

Electrical Machines - I
Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 12
Factors on Eddy Current Loss Depends

So, we were discussing Eddy current loss, we will continue that topic in lecture 12.

(Refer Slide Time: 00:21)

To find out an approximate expression for Eddy loss in a thin plate

RMS induced voltage in the loop

$$E_{\text{eddy}} = \sqrt{2} \pi f B_{\text{max}} (2x h) l$$

Eddy power loss in this loop

$$= \frac{E_{\text{eddy}}^2}{R_{\text{eddy}}} = \frac{2 \pi^2 f^2 B_{\text{max}}^2 4 x^2 h^2 l}{\rho (2x + 2h) / L dx}$$

$$= \frac{8 \pi^2 f^2 B_{\text{max}}^2 h^2 x^2 L dx}{\rho (2x + 2h)}$$

$$\approx \frac{\rho 2h}{4 \pi^2 f^2 B_{\text{max}}^2 h L} x^2 dx$$

And so, we understood no point in using solid block of iron because there will be so, many Eddy paths, Eddy current loss will increase. Because, that Eddy current loss is in any case is not going to transfer energy from primary to secondary which is our primary job. It will be locally there will be heat generated, core will be heated up and there will be efficiency will be poorer.

So, how to reduce that Eddy current loss? From common sense it look like that instead of using a solid block of iron you better use thinner plates, stack them together and get the sectional area. Therefore, it is now true that at least I have to use thin plates ok. Therefore, in this topic what I am going to do is this to find out an approximate expression for Eddy loss in a thin plate. So, if in one plate I can find out for a all other plates it will be same. So, this is the thing I am trying to get [FL]; to do this what we will be doing we have to do a little bit of mathematics, but that is very simple no field equation nothing.

So, consider a thin plate, let me draw it here and this thin plate I will draw slightly larger although its thickness is quite smaller. Thickness is of the order of 0.25 millimeter, 0.3 millimeter, but I am drawing it in a larger scale. So, that we can understand what is going on and suppose this is a single plate. And, it has got a length that side also got the point [FL]. In this thin plate what we will be doing? We will make x and y axis, I will assign this to be my x axis and this is my y axis and this is my origin, and suppose the thickness of the plate is tau is the thickness.

Now, I have to find out an approximate expression of Eddy current loss; to find that out what I will do is this, I will consider at a distance x from the origin a loop like this got the point. Let me complete then I will explain, see within that thin plate I have considered at a distance x, this distance is x. And, elementary thickness d x, why that d x is necessary will be clear.

Our plan is to find out what will be the Eddy current induced voltage in this loop and this loop exists in all the section. Here also it exist, all along the length this is suppose the length of that magnetic circuit whatever it is. If you unfold that core material that length over which that exist that is the somewhat the you got the point length of the magnetic circuit anyway. So, this is the thing. Now, in this considered loop of thickness d x, what will be the induced voltage RMS induced voltage in this loop will be that is Eddy is equal to $\sqrt{2} \pi f \phi_{max}$.

ϕ_{max} is what? The flux enclosed by this area, but that flux enclosed in the area can be written as B max which is calculated over the whole cross sectional area $\sqrt{2} \pi f \phi_{max}$ divided by overall area; $\sqrt{2} \pi f B_{max}$ into the cross sectional area that is this blue hatched area. What is that area is, this is 2 x at a distance x I have taken so, 2 x into suppose this is the height I just call it h here into h. So, $\sqrt{2} \pi f B_{max}$ into 2 x into h, it is ϕ_{max} into number of turns that is 1. Therefore, Eddy power loss sorry, Eddy power loss in this loop will be equal to if you neglect a reactance of this loop only resistance single turn.

So, Eddy square divided by resistance of this path, this will be the power loss B square by r what else because, this circuit is closed. Therefore, this I will write it now I will just put these values it will be $2 \pi^2 f^2 B_{max}^2$ into $4 x^2 h^2$. Now, the question is what will be these resistance? So, resistance of a path I will apply

that basic formula ρl by A . Now the question is what is ρ ? ρ is the resistivity of this core material, ρ is resistivity so ρ . What is l ? l will be how much the length of this path current path, it will be $2x$ plus $2h$ perimeter of this path divided by the cross-sectional area through which this current is flowing.

