Principles And Techniques Of Modern Radar Systems

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Lecture – 42
Detection in Radar Receiver
(Contd.)

Key Concepts: Introduction to the Doppler resolution constant, analytical expression for the Doppler resolution in both the time and frequency domains, product of the delay resolution time constant and the Doppler resolution constant.

Welcome NPTEL course on Principles and Techniques of Modern Radar Systems. In the last class we have seen the Delay Resolution Constant; today we will see the Doppler Resolution Constant.

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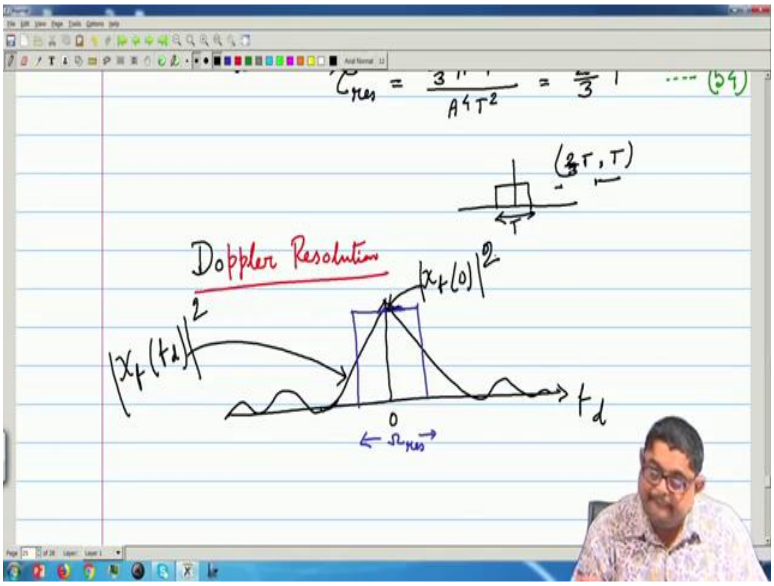

Now, for rectangular pulse the delay resolution constant came out to be two-third of T. Let us call that, it will require a number I think it is previous was 53. So, this should be 54, this should be named as 54. So, now we will come to the Doppler resolution concept. So, again the similar concept we know that, the ideally again I can say the x f f d that should be peaky or impulse about f d is equal to 0 and in all other places it should go to 0, but it does not happen.

So, again in general it will have this shape some arbitrary shape and again to put that equivalence, we will put a rectangular function here with the peak and the energy of the two will be constant. Energy is a fundamental parameter, so energy of these two graphs or area under these two graphs are same, and this difference is called the Doppler resolution constant. So, this peak we know, this is this peak is 0 and this function is this. So, again this is x f f d sorry, this is square. So, this is x f 0 square ok, square.

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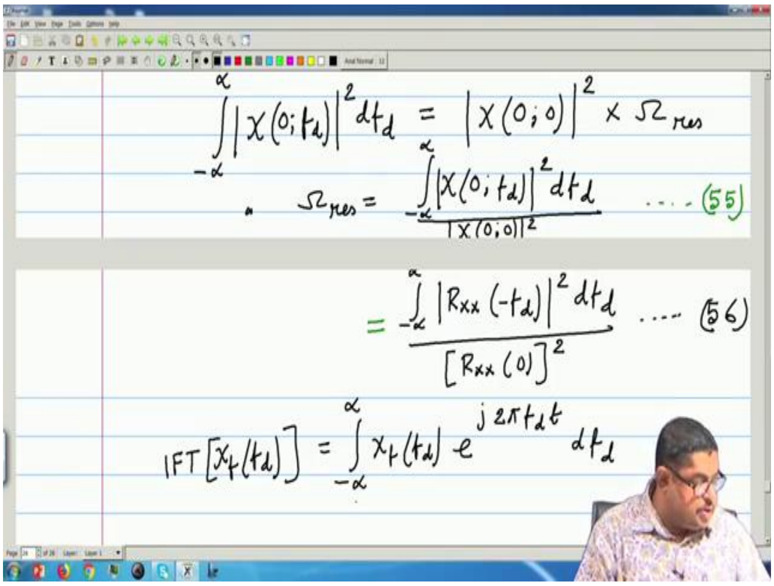

So, we can say that the minus infinity to infinity.

$$\int_{-\alpha}^{\alpha} |\chi(0;f_d)|^2 df_d = |\chi(0;0)|^2 \times \Omega_{res}$$

$$\Omega_{res} = \frac{\int_{-\alpha}^{\alpha} |\chi(0;f_d)|^2 df_d}{|\chi(0;0)|^2}$$

So, this again can be. So, first let me give it a number, I think earlier I have given 54, so this is 55. And in terms of, because ambiguity function may not be always known to other people. So, we should write it in terms of autocorrelation that minus infinity to infinity R x x and remember that in case of Doppler it is minus f d that we have derived earlier. So, this we can call equation 56.

$$= \frac{\int_{-\infty}^{\infty} |R_{xx}(-t_d)|^2 \, dt_d}{[R_{xx}(0)]^2}$$

So, now the question is again instead of doing this, if we can do it in the time domain that is an alternative one; if you know these expressions from this Doppler ambiguity function, then it is ok; otherwise again we will do some mathematical manipulation to have an alternate expression in terms of signal energy and also time domain things. So, for that I am doing some mathematical thing that we know what is the inverse Fourier transform of this x f f d; x f f d is nothing but this function, if we do not write it as a two dimensional function I generally write it like this. So, what is this?

