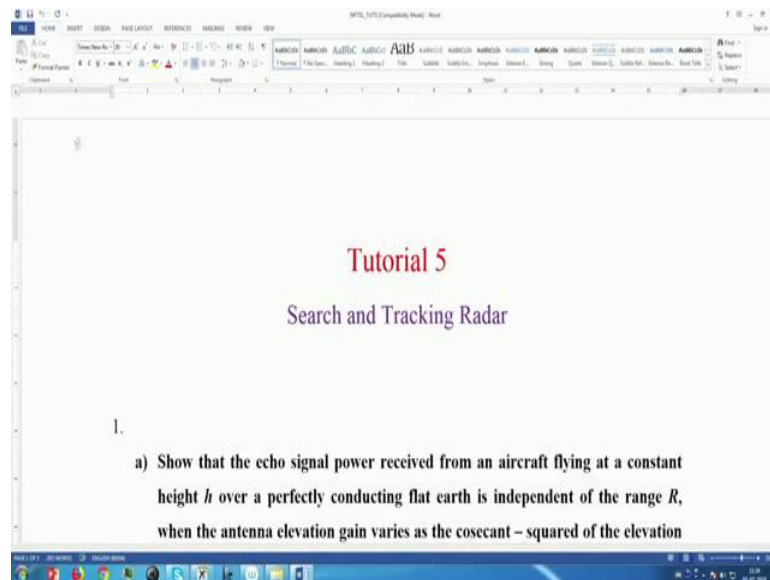
Principles And Techniques Of Modern Radar Systems
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Lecture - 32
Tutorial Problems on Search and Tracking Radar

Key concepts: Tutorial 5

Welcome to the NPTEL lectures on principles and techniques of modern radar systems. In previous few lectures we have discussed search radar and tracking radar, monopulse radar etcetera.

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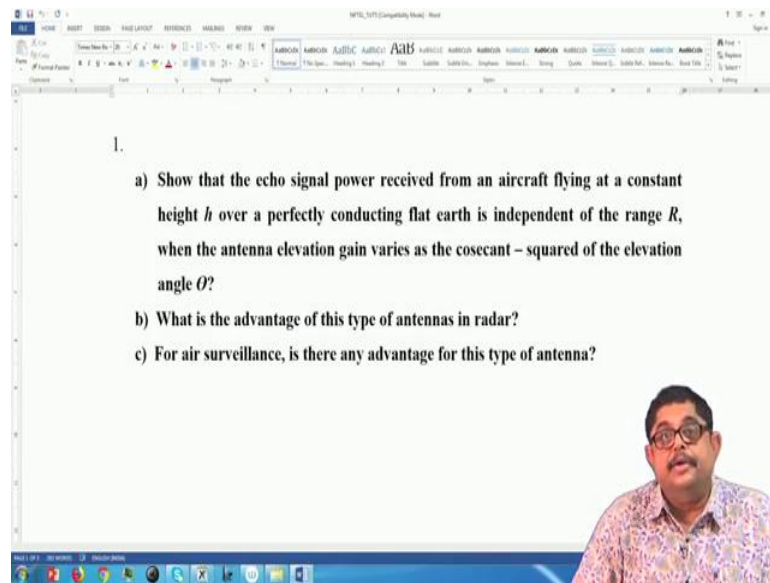
The image shows a screenshot of a presentation slide. The slide has a white background with a blue border. At the top, there is a navigation bar with various icons and text. The main content of the slide is centered and reads:

Tutorial 5
Search and Tracking Radar

1.
a) Show that the echo signal power received from an aircraft flying at a constant height h over a perfectly conducting flat earth is independent of the range R , when the antenna elevation gain varies as the cosecant - squared of the elevation

So, today we will see tutorial 5 on search and tracking radar. So, the first problem is an interesting problem and it is a practical problem.

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A screenshot of a presentation slide displayed in a software window. The slide contains a list of three questions. A man with glasses and a patterned shirt is visible in the bottom right corner of the slide, appearing to be the lecturer. The software window has a menu bar with options like FILE, HOME, INSERT, DESIGN, PAGE LAYOUT, REFERENCES, MAILINGS, REVIEW, and VIEW. The taskbar at the bottom shows various application icons.

