Principles And Techniques Of Modern Radar Systems
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Lecture – 39
Detection in Radar Receiver (Contd.)

Key concepts: Mathematical model for ambiguity function for a rectangular pulse, analysis of range ambiguity function for rectangular pulse

Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar Systems.

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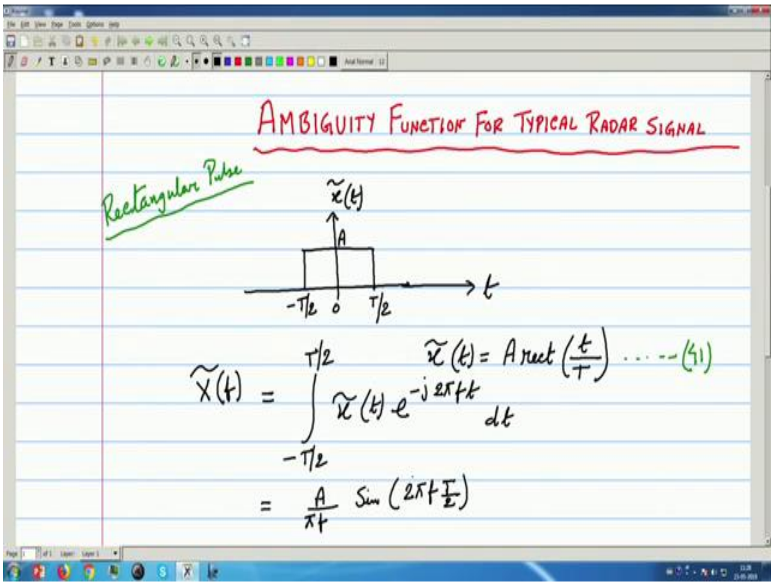

We were discussing the ambiguity function, we have seen the theoretical derivations for ambiguity function and today we will see ambiguity function for some typical radar signals. And then actually we will see that what is the concept of the delay resolution and the Doppler resolution fully; we have already introduced some of them qualitatively, but quantitatively we will find some metrics which will give us that.

So, first I start with this rectangular pulse its ambiguity function let us see rectangular pulse. Now, actually a radar sense a pulse train instead of a pulse; so actually we should have the real signal the pulse train also remember that it sends an RF signal whose amplitude is modulated as a pulse type of thing. Now the question is we have already posed the question that what is the best possible waveform; that means, whether a rectangular pulse is a good candidate from the resolution point of view.

Because match filter gave us that liberty that you can choose the signal, it does not matter as well as far as you keep the energy of the signal constant, it will be able to do the SNR maximization. So, we are now going deep into that question that what is the best possible pulse for giving best possible shape of the pulse for giving this resolution? So, in that we have gone through several lectures and we have found that ambiguity function of the signal; the width of the ambiguity function will give us the both the delay resolution and Doppler resolution.

So, we will now see that what is the shape of the ambiguity function for rectangular pulse and then we will see some one more pulse Gaussian if we instead of rectangular pulse which has a sharp rise and fall instead if we can have a Gaussian type of thing how the ambiguity function changes and from that we will come see some more interesting things.

So, first I start with the rectangular pulse and so the rectangular pulse waveform; please remember this is the transmitted signals envelop of the RF carrier. So, I am calling this x t as before and this is t. So, I am assuming that the on time of the pulse is capital T because tau is already we have taken as delay parameter. So, generally we say the on time as tau, but here we cannot do that because we have already introduced the tau for delay. So, let me take capital T as the; so that means, if this is 0, this will be T by 2 this will be minus T by 2 and let us take the amplitude A; so this is the waveform.

So, now first we will have to also this is the we can write in terms of the either in terms of the step function or something; you can write this thing that everyone will be able to do, but what is the or what is the Fourier transform of this? So, let me say that this is the complex envelope X t because actual X t is bandpass. So, we have already introduced the concept of complex envelope. So, this is the complex envelop X t thing and let me write first the; because equation number should be given.

