

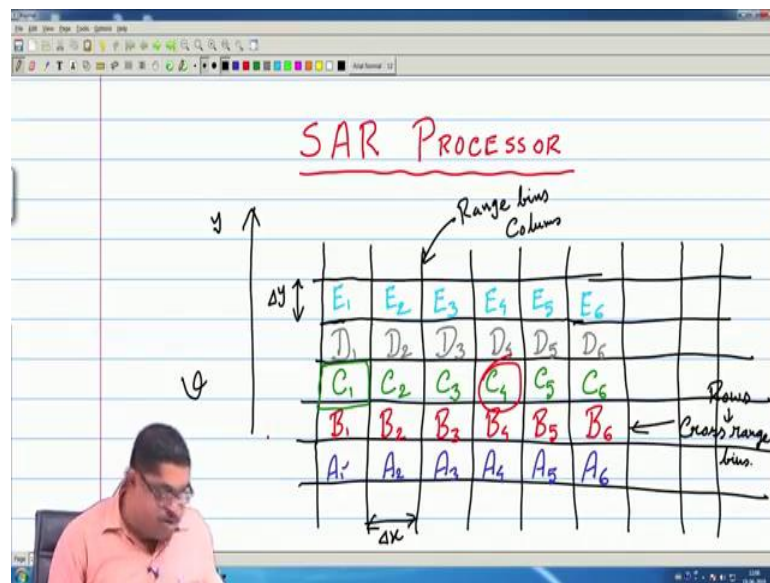
**Principles and Techniques of Modern Radar Systems**  
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**Lecture - 51**  
**SAR Processor**

**Key Concepts:** Description of SAR imaging processor, simplified block diagram of a SAR processor with tap-delay line, range migration in SAR processing

Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar System. We were discussing synthetic aperture radar processing. So, today we will see the SAR Processor; what is architecture of a SAR processor?

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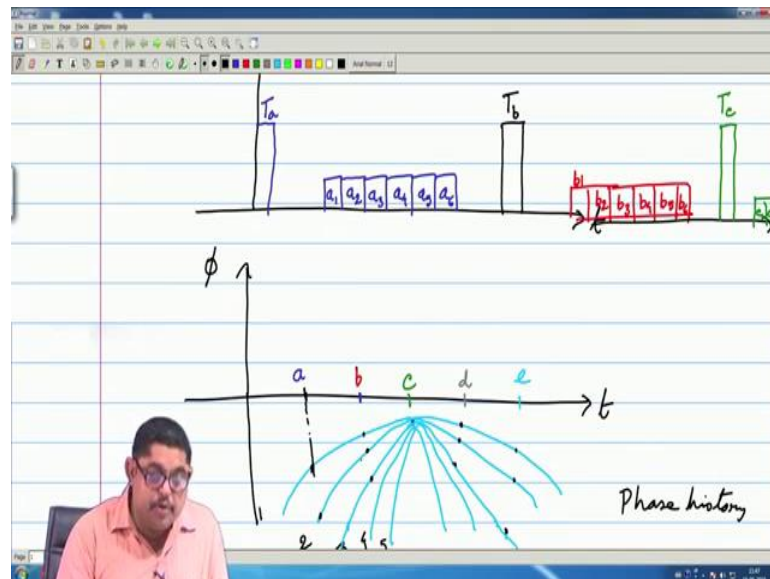
So, actually the signal processing ultimately, this synthetic aperture or pulse compression all are techniques for imaging radar. We are actually discussing imaging radar I have not said that before. But when we have fine resolution image fine resolution is achieved then actually we get image. So, let us consider that this part that a portion of the ground, which is marked like this.

So, we are imaging it. So, basically you see that we can divide that at one go from one of these bin. The signal is getting returned that we will discuss. And actually ultimately this is an image of the portion of the ground. So, I can say that these are the range bins. And I

will say that these are range bins and this lines this horizontal lines they are cross range bins. That means, you see that this is a range bin the first one first column second column is another range bin.

Because there are different ranges from the radar the radar is flowing in y direction with a velocity  $v$  and what is a cross range bins? Suppose the rows they are basically cross range. So, I can say rows are cross range bins and columns are the down range bins ok. So, this is the picture of a. So, actually how I got this thing for that we will have to see what the radar is doing the.