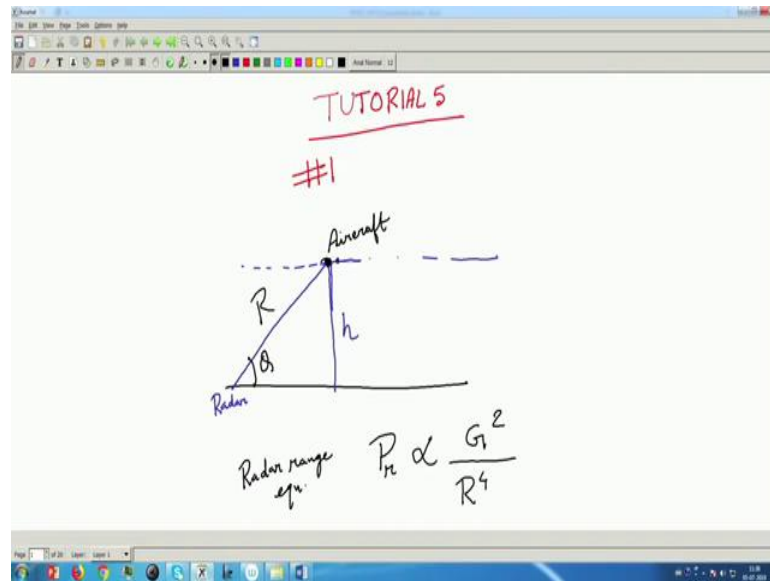
1.

- a) Show that the echo signal power received from an aircraft flying at a constant height h over a perfectly conducting flat earth is independent of the range R , when the antenna elevation gain varies as the cosecant-squared of the elevation angle θ ?
- b) What is the advantage of this type of antennas in radar?
- c) For air surveillance, is there any advantage for this type of antenna?

Actually this the first thing is show that the echo signal power received from an aircraft flying at a constant height h over a perfectly conducting flat earth is independent of the range R , when the antenna elevation gain varies as the cosecant-squared of the elevation angle θ ?

Now, this is an important thing actually this cosec squared pattern is generally desired from antennas and actually if you see any practical radar site these generally the antenna pattern is such and first we will show this then we will discuss this what is the advantage of this type of antennas in radar and actually surveillance radars they have these type of antennas. So, for air surveillance is there any advantage for this type of antenna. Actually this is a practical example of the application of theory. So, we will solve this problem note down this.

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So, tutorial 5, this is problem number 1. Now what is said that suppose this is the flat earth and this is an aircraft flying at a constant height. So, flying at a constant height means I can say that this is the constant height. So, what is the height? Height from any position height is h constant height and so range of the aircraft from the radar let this is the radar and this thing is the aircraft and this angle is the elevation angle theta.

So, what the question is saying that show that P_r is independent of the range, but we know from radar equation P_r is you can find that from radar range equation. P_r it is obviously, proportional to transmit power, etcetera but those are not varying I can say that it is proportional to the gain of the antenna square I am assuming a monostatic antenna and we know that it is proportional to R to the power inversely proportional to R to the power 4 R is the range.

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Radon range eqn. $P_r \propto \frac{G^2}{R^4}$

$$G \propto \text{Cosec}^2 \theta$$

$$G = G_0 \text{Cosec}^2 \theta$$

$$P_r \propto \frac{G_0^2 \text{Cosec}^4 \theta}{R^4} = k \underset{\text{const.}}{G_0^2} \frac{(R/h)^4}{R^4}$$

$$= k G_0^2 \frac{1}{h^4} = \text{constant}$$

So, this is the range R and also it is said that the antenna gain function that is proportional to cosec square theta. This is the meaning. Now so, I can write that G is some constant G naught cosec square theta. So, I can put this there that P r will be then proportional to in place of G I can put G naught square cosec 4 theta by R 4. So, these I can say constant G naught also is absorbed or let is capital.

So, this is a constant of proportionality G naught square then I have this cosec 4 theta by R 4. So, what is cosec 4 theta from here? What is sin theta; sin theta is h by R. So, cosec theta is R by h. So, I can put that k G naught square then R by h whole to the power 4 by R 4. So, what it gives me k G naught square then simply 1 by h to the power 4. So, you see h is a constant aircraft is flowing. So, power received this is obviously constant. So, I can say this is nothing, but a constant that was we supposed to put.

So, this is the advantage that if we choose the antennas radiation pattern or gain pattern to be cosec squared theta then the always the whatever may be the elevation angle the power received is always same.

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1.