So, what is x t? I can write that it is A into rectangular t by capital T. So, this I should last we have seen equation 40. So, today let me give it equation 41.

$$\tilde{x}(t) = A\, rect\left(\frac{t}{T}\right)$$

So, now, also let us see what is the Fourier transform of this pulse because in delay resolution that will help. So, what is the Fourier transform? So, that I think all of you will be able to write or otherwise we can do it. So, except we know it will be minus T by 2 to plus T by 2; then x t; e to the power minus j 2 pi f t d t. So, this will be able to do that this will be can I write it as A by pi f; sin of 2 pi f T by 2.

$$\tilde{X}(f) = \int_{-T/2}^{T/2} \tilde{x}(t)\, e^{-j 2\pi f t}\, dt$$
$$= \frac{A}{\pi f} \sin\left(2\pi f \frac{T}{2}\right)$$

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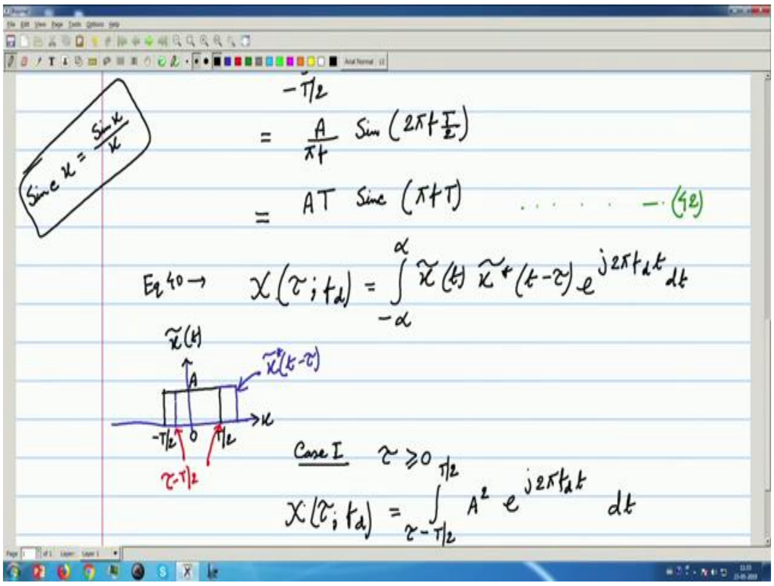

So this; I want to write it as a sinc form all of us know that the thing is sinc. So, my definition of sinc is like this there are various definitions all also sinc of x; I write it as sin of x by x. In some books you will see that it is written as sin pi x by x like that, but this is our definition. So from this, I can easily write it as A; so this one will be pi f T and if I make it sinc, then here it will be A into T into sinc pi f T; so this is well known. So, let me give this as equation number 42.

$$\tilde{X}(f) = AT \ \text{Sinc}\,(\pi f T)$$

Then let us see from; I am going to find ambiguity function. So, that is our equation 40; so from equation 40; I can write that the ambiguity function tau f d that is minus infinity to infinity; x tilde t, then x star t minus tau e to the power j 2 pi f d t; d t. So, this then again I can evaluate this. So, you see this is something like correlation; so this tau now needs to be said because I have a finite pulse on time t; so we need to draw a picture.

So, let me choose some colours that let us say that these tau actually that can be broken into two parts; that this tau can be greater positive or tau can be negative also and tau can be positive. So, if tau is positive then what is the picture my original signal is this; this is my minus T by 2, this is T by 2, this is in time domain x, this is my x t. Now what will be my delay? So, t is tau is positive so; that means, my signal will be something like this.

So, this is x; t minus tau and actually conjugate will also be same because it is a real signal; so I can say this is the thing. Now, then you can see that they exist; so from; so the product that will then exist from this point and it will extend up to this point. So, if I take these; then I can write that the same thing X. The ambiguity function is extending from minus; what is this point? Let me say that in red or first let me put that; this is my 0.

So, what is this point? So, this will be; obviously, tau minus T by 2 and this point is T by 2, so no problem. So that means, the lower limit will be tau minus T by 2, the upper limit is T by 2 and the product is this part is A. So, both of them are having magnitude so; that means, it will be A square, then e to the power j 2 pi f d t; dt.