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Let us say the time diagram. So, radar at let us say this point is sending a transmitted pulse  $T_a$ . I am calling it  $T_a$  that a subscript. So, if I now see these in response to this from different range bins, I can consider that one this bin the; that means, one this rectangular portion that is an object. So, for me this is the resolution that the whole thing in this range they will give a return. Let me call that return as  $A_1$ ; similarly from the next range bin I have got the return  $A_2$ .

The radar is just looking straight at this row, the bottom row of these thing. So, let me come back what we are getting. So, I can say that with this I am getting this is green color. So, let me I write in blue color. So, after sometime I will start getting the returns I am drawing it like them. I have let us consider 6. So, 6 range means. So, I will get 6

returns 1, 2, 3, 4, 5, 6. Let me call them small a 1, a 2, a 3, a 4, a 5, a 6. So, in the SAR processor, I have this one by one sequentially they are coming in time.

So, I am getting that and storing it. After I have got these then the next pulse again is transmitted. So, let us say that at this point I am sending another transmitted pulse let me call it b T b. So, what I will get in return that after sometime I will get. Now the radar has moved before transmitting these b pulse, it is in front of the row given by b.

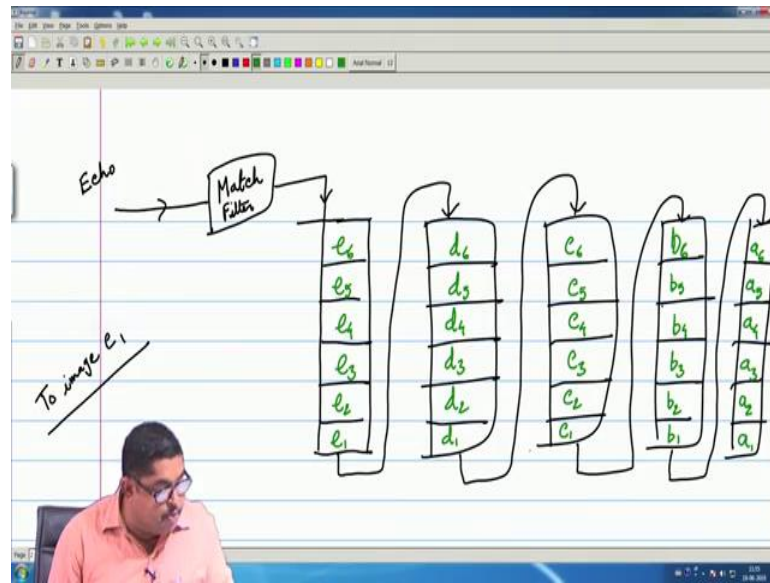
So, I will get the 6 such returns, let me call them b 1, then b 2, b 3, b 4, b 5 and b 6 ok. Then there after sometime what is the color let me see the c 1 T c is transmitted. So, in response there will be again those three things so, c 1, c 2 etcetera. So, this is the thing ok. Also for we have previously seen in yesterday that at different down ranges you have different phase histories.

So, I can show that also that the in the memory of the processor the phase histories are there. So, phase histories are such let me go they are negative. So, I can say that for there will be. So, phase history again is with respect to time. So, this history I know and I know that. Suppose there will be a point called a for point called b then there will be a point called c let me see the color a b c d e then d then e. And the phases will be parabolic we have already seen that. So, for different ranges we know that let us say that actually we are now trying to image the point c.

So, that is why in the near c; that means, that t is equal to 0; here the all the phases are going to 0. And here if I project then I will get let me call this curve as 1, this curve as 2, this curve as 3 like that I will have 5 a 4; a 1, a 2, a 3. So, 6 such curves will be there 1, 2, 3. And what is the phase for these a 1? This is this a 1, what is the phase for point a? This is actually a 2's phase then. So, if I go here somewhere here will be a 3 phase etcetera. So, I can check these that here all are here. So, processor can easily find out from this stored graph that.

So, also let me draw more 1, 2, 3. So, this will be 4, 5 etcetera ok. So, this is the phase history. I can write this is the phase history part. So, from that while required it can find these.

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Now, let us see that the signal is coming returned. I will say that echo is coming there is a match filter. So, match filter is detecting and from that actually. So, if I see again that timing diagram first I will get a 1 then a 2 a 3 a 4 a 5 a 6 then I will get b 1 b 2 b 3 b 4 etcetera. So, what is done in the processor thing there are delay elements. So, there are 7 several sequential delay elements.