Now, what is the here requires some observation that is what is the cross this loop as I told you exist all along this length here also it exist. So, and current at a given instant of time will flow this is the cross sectional area because, the loop exist here also for all the this is not for this single fellow. Loop is positioned at a particular x and it is here also and current Eddy current path will be what? It will be perpendicular to this area got the point.

So, ρl by area will be then the area of this rectangle. What is the end of this rectangle? The length of this iron piece and this will be then if I call this length to be L which I wrote earlier suppose, this length is L up to whatever distance is both. So, this is L into d ; L into d x this is the cross sectional area therefore, this is the thing. [FL] Now, you see as I told you we would like to find out an approximate formula ok, to have a feeling on what factors Eddy current depends like that.

Now, since it is a thin plate the value of τ is much smaller than your this finite dimension h or l whatever it is. The first assumption is this one, this equal to I will write it as it becomes $8 \pi^2 f^2 B_{\max}^2 h^2 x^2$, this is these thing divided by this one. I will write it as this is approximately equal to this $2x$; see x is a subset of τ , τ is already small compared to h . So, there will be no harm if you write it as ρ into $2h$, this $2x$ term can be neglected compared to $2h$ divided by $L dx$ which I will put up like this exactly.

So, this then will be equal to $4 \pi^2$ these two \max $4 \pi^2 f^2 \omega$ my god B_{\max}^2 into h^2 , h^2 no h one h goes out. So, h it becomes h and then x^2 into L into L into $x^2 dx$; it is now a manageable expression and divided by ρ of course, isn't it. So, this will be this loss. So, this is I will say this is dP eddy on this elementary loop and this portion I will just copy it and carry on.

(Refer Slide Time: 14:41)

$$dP_{\text{eddy}} = \frac{4\pi^2 f^2 B_{\text{max}}^2 h L}{\rho} x^2 dx$$

$$P_{\text{eddy total}} = \frac{4\pi^2 f^2 B_{\text{max}}^2 h L}{\rho} \int_0^{\tau/2} x^2 dx$$

$$= \frac{4\pi^2 f^2 B_{\text{max}}^2 h L}{\rho} \frac{\tau^3}{8} \times \frac{1}{3}$$

$$= \frac{1}{6} \pi^2 f^2 B_{\text{max}}^2 h L \tau^2$$

$\rho \approx 0.25 \text{ to } 0.50 \text{ m}\Omega$

$P_{\text{eddy total}} = \frac{1}{6} \pi^2 B_{\text{max}}^2 f^2 \tau^2 \times \text{volume of iron}$

higher resistivity

Approx Expression

w/ per unit vol.

So, we got from the previous slide this is the thing therefore, this is the thing. Now the question is how to get P eddy total? So, P eddy total in that plate it will be equal to these are constant $4\pi^2 f^2 B_{\text{max}}^2$ into h into L and this is $x^2 dx$ and this is to be integrated.

So, I go to previous page; the question is what will be the limit of integration? See at a distance x , I have considered the full loop. So, 0 to $\tau/2$ will be the limit this is $\tau/2$, where τ is the thickness of the plate so, that will be the limit. So, 0 to $\tau/2$ that is all this will be the thing divided by ρ of course, is there.

So, next step is straightforward what you do $4\pi^2 f^2 B_{\text{max}}^2 h L$ x^3 by 3 . So, it will be τ^3 by 8 minus 0 and this is by ρ is that correct into one-third x^3 by 3 into one-third. So, the this is 1 by $6\rho \pi^2 f^2 B_{\text{max}}^2 h L$. This τ^3 I will write it like this $h L \tau$ into τ^2 and then I can write it as 1 by $6\rho \pi^2 B_{\text{max}}^2 f^2 \tau^2$ into $h L \tau$ is the volume of the volume of iron. So, the P eddy total is given by this.

Now, going to the previous page you also notice I have assumed another thing. When I varied I have considered the loop not like this, but it covers as you vary x it covers the whole area then you are through ok. But, not only that when x varies this portion remains same as if I have calculated many times that area. But anyway, whatever I am estimating I will be estimating slightly higher it is no harm. So, there is another approximation are

you getting, another loop you consider; you will consider it like this ok. This power loss here, power loss here and once again this power loss, this power loss.

But, that portion is always small because I am talking about tau which is far less than h. Therefore, this is a very handy approximate expression mind you approximate, it will serve my purpose approximate expression for Eddy loss. And, I know many things Eddy current loss then depends upon the maximum value of the flux density squared, supply frequency squared and thickness square.