<u>Case I</u> $\quad \tau \geq 0$

$$\chi(\tau; f_d) = \int_{\tau - T/2}^{T/2} A^2 \, e^{j 2\pi f_d t} \, dt$$

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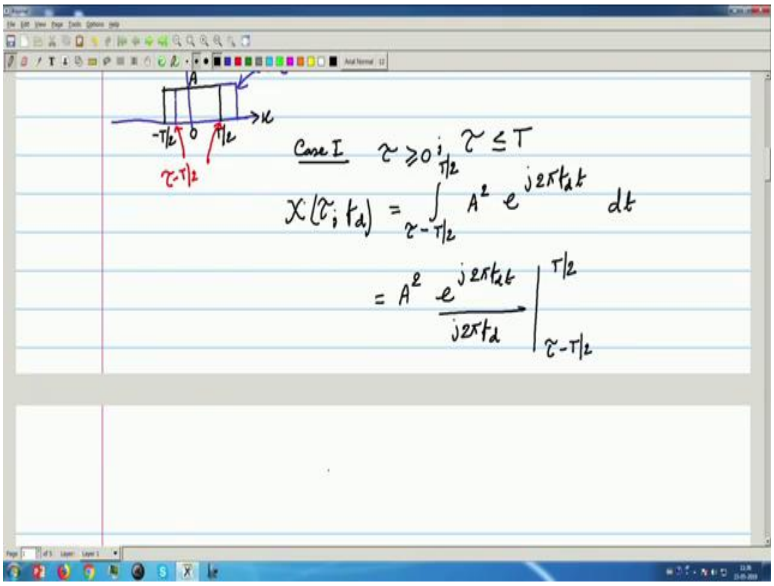

So, this you can evaluate that we know e to the power j this thing. So, after integration with respect to t, this will be that I can write A square e to the power j 2 pi f d t by j 2 pi f d and as a I can put the limits; tau minus T by 2 to T by 2.

$$= A^2 \left. \frac{e^{j 2\pi f_d t}}{j 2\pi f_d} \right|_{\tau - T/2}^{T/2}$$

And also I should say that this tau; please see this picture that this tau that if tau becomes greater than capital T that time this lower end will come here.

And so, it is tau is greater than T; that means, T will be here and the x star T minus tau; its lower end that will be just falling on this T by 2 point; so after that the product will be 0. So, I should also say that this tau is greater than 0 and also this thing is true for tau that less than equal to T; so within this limit this thing is coming. So, this you know that I can manipulate that because I can write it in terms of sin function.

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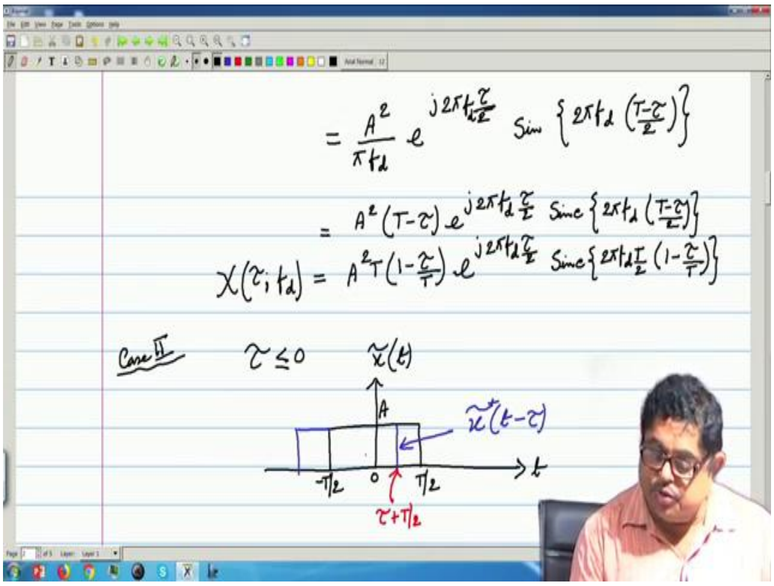

So, if you do that; I am jumping those steps because there is nothing new there. So, we will see that it will come as pi f d; e to the power j 2 pi f d T by 2 sorry tau by 2; that means, you would generally were for this manipulation, you will have to take this out and then you get a beautiful sine type of thing. So, that is sin 2 pi; f d; T minus tau by 2. Now this again you see can be written in sinc form; so you do that manipulation and you can immediately write it as A square by T minus tau; e to the power j 2 pi f d; f, here that f d is missing f d; tau by 2, then sinc 2 pi f d; T by 2 into 2 pi f d; 2 pi f d; T minus tau by 2. So, this thing finally, I am writing it in a somewhat more better form because I know the answer, so I am writing it as; I can always write this thing as; I can take this T out and this is 1 minus tau by T, then e to the power j 2 pi f d tau by 2; sinc of 2 pi f d T by 2; 1 minus tau by T.