Suppose I am trying to map these. Here if you one see you can see that how many cross range bins I have the rows are cross range. So, I have 5 rows. So, I will keep 5 such vertical blocks and from the match filter it comes. So, and there are several memory elements here. And when one signal comes echo comes, the whole thing gets a shift. So, actually after these it is shifted here, after these it is shifted here, after these it is shifted here, after these it is shifted here. Now, here I two image c 1 actually you can see that the central element of these zone that in these image.

We are trying to find out these portions c 1. So, this is the SAR focusing point we are trying to create the image of c 1 ok. So, this is that picture. So, what I will get, I will to create c 1 I will have to get all the images. So, again I am repeating the transmitted pulses are T a, T b, T c etcetera. The received signal consist of returns from 6 range bins.

They are resolved by down range processing system that is different already we have seen the down range processing previously. In case of Doppler field there can be Doppler filters or there can be various range gates. So, by that they have been resolved into 6

different range bins. Then they are match filtered to get range a point range compression reducing to a point by match filter when SNR is maximized.

So, let the  $g$   $p$   $r$  the sorry, the radar that is in line with the  $a$  row then it is transmitting  $T_a$  then it goes to  $b$  row and transmits  $T_b$ ; then it goes to  $c$  row transmit  $T_c$  etcetera and repeats the signal. Now, one thing you see that the read suppose it is transmitting  $T_b$ , it is getting the returns  $b_1, b_2$  etcetera. However, one thing we will have to keep in mind that the return at range  $b_1$  does not come from area element  $b_1$  only.

Because of the wide antenna beamwidth we are not having a laser beam we are having a microwave antenna it has a finite beamwidth. So, because of these non linear beamwidth actually any microwave antenna will have a finite beamwidth. So, neighboring area elements in the same range; that means, for  $b_1$  actually what we are getting we are calling it  $b_1$ , but actually it is the return.

Let us say that beam width is such that  $a_1$  and  $c_1$  are also covered. So, actually it is sum of this returns from this three bins; that means,  $a_1, b_1, c_1$  three are coming. When I am getting  $c_1$  actually I am getting  $d_1, c_1, b_1$ , three. If I assume that beam width is such that three adjacent cross range bins are covered. So, also contribute to the return mark  $b_1$ .

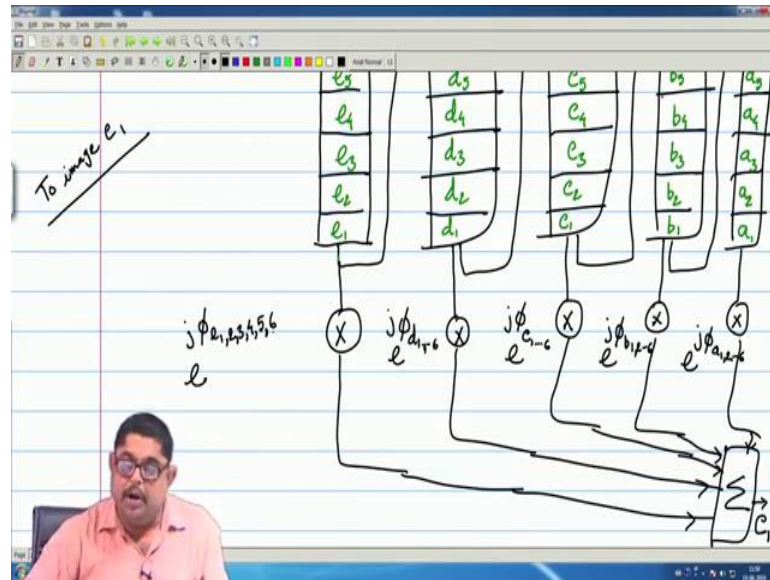
So, I am again repeating that because of the wide antenna beam width, neighboring area elements in the same range also contribute to the return mark  $b_1$  this is true for every range cell. The processors job is now it will clean up those returns from  $a_1$  and  $c_1$  and make it  $b_1$  or when it is doing for  $c_1$  it will have to clear for  $d_1$  and  $b_1$  returns etcetera that it will do by that phase things.