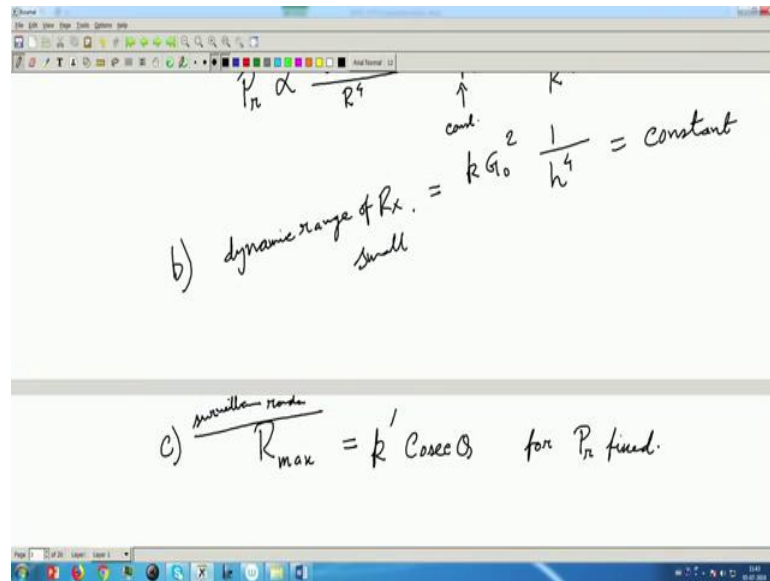
- Show that the echo signal power received from an aircraft flying at a constant height h over a perfectly conducting flat earth is independent of the range R , when the antenna elevation gain varies as the cosecant – squared of the elevation angle θ ?
- What is the advantage of this type of antennas in radar?
- For air surveillance, is there any advantage for this type of antenna?

So, the second part of the question says what is the advantage of this type of antennas in radar a very good question? So, we will say that if received signal is independent of range that is a huge advantage because; that means, for long range and short range I am receiving same power.

So, that means, a dynamic range that is required from the receiver that is small because I have a constant (Refer Time: 08:14) thing noise is varying depending on range etcetera and but I have the power received is not changing. So, my requirement of dynamic range from the receiver is a so if we want to have more dynamic range from a receiver we will have to design it more sophisticatedly and that cost increases. So, that is an advantage.

Now, more than that actually the part c for air surveillance is there any advantage for this type of antenna? This is an interesting question actually with this genesis of these came that we went to a we have in our IIT Kharagpur nearby there is a Salua radar we went for a visit and suddenly one student asked this question and from there the students also understood that what is the benefit of this. So, I can say that let us discuss that very very important for air surveillance.

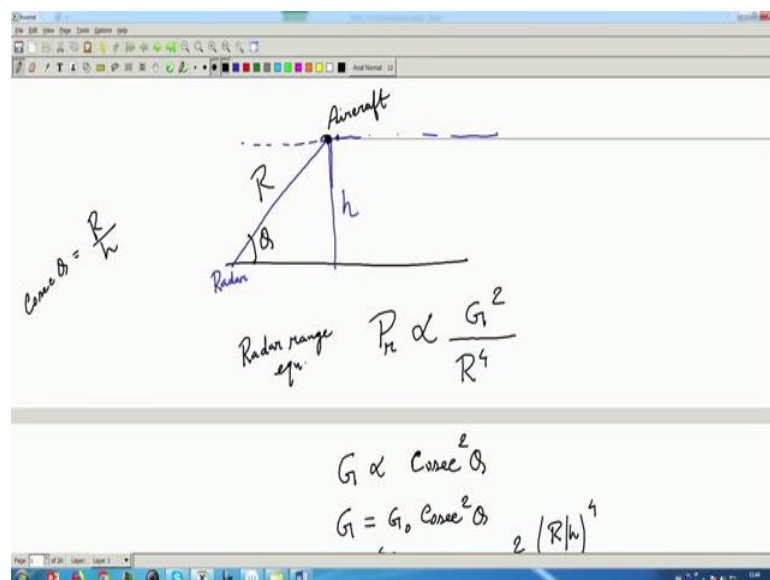
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So, part c; so, b part I am saying that dynamic range small receiver small. So, that is the advantage, but this is more important that in air surveillance radar.

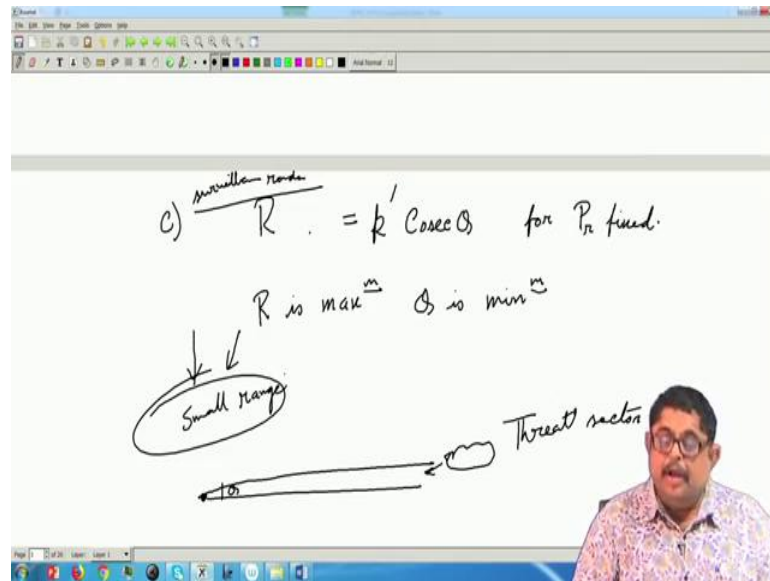
Now, from this equation you can see what is R max; R max is let us say some constant k dash into cosec theta for P r fixed from the same diagram you can see that.