$$= \frac{A^2}{\pi f_d} e^{j 2\pi f_d \frac{\tau}{2}} \operatorname{Sin}\left\{2\pi f_d \left(\frac{T-\tau}{2}\right)\right\}$$

$$= A^2 (T-\tau) e^{j 2\pi f_d \frac{\tau}{2}} \operatorname{Sinc}\left\{2\pi f_d \left(\frac{T-\tau}{2}\right)\right\}$$

$$\chi(\tau; f_d) = A^2 T\left(1-\frac{\tau}{T}\right) e^{j 2\pi f_d \frac{\tau}{2}} \operatorname{Sinc}\left\{2\pi f_d \frac{T}{2}\left(1-\frac{\tau}{T}\right)\right\}$$

You see that basically what I am doing? I am introducing that duty factor because that is the specification for pulses. So, instead of writing tau I am writing tau by T that is why this manipulation. Now, this is a for tau greater than tau positive; that means, delay is positive; now I can have a case II also that case II where tau is negative; tau is negative. So, again I can draw that because the limits needs to be found out; actually any correlation time, you need to have these that tau. You should actually tau can take any value from minus infinity to plus infinity, but since the given signal is finite; so you need to break that.

So, I am again drawing the; this is your x t, this is t, this is A, this is minus T by 2, this is plus T by 2. And let us take what happens if tau is negative, then you know that it will extend somewhere here and it may come here. So, this is the new pulse; so I can write that this is x star t minus tau. So, now what will be the limit here? So, this portion you can find out that; so what is tau?.

So, the midpoint of this is tau; that means, this pulse that blue pulse, the midpoint is you can say tau. So, this thing we need to find out actually; this point this point will be what? Tau plus T by 2; this I should have written it in red, that this point is tau plus T by 2 with respect to the 0 here and so now the; our integral will be having some nonzero values from minus T by 2 to tau plus T by 2.

And if tau becomes the negative but if it is more negative than minus T, then this blue pulse will be such that it is just having that it is this part that the forward path you can say; forward edge that will just fall here, so after that there will not be any their correlation function will be 0.

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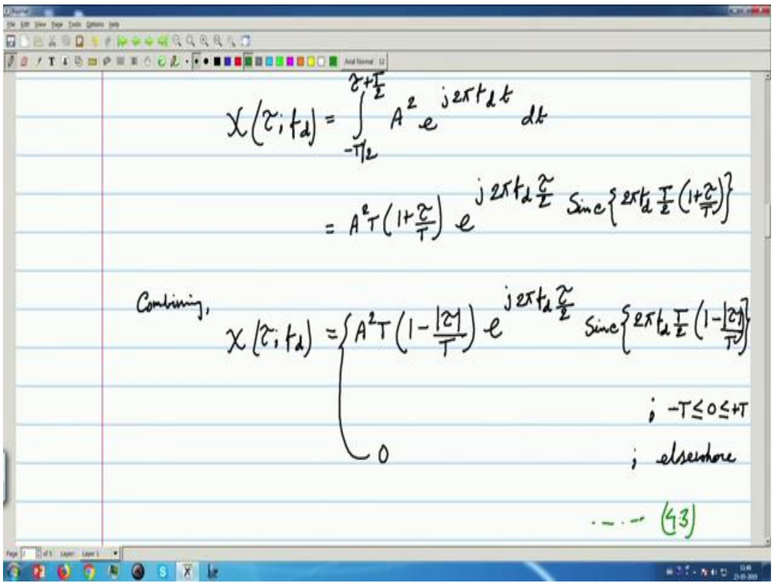
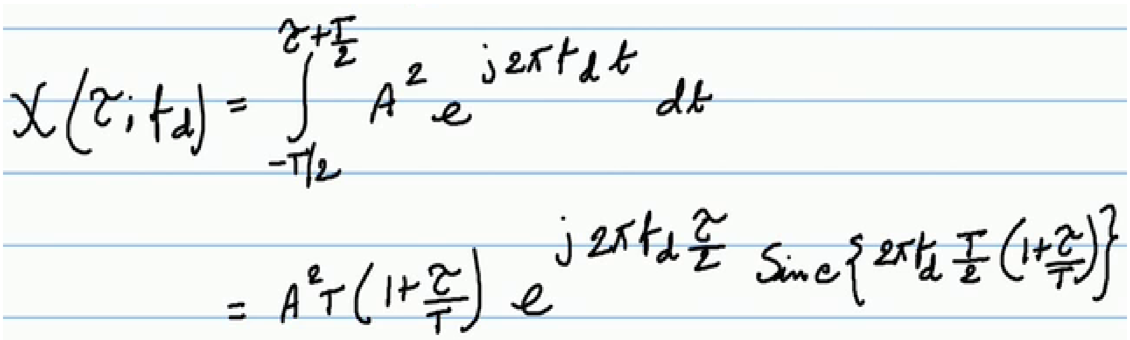