Now, in the processor, the match filter output enters a this thing. So, when I am trying to image  $c_1$ , the first one came is  $a_1$  then I have got  $a_2$  then I have got  $a_3$  then I have got  $a_4$  then I have got  $a_5$  then I have got  $a_6$ . Then what is here I have got  $b_1, b_2, b_3, b_4, b_5, b_6$  sorry  $b_6$ . Here I have got  $c_1, c_2, c_3, c_4, c_5, c_6$ .

Here I have got  $d_1, d_2, d_3, d_4, d_5, d_6$ . Here I have got  $e_1, e_2, e_3, e_4, e_5, e_6$ . So, when  $e_6$  as entered you see the bottom row there we have the returns from  $e_1, d_1, c_1, b_1, a_1$ . So, in the processor the match filter output enters a tap delay line this is also called transversal filter, when all the range bins with subscript one is at the last delay

then, central element of that range who is the central element here  $c_1$  that is output. So, the parallel output from the taps consist of equally delayed returns from all the pulses the first range bin.

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Now, each output also you see this thing then comes with a multiplier phase multiplier. So, this will come with a phase multiplier, this will come with a phase multiplier, this will come this will also come so, all are phase multiplier. And what is the phase multiplier? This for this is the phase history curve is used  $e$  to the power  $j \phi$ . So, this is  $e$  so, it can be  $e_1, 2, 3, 4, 5, 6$  any of these it will take appropriately ok.

Similarly, here it will take  $e$  to the power  $\phi d_1$  to 6. Here it will take  $e$  to the power sorry  $j \phi j \phi c_1$  to 6 any of this will be taken. So, actually there will be comma here similarly  $e$  to the power  $j \phi b_1, 2$  etcetera up to 6 and here  $e$  to the power  $j \phi a_1, 2$  up to 6. So, this multiplies phase multipliers will come this will come, this will come, this will come and this will come.

And then this whole thing is summed and who is the output of that this output is  $c_1$ . So, basically this is the correlation receiver type of implementation of the match filter. So, I can say again that each output is multiplier by a complex weight that compensates for the phase shift associated with that particular column you can see phase history.

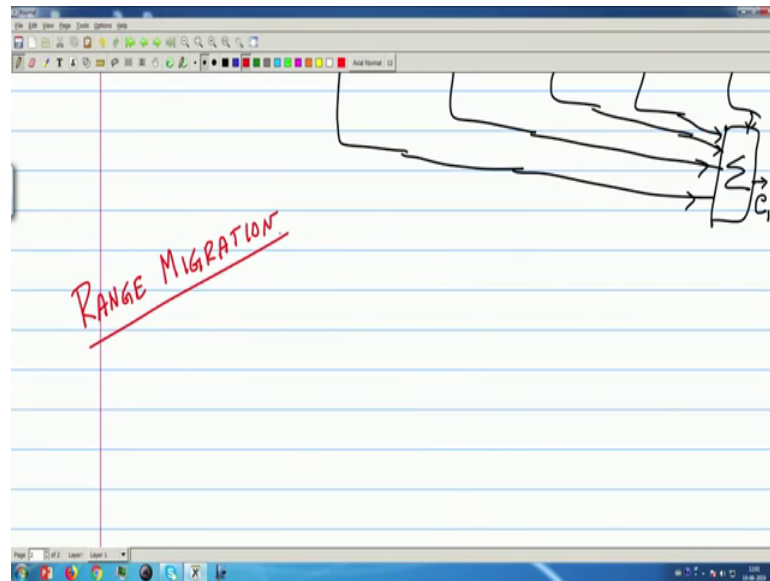
So, we will the reverse of that will compensate; that means, if I am negative I will compensate here by plus j. Actually previously you have seen that we have a space phase due to that range as something from the phase history from this phase history graph I can find out for b 1 what is the phase history. In the phase multiplier plus of that; that means, opposite of that is getting multiplied.

The a to e columns will be multiplied by the phases corresponding to the curve marked 1. Then the next bin returns are moved to last tap then the weights will be changed to those of curve 2. So, then we will get let us say c 2 then the one more thing will come. So, the that time e 3, d 3, c 3, b 3, a 3 will be there accordingly the curve three phase values will be taken by these phase multipliers. And then output will be c 3 like that c 1, c 2.