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What is R; R is obviously, some constant into cosec theta. I can write here what is cosec theta; cosec theta is R by h. So, R is some constant h is constant into cosec theta. So, R will be maximum when cosec theta will be maximum. This cursor is not visible to me.

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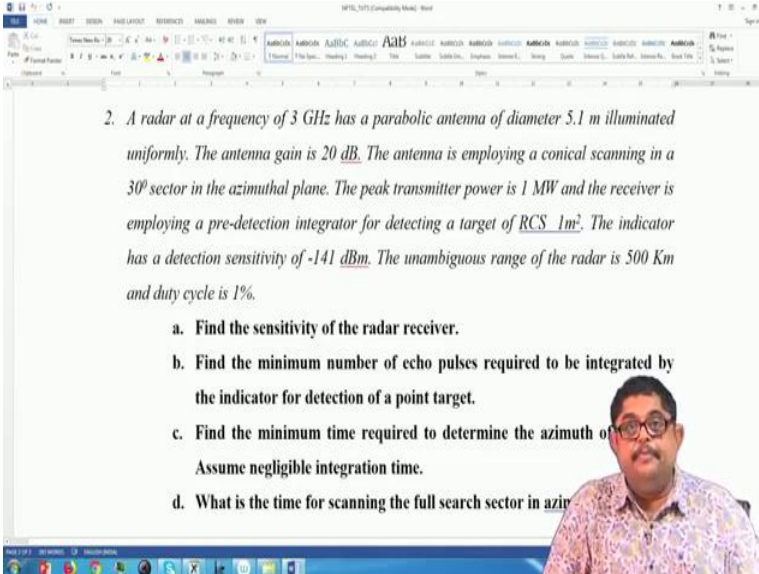
So, we can say that or first I will write R then I will say that R is maximum when cosec theta is maximum; that means, theta is minimum. Now you see that theta minimum elevation angle minimum. So, this is the radar and minimum elevation angle; that means, those which are coming suppose an aircraft is coming from here this aircraft approaching actually the near the radar the aircrafts try to be low, so that they do not get detected.

So, this is called the threat sector. So, in the threat sector you see that the radar will be able to see at the maximum range because R is maximum when theta is minimum theta is. So, an enemy aircraft is coming from a very low observation angle, very low elevation angle theta. So, there the surveillance radar can see at the maximum point. So, in the threat sector we can say that it has a very high range whereas, you see that no aircraft will come from this direction or even this direction because these are the points where easily they will be detected. So, this is a non threat sector.

Generally all aircrafts will try to come like this angles. So, here you can have small ranges it does not have any problem. So, in all these zones small range is not a problem. So, who can give this benefit? That benefit comes from that you see, but still the power is same range is more, but power is same. So, that is why cosec square pattern generation is a very interesting challenge to an antenna engineer; all modern antenna systems like phased array antenna etcetera they try to generate this cosec square pattern. So, that you can have a low flying enemy aircraft can be detected from a large range.

So, this is an interesting problem and this cosec square pattern is a. So, let us go to our next problem 1 we have seen now let us come to the second problem. So, this problem is over.

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The image shows a presentation slide with a video feed of a presenter in the bottom right corner. The slide contains the following text:

2. A radar at a frequency of 3 GHz has a parabolic antenna of diameter 5.1 m illuminated uniformly. The antenna gain is 20 dB. The antenna is employing a conical scanning in a 30° sector in the azimuthal plane. The peak transmitter power is 1 MW and the receiver is employing a pre-detection integrator for detecting a target of RCS 1m². The indicator has a detection sensitivity of -141 dBm. The unambiguous range of the radar is 500 Km and duty cycle is 1%.

- Find the sensitivity of the radar receiver.
- Find the minimum number of echo pulses required to be integrated by the indicator for detection of a point target.
- Find the minimum time required to determine the azimuth of the target. Assume negligible integration time.
- What is the time for scanning the full search sector in azimuth?

Now let us see the second problem a radar at a frequency of 3 gigahertz has a parabolic antenna of diameter 5.1 metre a huge antenna because 1 metre antenna you will see in our home dish TV roughly 1 or 80 centimeter 90 centimeter that antenna but 5 meter antenna in say huge antenna in sophisticated radars air surveillance radars we will see that illuminated uniformly.