So, now I can with this; we can come back and write the ambiguity function. So, basically these limits, then will be minus T by 2 to tau plus T by 2, then A square and e to the power j 2 pi f d t; d t. So, this as before you can again integrate this and finally, you can write it as; I am writing at one step the final thing again A square T; 1 plus tau by T, e to the power j 2 pi f t tau by 2; sinc of 2 pi f d; T by 2, 1 plus tau by T.



So combining I can write the ambiguity function for the rectangular pulse and that is A square T; in one case 1 minus T by tau, this step and 1 minus was for tau positive, 1 plus is for tau negative. So; obviously, this I can write in terms of mod that this is 1 minus mod of tau by T, then e to the power j 2 pi f d; tau by 2 sinc, 2 pi f d T by 2. Again here 1 minus tau by T. So this is an quite interesting thing and we can give it that.

So, this thing I can also write that this is for what? What is the range of tau for which this is true? That this is I can write for minus T greater than equal to 0, greater than plus T and I can say that is 0 elsewhere.

$$\chi(\tau; f_d) = \begin{cases} A^2 T\left(1-\frac{|\tau|}{T}\right) e^{j2\pi f_d \frac{\tau}{2}} \operatorname{Sinc}\left\{2\pi f_d \frac{T}{2}\left(1-\frac{|\tau|}{T}\right)\right\} & ; -T \leq 0 \leq +T \\ 0 & ; \text{elsewhere} \end{cases}$$

So, this is the definition of the ambiguity function of a rectangular pulse and this is an important thing because I will now discuss on this.

So, this equation I am saying equation 43 ok. So, this is the two dimensional ambiguity function; it can be easily plotted for various values of tau and various values of f d; you see a function of tau and f d. So, it can be plotted as a 3 D plot where the ambiguity function magnitude what says tau and f d can be plotted, but more instructively; you see we can easily find what is the range ambiguity function?

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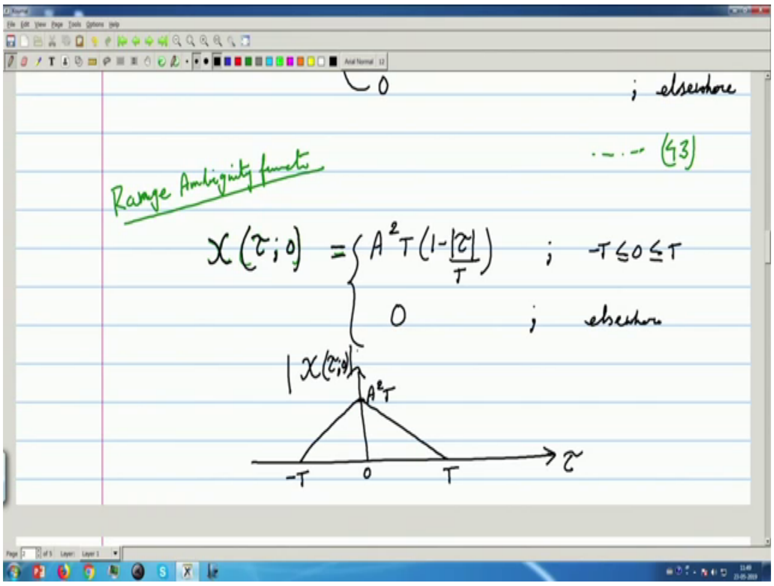

So, this is the combined ambiguity function, but I can write range ambiguity function from here; so, range ambiguity function is nothing, but you put f d 0. So, in this equation if you put f d 0; what you get? You see this thing will go because this will become 1, then in sinc case it will be 0. So, sinc of 0 is 1; so it is basically nothing, but sorry this is A square T; 1 minus tau by T; minus T to T 0 elsewhere.

$$\chi(\tau;0) = \begin{cases} A^2 T\left(1-\frac{|\tau|}{T}\right) & ; \quad -T \leq 0 \leq T \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

So, this is the range ambiguity function and if I draw it; so it is now can be drawn as a 2 D plot because it is a basically one dimensional function.