So, finally, you see that we get these things C 1, C 2, C 3, C 4, C 5, C 6. Then I will get D 1, D 2, D 3, D 4, D 5, D 6. Then E 1, E 2, E 3, E 4, E 5, E 6 by that it mapped already previously previous coming to our C 1 already B 1, B 2, B 3, B 4, B 5, B 6 have been done before that A 1, A 2, A 3, A 4, A 5, A 6 they have been done. So, the processor remembers the 6 phases in the phase history curve the phase curves are different for different ranges that we are saying that is why the 6 such a things.

So, this is actually what the processor does. Now here there is a thing that we want to discuss. So, its simple actually we will have to write a program, where this thing can be done sequentially. And with this processing you get the outputs c 1, c 2 etcetera. So, by that the whole thing gets filled up. So, initially we have the returns from the radar with this phase processing. We are getting the images that images will put to the appropriate bins.

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Now, there is a problem that is called range migration in SAR processing range migration. You understand the meaning of the term migration now why it comes? Now here we have assumed that a return from a point in the ground remains in the same range bin during the entire length of the synthetic processor.

What is that mean you see these. Here we are saying that the we are sending a pulse you see  $T_b$  we are sending we are getting  $b_1, b_2, b_3$  etcetera. So, I am saying that the return from  $b_1$  that is contained in  $b_1$  return from  $c_1$  that is contained in  $c_1$ . I said that cross range there are other due to the finite beam width others are coming. But I am the when I am getting the return from for  $b_1$  I am nothing is coming from  $b_2$  that is not there that is ok.

Now, the point is actually you see due to the movement of the radar there is a relative velocity there is a relative velocity in the whole thing. So, whole Doppler processing there is that is why there is a Doppler, Doppler processing means there is a change in range based on a range change. We have seen that the phase is coming due to the change of range.

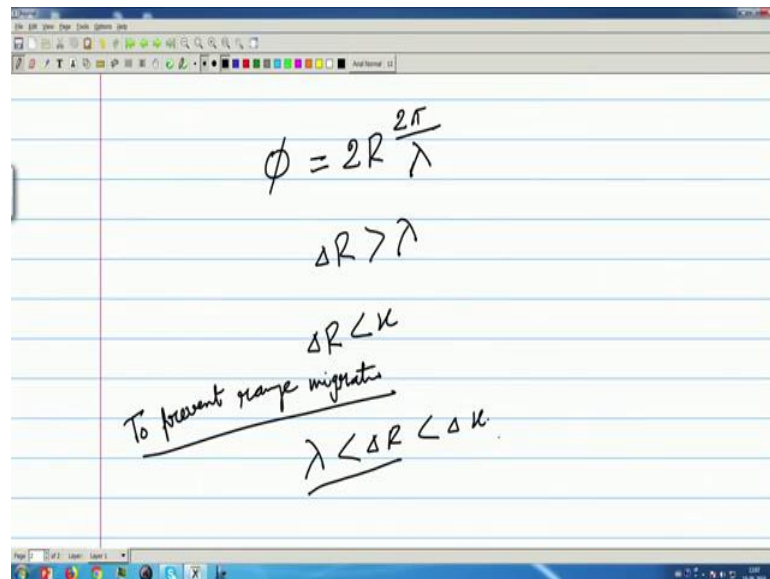
But we want that range change should be such that throughout this processing  $b_1$ 's return will always come to  $b_1$  it will not come to some other range, but there is a range change. So; that means, I want that there will be a range change because these  $b_1$  point when I



am looking from extreme angle the range has increased. But that increase should not be such that it goes to the next range bin. So, you can think of suppose this c 4 point.

So, I want that when I am looking from this bottom part there is a maximum range change and when I am looking straight from this c row that is the minimum range. So, this is the two extremes of the ranges. This range change should not be such that is more than one range bin if that is there that is called range migration. So, c 4 will come to let us say 2 or 1 etcetera. So, actually this is a paradox of the whole processing that, we want that there should be a due to the movement of the radar. There should be a range change, but that range change should not go to the next range bin.

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So, we have already seen that what is the space phase associated with that change of range  $2R \frac{2\pi}{\lambda}$  here  $R$  is a function of time so,  $\phi$  is also known.

$$\phi = 2R \frac{2\pi}{\lambda}$$

So, to generate a good resolution in phase change so, that sufficient Doppler shift is generated. We require that  $\Delta R$ ; the change in range should be from this equation you can see this should be much larger than  $\lambda$ .