The antenna gain is given, the antenna is employing a conical scanning in a 30 degree sector in the azimuthal plane. In the azimuthal plane it is just showing covering 30 degree because that is the zone generally all 360 degree need not be covered because you know from which direction the surveillance needs to be done. So, that is why we will see that this surveillance radar they have a sector.

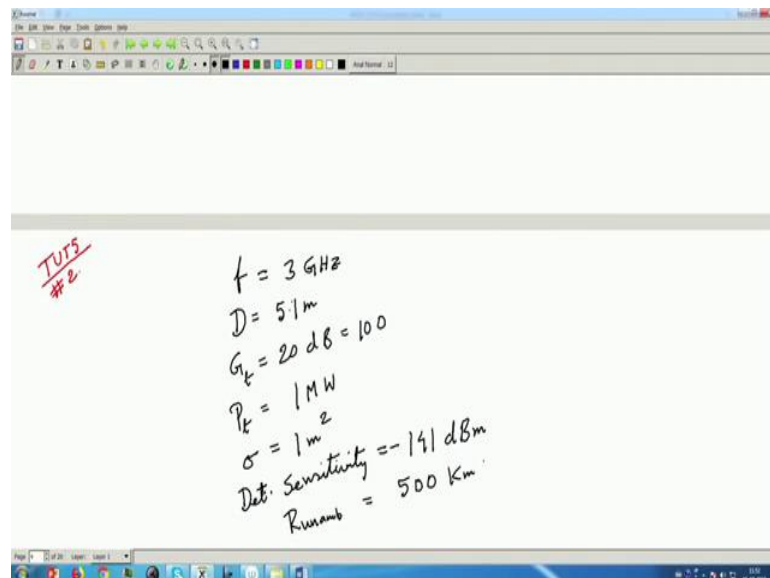
So, 30 degree sector in the azimuthal plane it is covering. The peak transmitter power is given and the receiver is employing a pre detection integrator for detecting a target of RCS 1 meter square ok. The indicator has a detection sensitivity of so much thing that means the instead of receiver sensitivity this time detector sensitivity is given. The unambiguous range of the radar is 500 kilometer and duty cycle is 1 percent.

So, you know the unambiguous range of the radar is said means basically the PRF is said and duty cycle said etcetera. So, pulse on time also we will be able to find now first thing is find the sensitivity of the radar receiver. So, this type of problem then find the minimum number of echo pulses required to be integrated by the indicator for detection of a point target; obviously, we will can easily find out from the required SNR how much it will then find the minimum time required to determine the azimuth of a target this is the job of a scanning antenna.

So, because it will have to determine the azimuth and elevation. So, here elevation is not necessary, but azimuth. So, what is the time required to determine the azimuth of a target? This is important for fire control etcetera exercises, because if the least time you have to determine you can activate your fire control. When you can control your fire and here for simplicity assume negligible integration time; that means, for integrating let us say that though there is a finite integration time.

Let us assume that the integration time is 0. What is the time for scanning the full search sector in azimuth 30 search sector; so, it is a very relevant problem. So, let us see its solution. So, this will be problem 2. So, I can say tutorial 5 problem 2.

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So, let us see what is given? Given is f is given 3 gigahertz then D is given 5.1 meter then G_t is given 20 dB, 20 dB means absolute value 100 then P_t is given 1 megawatt

then sigma is given 1 meter square RCS then detector sensitivity that is -141 dBm, and also said R unambiguous is 500 kilometer.

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Handwritten derivation for radar receiver sensitivity:

$$a) \lambda = \frac{0.3}{3} = 0.1 \text{ m}$$

$$P_{r \text{ min}} = \frac{P_t G_t^2 \lambda^2 \sigma}{(4\pi)^3 R_{\text{max}}^4} = \frac{10^6 \times (100)^2 \times (0.1)^2 \times 1}{(4\pi)^3 \times (5 \times 10^5)^4}$$

$$= 8.06 \times 10^{-19} \text{ W}$$

$$= -151 \text{ dBm}$$

So, now we can start the first part. What is the first part? Sensitivity of the radar receiver. So, from the given data frequency is given. So, lambda is 0.3 by 3 that is 0.1 meter. So, we can find P r minimum that is the sensitivity that will be P t G t square lambda square sigma by 4 pi whole cube R max the moment I put max. So, this is sensitivity.

So, these if you put the value the 10 to the power 6 into G t is 100 square into lambda is 0.1 whole square into sigma is 1 by 4 pi whole cube and this is 5 into 10 to the power in meters whole to the power 4. So, these will be minus. This is actually; write the absolute value first 8.06 into 10 to the power minus 19 watt. So, if you convert it to dBm it will be -151 dBm ok.