This is minus infinity to infinity x f f d, e to the power j 2 pi f d t d f, please remember here running variable is Doppler. So, that is why this is f d and running variable is f d.

$$IFT[\chi_f(f_d)] = \int_{-\infty}^{\infty} \chi_f(f_d) \, e^{j 2\pi f_d t} \, df_d$$

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$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{X}^*(f) \, \tilde{X}^*(f - f_d) \, df \, e^{j 2\pi f_d t} \, df_d$$

$$= \int_{-\infty}^{\infty} \tilde{X}^*(f) \int_{-\infty}^{\infty} \tilde{X}(f - f_d) \, e^{j 2\pi f_d t} \, df_d \, df$$

Put $f - f_d = z$

$$= \int_{-\infty}^{\infty} \tilde{X}^*(f) \int_{-\infty}^{\infty} \tilde{X}(z) \, e^{j 2\pi (f - z) t} \, (-dz) \, df$$

$$= \int_{-\infty}^{\infty} \tilde{X}^*(f) \, e^{j 2\pi f t} \int_{-\infty}^{\infty} \tilde{X}(z) \, e^{-j 2\pi z t} \, dz \, df$$

So, again we have earlier defined this function. So, we can put that definition. So, in terms of the spectrum, so again I can take because f d is the running variable. So, X f I can take out.

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{X}^*(f)\,\tilde{X}^*(f-f_d)\,df\; e^{j2\pi f_d t}\,df_d$$

$$= \int_{-\infty}^{\infty} \tilde{X}^*(f) \int_{-\infty}^{\infty} \tilde{X}^*(f-f_d)\, e^{j2\pi f_d t}\,df_d\,df$$

Now we have to put a change of variable. So, d f has come in; now change of variable put this f minus f d is equal to let us say z.

So, if you do that, you will get something like this, in the second integral put here do not put it here unnecessary. So, we will get that minus infinity to infinity oh this is corrected, this is X z e to the power j 2 pi f minus z t then minus d z d f. This is as it is and from here also this f term can be taken out j 2 pi f t. And so, we are left with here this X z e to the power minus j 2 pi z t d z then d f.

Put $f - f_d = z$

$$= \int_{-\infty}^{\infty} \tilde{X}^*(f) \int_{-\infty}^{\infty} \tilde{X}(z)\, e^{j2\pi(f-z)t}\,(-dz)\,df$$

$$= \int_{-\infty}^{\infty} \tilde{X}^*(f)\, e^{j2\pi f t} \int_{-\infty}^{\infty} \tilde{X}(z)\, e^{-j2\pi z t}\,dz\,df$$

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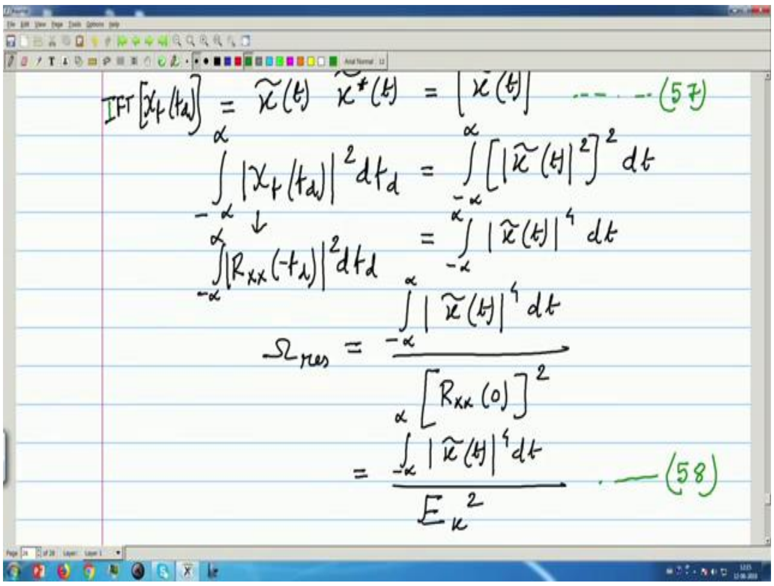

Then again you can have a substitution here, because it is a frequency domain term. So, you can one more substitution is required, put minus z is equal to u. And so, you will get the again in this one only, do not do it here unnecessarily that would not help anything. So, here it will be x minus u e to the power plus j, so it is we can write it as this whole part we can write as, x of t let us see this is the basic definition and then there will be d f.

$$= \int_{-\infty}^{\infty} \tilde{X}^*(f) e^{j2\pi f t} \, \tilde{x}(t) \, df$$

This is simple because spectrum is symmetric, so x minus z you can again put x plus z because all for real signals all spectrum is symmetric. So, you should write here that symmetric spectrum for real signal and radar signal is always a real signal. So this x t I can take out and what is this, this is what is left is the inverse transform of. So, this is nothing, but as simple as that.

$$= \tilde{x}(t) \, \tilde{x}^*(t) = |\tilde{x}(t)|^2$$

So, now, we have got an important relation and these will now put to, so I think we should give the numbers where are the last 56.