But in order to remain in the same range bin, the range change has to be smaller than the range resolution element of the image what is our range resolution element? So, let me

call that this is  $y$ . So, I can say that this distance let me call  $\Delta y$  this is our range resolution cross range resolution and this is our down range resolution.

So, what I want this I want, but I also want that the total  $\Delta R$  that should be less than  $x$ . That means, to prevent migration range migration the condition is that  $\lambda$  should be less than  $\Delta R$  should be less than  $\Delta x$  ok.

$$\lambda < \Delta R < \Delta x$$

Now, this is generally met because it is easy to make these compared to  $\lambda$  the  $\Delta R$  should be larger, but this one whether it means? Now let us do some calculation so, because we know the expression of range change.

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$$\theta_0 = 90^\circ$$

$$R = R_0 + \frac{v^2 t^2}{2 R_0} - \frac{v^4 t^4}{8 R_0^3} + \dots$$

$$\frac{vT}{2} = L$$

$$R_{\text{MIN}} = R_0$$

$$R_{\text{MAX}} = R\left(t = \pm \frac{T}{2}\right)$$

$$= R_0 + \frac{v^2 T^2}{8 R_0}$$

$$\Delta R = R_{\text{MAX}} - R_{\text{MIN}} = \frac{v^2 T^2}{8 R_0} = \frac{L^2}{8 R_0}$$

So, you can easily do the calculation. So, what is the expression for range for a broad side SAR; that means,  $\theta_0$  is 90 degree we have seen what is the expression for  $R$ ?  $R$  is equal to  $R_0$  plus  $v^2 t^2$  by  $2 R_0$  minus  $v^4 t^4$  by  $8 R_0^3$  etcetera.

$$R = R_0 + \frac{v^2 t^2}{2 R_0} - \frac{v^4 t^4}{8 R_0^3} + \dots$$

So, what is my minimum range that we know, if I put  $t$  is equal to 0; that will be minimum range  $R$  naught.

$$R_{\text{MIN}} = R_0$$

And what is my maximum range change? That will be at the extreme point. So, when  $R_{\text{MAX}}$  is  $R$  is  $t$  is equal to plus or minus  $T$  by 2. So, if I put this, it will be  $R$  naught. Let us take the only up to the square term so, plus  $v$  square  $T$  square by 8.

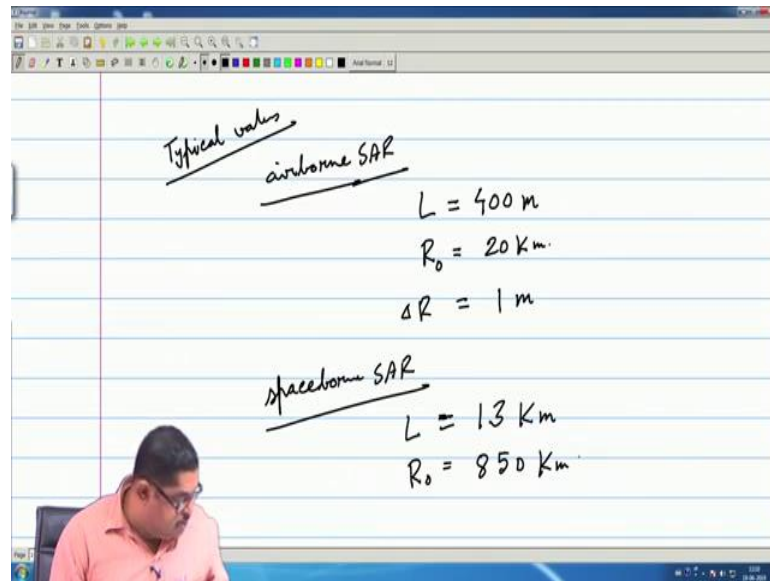
$$\begin{aligned} R_{\text{MAX}} &= R \left( t = \pm \frac{T}{2} \right) \\ &= R_0 + \frac{v^2 T^2}{8 R_0} \end{aligned}$$

Now, what is this  $v$  into  $T$  by 2, that is  $v$  square  $T$  by  $v$  square by 2. So, these if you do it will be and that will. So, what is my  $\Delta R$ ? It's  $R_{\text{MAX}}$  minus  $R_{\text{MIN}}$ . So, if I do that, it will be  $v$  square capital  $T$  square by 8  $R$  naught. So, that is  $L$  square by 8  $R$  naught  $P$  into  $v$   $T$  by 2, that is  $L$ ; if I put that this thing comes.

$$\Delta R = R_{\text{MAX}} - R_{\text{MIN}} = \frac{v^2 T^2}{8 R_0} = \frac{L^2}{8 R_0}$$

So, now, let us take some typical values what are typical values.