So, first part is the what is the receiver sensitivity we got that this is the minimum that will come. Then find the minimum number of echo pulses required to be integrated by the indicator for detection of a point target. Now detection sensitive detector sensitivity is given as -141 dBm whereas, receiver sensitivity is this, so that means, by integration we will make up this 10 dB. So, I can say that 10 dB.

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$$P_{\min} = \frac{(4\pi)^3 R_{\max}^4}{(4\pi)^3 \times (5 \times 10^5)^4}$$
$$= 8.06 \times 10^{-19} \text{ W}$$
$$= -151 \text{ dBm}$$

b) $10 \text{ dB} \Rightarrow 10 \text{ no. of pulses}$

c) $\text{prf} = \frac{c}{2R_{\text{unamb}}} = 300 \text{ Hz}$

$T = \frac{1}{300} \text{ sec}$

Now, SNR improvement will be required. Now to get 10 dB means I will have to have ten number of pulses because n number of pulses give me an improvement of n dB. So, 10 number of pulses, this power ratio. So, 10 so; that means, answer to part b is find the minimum number of echo pulses required to be integrated by the indicator for detection of a point target.

So, that will be 10 number of pulses because this is -151. Detector sensitivity is -141. So, who will give me an improvement of 10 in terms power received? So, that is by integrating 10 number of pulses. We know that noise power will not change by integration because noise is a random signal. So, 10 such noise powers if you add it will not change whereas, for deterministic signal that gives you if you add 10 times that will be 10 times added. So, now part c, find the minimum time required to determine the azimuth of a target.

Now, first find the prf it will come from that unambiguous range. So, you know prf is C by $2R_{\text{unambiguous}}$, unambiguous range given. So, that will be 300 Hertz. So, time what is a pri; T is 1 by prf. So, that is 1 by 300 second.

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b) $10 \text{ dB} \Rightarrow 10 \text{ no. of pulses}$
c) $\text{prf} = \frac{c}{2R_{\text{max}}} = 300 \text{ Hz}$
 $T = \frac{1}{300} \text{ sec}$

For conical scanning, in azimuth, minimum two beams points are required.
At each position of the beam $\rightarrow 10$ pulses are sent

Now, for conical scanning here it is set conical scanning. So, I can say that for conical scanning. When we discuss conical scanning; I do not in azimuth, in azimuth minimum 2 beam points are required; minimum 2 beam points because for squint beams from that 2 it can say that in which direction it should move, minimum 2 beam points are required.

Now, at each position there are integration of 10 pulses. So, at each position of the beam 10 pulses are sent. So, for two positions there will be 20 things 20 pulses.

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For conical scanning, in azimuth, minimum two beams points are required.
At each position of the beam $\rightarrow 10$ pulses are sent

min time to determine azimuth = time to send 20 pulses + one pulse on T_{int}
 $= 20 \times \frac{1}{300} + \left(\frac{1}{300} \right) \left(\frac{1}{100} \right)$
 $\approx 66.67 \text{ msec}$

So, minimum time to determine minimum time to determine azimuth that will be 20 pulses. Now each pulses pri is 1 by 300. So, this is in second plus. So, I can say time to send 20 pulses plus 1 pulse on time because first pulse on time at least should be given, so that I can say 1 by 300 by this is the reason it has been said about the duty cycle.

The pulse on time is this pri then duty cycle is 1 percent; that means, for this much time. So, this total is second. So, if you do that roughly it is 66.67 millisecond. So, to determine 1 azimuth position this will be the time required and let us do the last thing that what is the time for scanning the full search sector in azimuth.

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The image shows a whiteboard with handwritten mathematical calculations. The top part shows a calculation for time in seconds, which is then converted to milliseconds. The bottom part shows a calculation for beam width (BW) in degrees, followed by a note on the number of beam positions and pulses required, and finally the total scan time for the entire sector.

$$= 20 \times \frac{1}{300} + \left(\frac{1}{300} \right) \text{ sec}$$

$$\approx 66.67 \text{ msec}$$

d) $BW = \frac{51 \times 0.1}{5.1} = 1^\circ$

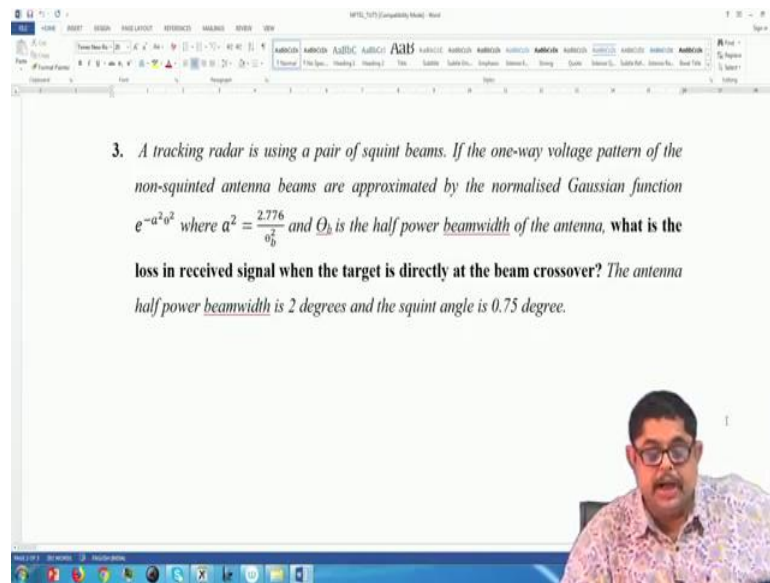
30 beam positions are required.
 $30 \times 10 = 300$ pulses are required