So, I can say that this will be this is a well-known function; all of us for match filters, generally this is shown and what is this value; if this is 0; this is T, this is minus T, this is A square T; so it is a triangular function. So, basically we know that match filter output for a rectangular pulse comes as triangular, but here we are calling that the magnitude; obviously, this is the sorry; this is the magnitude part; so you should mention that the magnitude that triangular function well-known thing.

Now what is that; so here you see that range ambiguity function. So, here if I say that from this peak value; it is not very peaky as you are seeing that its roll off is not very sharp; it is coming down. And so I can say that the width will be something like because we are saying that, what is the width delay width of the ambiguity function in range dimension? It will be 2 T; so, twice the pulse on time.

So, if 1 microsecond I have then that; so my resolution will be something related to that; that tau you need to convert etcetera. Similarly, let us see the frequency correlation function; so, frequency correlation function; frequency correlation function.

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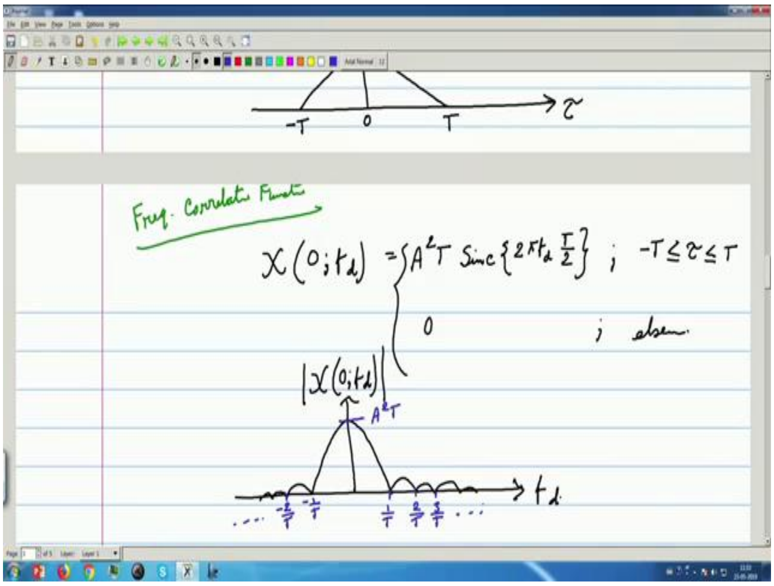

So, for that in the ambiguity function; I will have to put the delay part 0 and it will be a function of f d only and that from the expression you can see that what is that. So, this thing you tau is 0. So, A square T into then again here you put tau 0, so this is again 1. And here you put tau 0, so it is a sinc function and if I take the mod of that; this mod will this thing will go. So, it is A square T into the sinc type of thing, so I can write that expression.

So, that will be A square T; then sinc of 2 pi f d T by 2 for minus T tau less than T and 0 elsewhere ok.

$$\chi(0;f_d) = \begin{cases} A^2T\ \mathrm{Sinc}\left\{2\pi f_d \frac{T}{2}\right\} & ; \ -T \le \tau \le T \\ 0 & ; \ \text{elsew.} \end{cases}$$

So, we can draw this thing that frequency correlation function verses f d. So, this plot will be a sinc plot and I am taking the magnitude. So, sinc what there is no negative part. So, it will have side lobes; diminishing side lobes in the cases and what is this value? This value is A square T and also let us mark this first null.

So, you can find that these the fast null will come at 1 by T, the next one is 2 by T, 3 by T etcetera and this is minus 1 by T minus 2 by T etcetera. So, this now actually we have got this. So, we will discuss the implications of these from the next thing because we actually wanted that ideally this should have been a delta function type of thing, but it is not; obviously, it is not and we will discuss more on these in the next class.

Thank you.