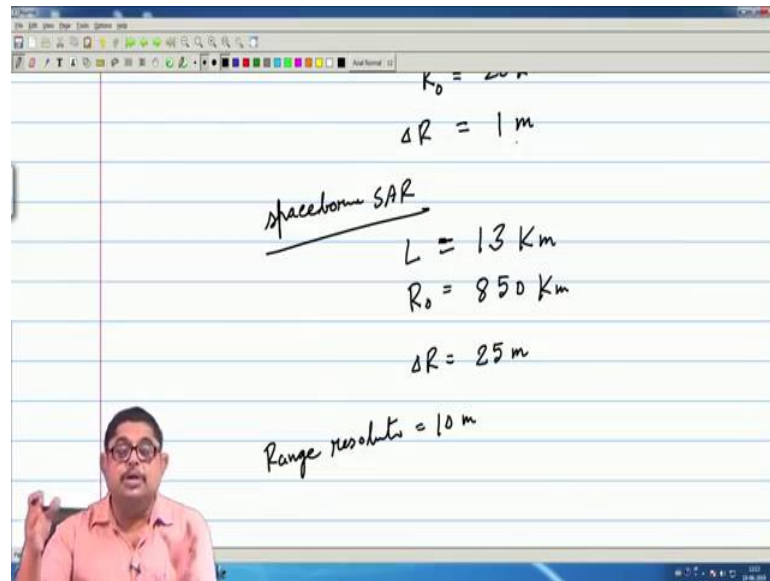
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Now, there are generally two varieties of SAR one is airborne SAR. Airborne SAR means the radar is flown with a on an aircraft and aircraft moves. So, typically I can take some typical values of Indian made SARs the synthetic aperture length is 400 meter, the range  $R_0$  is 20 kilometer. So, in that case the  $\Delta R$  that will come to be 1 meter.

If you do the calculation with these  $\Delta R$  that  $L^2$  by  $8 R_0$  that will give you that. And for spaceborne SAR, this values will be different typically in one of our Indian SARs the  $L$  is 13 kilometer. Think of synthetic aperture length is 13 kilometer,  $R_0$  is lower orbit satellite 850 kilometer.

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So, that delta R comes out to be if you do the calculation it will be 25 meter. So, suppose the range resolution range resolution in both the cases is 10 meter. So, in case of airborne SAR you see that the maximum range change that is 1 meter. So, if you have a range bin of 10 meter range resolution so, its not a problem only one tenth of that is getting changed.

But for space borne SAR you can see that the ranges they can change by 3 bins. So, and; obviously, a SAR with a squint angle non 90 degree for that the delta R will be higher values; because, this is the minimum delta value you have seen. So, the SAR processor should be capable of changing that actually that is done. Because you know your system so, at which ranges you will get the range migration that you know. So, you can correct for that. So, before actually doing all those what we have seen SAR processor it will have to correct for that range migration; once corrected then the whole data is okay and then you can work with that ok.

So, with that we end the synthetic aperture processing its a very nice processing. You see simply by creating a relative motion of the radar with respect to the ground target you can have the resolution. So, only thing is how much synthetic aperture you will keep that you will have to decide based on that you can generate any type of resolution.

So, its a nice say. So, we have seen how to improve resolution and with that we conclude the discussion on imaging radar. Because we have seen how to improve down range how

to now with SAR we are seeing how to improve cross range. So, we now know how to make a radar image resolution wise fine. So, that it comes closer to the optical image sort of thing that is the imaging radar. And that is the satellite based or aircraft based microwave imaging is something like this ok.

So, with this we conclude next time we will revisit. In our next class we will revisit our the basic radar equation model. And we will modify that model to have more practical make that model more practical. We started with their model we said we will first develop all this. So, all the various technologies we have seen. Now, we will revisit that model, we will make some realistic changes to that model. So, that model works for most of the practical scenario.

Thank you.