Scan time for entire sector = 1 sec

Now, for that some antenna knowledge is required; beam width of the antenna in the azimuthal plane is given by this. So, beam width is 51 into 0.1 by 5.1 in degrees. So, this comes to be 1 degree. So, 1 degree 30 degrees to be such; so, 30 beam positions are required. Now each in each beam position you need to send 20 pulses so; that means, 30 into 20, each beam position 10 pulses we send sorry 30 into 10 that is 300 pulses are required.

So, scan time for entire sector is 300 pulses 1 pulse is 1 by 300, so 1 second. So, in 1 second you can scan the; you see logical if you think then these type of problems can be answered. Let us see the third problem.

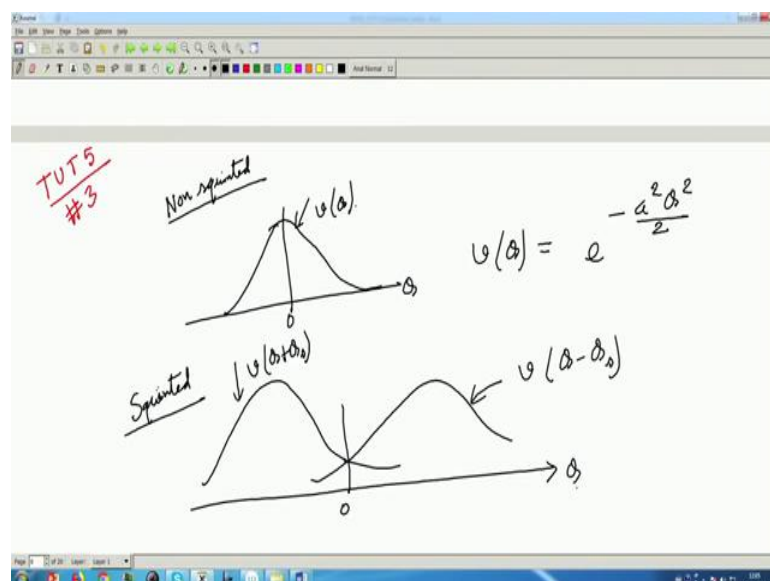
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3. A tracking radar is using a pair of squint beams. If the one-way voltage pattern of the non-squinted antenna beams are approximated by the normalised Gaussian function $e^{-a^2\theta^2}$ where $a^2 = \frac{2.776}{\theta_b^2}$ and θ_b is the half power beamwidth of the antenna, what is the loss in received signal when the target is directly at the beam crossover? The antenna half power beamwidth is 2 degrees and the squint angle is 0.75 degree.

This is an important problem a tracking radar is using a pair of squint beams; obviously, if the one way voltage pattern of the non squinted antenna beams are approximated by the normalised Gaussian function $e^{-a^2\theta^2}$ where a^2 is a constant and given by it is related to the half power beam width. So, what is the loss in received signal when the target is directly at the beam crossover. We have said that since we are having a squint beam, so when the target is at the beam cross over there is an attenuation of the signal, so that we will calculate now. The antenna half power beam width is 2 degrees and the squint angle is 0.75 degree. So, let us see this.