So, I can give it a number this is 57 what is it, this is the IFT of x f f d that is this.

$$\mathrm{IFT}[\chi_f(f_d)] = \tilde{x}(t) \, \tilde{x}^*(t) = |\tilde{x}(t)|^2$$

So, we can say that Fourier transform of this x t square will be these. So, now, coming back to our a thing that, what is minus infinity to infinity, because this is in our Doppler resolution constant numerator, so that we can say that this is in time domain it is nothing, but.

$$\int_{-\infty}^{\infty} |\chi_f(f_d)|^2 \, df_d = \int_{-\infty}^{\infty} \left[|\tilde{x}(t)|^2 \right]^2 dt$$

And also, now this also can be written in terms of cross correlation as autocorrelation that R x x minus f d whole square d f d.

$$\int_{-\infty}^{\infty} |R_{xx}(-f_d)|^2 \, df_d = \int_{-\infty}^{\infty} |\tilde{x}(t)|^4 \, dt$$

So, we can easily write that the Doppler resolution constant is nothing, but minus infinity to infinity by we know R x x 0 this is e x. So, it is simply the E x square.

$$\Omega_{res} = \frac{\int_{-\infty}^{\infty} |\tilde{x}(t)|^4 \, dt}{[R_{xx}(0)]^2} = \frac{\int_{-\infty}^{\infty} |\tilde{x}(t)|^4 \, dt}{E_x^2}$$

So, you see in both the resolution constant we have the E x square, only in one case it is x f whole to the power 4, in this case is x t whole to the power 4. So, actually this I can give it a number, what is this number yeah 57. So, this will be 58, equation number 58. So, we can do it for single rectangular pulse.

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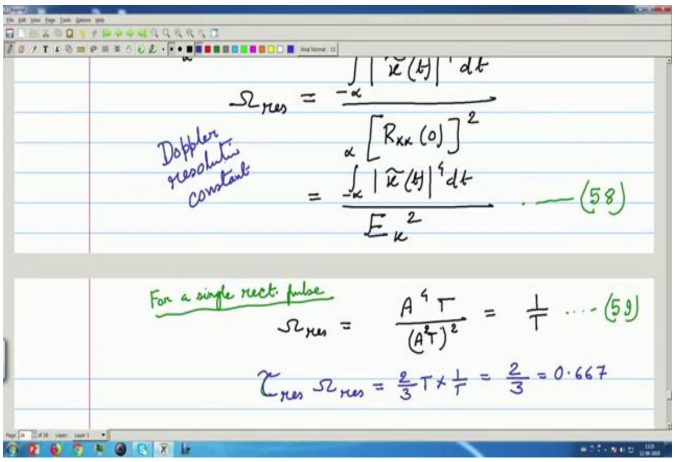

So, for a single rectangular pulse we can find this that, what will be this x t, we know the value of x t it is a rectangular function. So, these, so I can say this will be A to the power 4 T and the energy of the signal is A square T whole square. So, this will be 1 by T.

$$\Omega_{res} = \frac{A^4 T}{(A^2 T)^2} = \frac{1}{T}$$

So, we can give it a name this is 59. So, single rectangular pulse, the Doppler resolution constant is 1 by T and the delay resolution constant we have seen it to be two third T.

So now, what is the product of these two? So, it is instructive now to make this product that, this product comes out to be two third T into 1 by T. So, it is two third; that means, 0.667 ok.

$$\tau_{res}\,\Omega_{res} = \frac{2}{3}T \times \frac{1}{T} = \frac{2}{3} = 0.667$$

So, you see that the effect of all this is resolution in delay and resolution in Doppler their product is constant. So, if you want to have a fine resolution in one domain, the other will always be suffering. So, this is a drawback; that means, because if you want to make one smaller the other will increase. So, that is a very bad thing, because we want measurement and we want both of these measurement to be precise.

Actually we have seen earlier also that in ambiguity function if we have the curve that, in range it is giving very good resolution; but we are getting a very poor Doppler resolution. So, if we use a single rectangular pulse this is the case. In next classes we will see more realistic cases that a number of a pulse train; but still we will see that we cannot overcome this, then we will finally, see a technique called linear frequency modulation or chirp where we can overcome this, because these things should not be a constant. If this thing be a constant then we are at a trouble; but if we have this at a designer's hand, then we can have both these resolutions improved to whatever extent the physical hardware permits, so that we will see in the next class.

So, today what we have seen, we have seen the two important concepts in resolution; one is the delay resolution constant, and another is the Doppler resolution constant. This is Doppler resolution constant. So, if we know the expression for any transmitted wave form, we can find that these two things and these actually determines the resolution of the radar detection

Thank you.