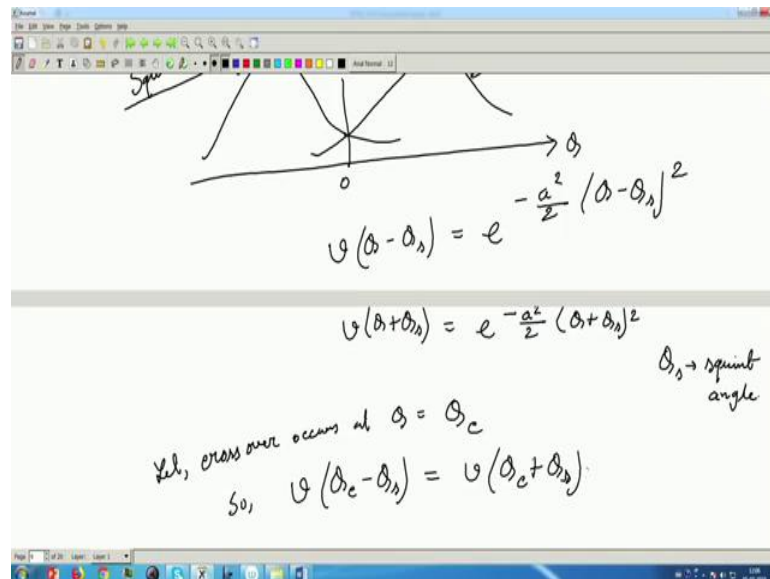
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So, let us solve this problem tutorial 5 problem 3. So, first let me draw the non-squinted pattern because Gaussians pattern. So, we will have the pattern like this. This is versus theta and this is our v theta this is 0. So, it is given that what is the form of v theta e to the power minus a square theta square by 2.

Now, when it is squinted pattern; let me draw that that one pattern will be something like this, the other pattern will be something like this and this one will be called v let us say theta minus theta squint this is v theta plus theta s and this is the crossover point. So, again this is theta etcetera.

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So, I can easily write the expression for v theta minus theta s that will be e to the power minus a square by 2 theta minus theta s square and v theta plus theta s is e to the power minus a square by 2 theta plus theta s square where theta s, what is theta s; theta s is the squint angle, as simple as that. Now let crossover occurs at theta is equal to theta c at such angle. Now what is crossover? We know that two patterns; two squinted patterns they are equal. So, we can say v theta c minus theta s will be equal to v theta c plus theta s.

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$$V(\theta_c + \theta_s) = e^{-\frac{a^2}{2}(\theta_c + \theta_s)^2}$$

$\theta_s \rightarrow \text{squint angle}$

Let, crossover occurs at $\theta = \theta_c$.

So, $V(\theta_c - \theta_s) = V(\theta_c + \theta_s)$

$$e^{-\frac{a^2}{2}(\theta_c - \theta_s)^2} = e^{-\frac{a^2}{2}(\theta_c + \theta_s)^2}$$

$$\theta_c = 0$$

So, if you put those expressions; that means, e to the power minus a square by 2 theta c minus theta s whole square is equal to e to the power minus a square by 2 theta c plus theta s whole square.

So, solving this we can say that theta c will be 0; that means, crossover is at theta is equal to theta c is equal to 0. So, what is the loss in signal power?

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Loss in signal power = $\frac{1}{|V(\theta_c - \theta_s)|^2}$

$$= \frac{1}{|V(-\theta_s)|^2}$$

$$= \frac{1}{\left[e^{-\frac{a^2}{2}(0 - \theta_s)^2} \right]^2}$$

So, loss directly I could have said initially from symmetry, but I have shown analytically that this is the way you should do analytically always; loss in signal power is definitely

actual patterns are normalized. These are Gaussian normalised pattern. So, one is the maximum, but actually I am getting v of minus theta c minus theta s. So, I can, sorry let me write because later you will understand v theta c minus theta s magnitude and that is 1 by v of theta c is 0. So minus theta s magnitude our signal power; so, power means this is to be squared so that means, 1 by e to the power minus a square by 2 0 minus theta s square whatever will come.

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The image shows a software window with a white background and a toolbar at the top. The window contains handwritten mathematical equations in black ink. The equations are as follows:

$$= \frac{1}{\left[e^{-\frac{a^2}{2} (0 - \theta_s)^2} \right]}$$

$$= e^{-a^2 (\theta_s)^2}$$

$$= 0.6767 = -1.7 \text{ dB}$$

So, that will be e to the power minus a square theta s square and so, if you find this; this value will be 0.6768. So, that is -1.7 dB. So, loss in signal power because power pattern was given.

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$$= 0.6767 = -1.7 \text{ dB.}$$
$$\text{Loss in signal voltage} = \frac{1}{|G(\theta_s)|} = e^{-\frac{a^2 \theta_s^2}{2}}$$
$$= 0.8227 = -1.7 \text{ dB}$$

So, loss in signal voltage that will be 1 by v minus θ s and so, it will be e to the power minus a square by 2θ s square.

So, if you put this value it will be again 0.8227 and that is also -1.7 dB. So, both power and voltage they will be having this loss. So, this is a good thing that how much loss you are suffering for squinting the beam this but actually you are gaining in the sensitivity because if you do not have that your sensitivity will be very poor.

So, measurement will be meaningless that is why we are ready to suffer this voltage loss at crossover but that shows us that the detection is perfect with a very high sensitivity. So, it is more important to have high sensitivity than having encountering some loss ok. So, this is all about tutorials on tracking and searching radars etcetera that means, angle determination radars ok.

Thank you.