Remember what is the fun of expressing it in this way; it is because of the fact I have considered while deriving this with a single plates. There will be multiple plates and I know oh it can be expressed in this way.

Therefore, if you have used to more number of plates then total volume is to be multiplied are you getting that is also crucial to note. So, I will say better right P eddy power loss per unit volume is equal to $\frac{1}{6} \rho \pi^2 B_{max}^2 f^2 \tau^2$ per unit volume per.

So, many Watt per unit volume and we say that then Eddy current loss will be proportional to the square of the thickness of the plate. See all these things the argument was if you use a solid block of iron Eddy current loss is bound to be more. So, many Eddy paths instead of that you use the Eddy currents, it will be only confining here.

Because, the next plate is insulated from the other, if you do not use insulated plates then the story remain same. The Eddy paths will be more are you getting, that is why the insulation is necessary therefore, we know that. So, how to reduce then eddy current loss, because it is a wasteful energy transformer will be on 24 hours and eddy losses will always take place. If that be the case you try to use such a material whose resistivity is higher, rho should be made higher at least. This is apparent use small tau that is one factor, thickness of the plate should be tau.

(Refer Slide Time: 22:25)

$$dP_{\text{eddy}} = \frac{4\pi^2 f^2 B_{\text{max}}^2 h L}{\rho} x^2 dx$$

$$P_{\text{eddy total}} = \frac{4\pi^2 f^2 B_{\text{max}}^2 h L}{\rho} \int_0^{\tau/2} x^2 dx$$

$$= \frac{4\pi^2 f^2 B_{\text{max}}^2 h L}{\rho} \left[\frac{x^3}{3} \right]_0^{\tau/2} = \frac{4\pi^2 f^2 B_{\text{max}}^2 h L}{\rho} \cdot \frac{\tau^3}{8} \cdot \frac{1}{3}$$

$$= \frac{1}{8} \cdot \frac{4\pi^2 f^2 B_{\text{max}}^2 h L \tau^3}{\rho}$$

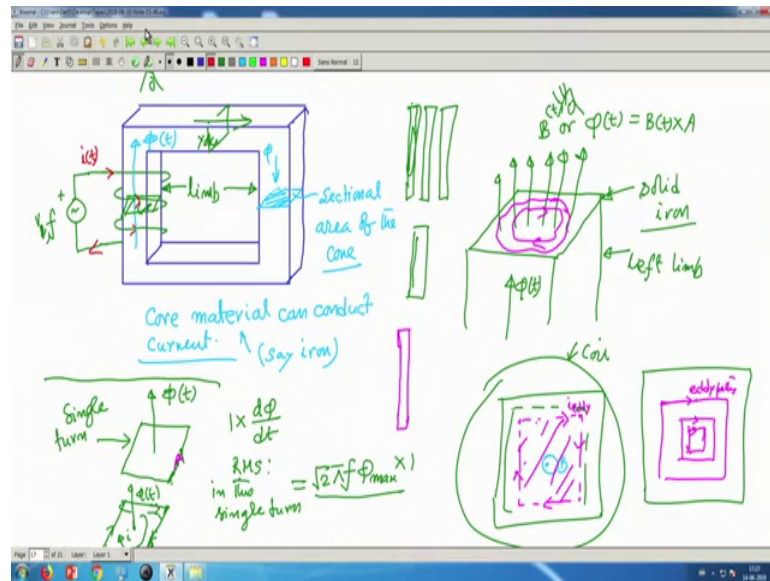
Approx Expression
 $\propto \text{Volume of iron}$

It cannot be too small otherwise those thin plates you cannot further make it smaller and smaller, it will become then brittle you know. So, the order of the thickness used is maybe 0.25 to 0.5 millimeter for 50 Hertz transformer tau order of tau. And, transformer size if you see they may be of the order of distribution transformer, if you look at their heights may be in meters.

So, the assumption that h and l compared to those values tau is very small is pretty good I mean nothing like that. But it gives so, do not use just iron, try to use a better material that is why people use silicon steel; had some small quantity exact percentage I do not do not remember. You can see a design book at silicon to ordinary iron steel and make a special type of a iron which will give you higher resistivity to reduce the loss. And, also try to do not try to use too high f B max which will also reduce and remember it is proportional to frequency square.

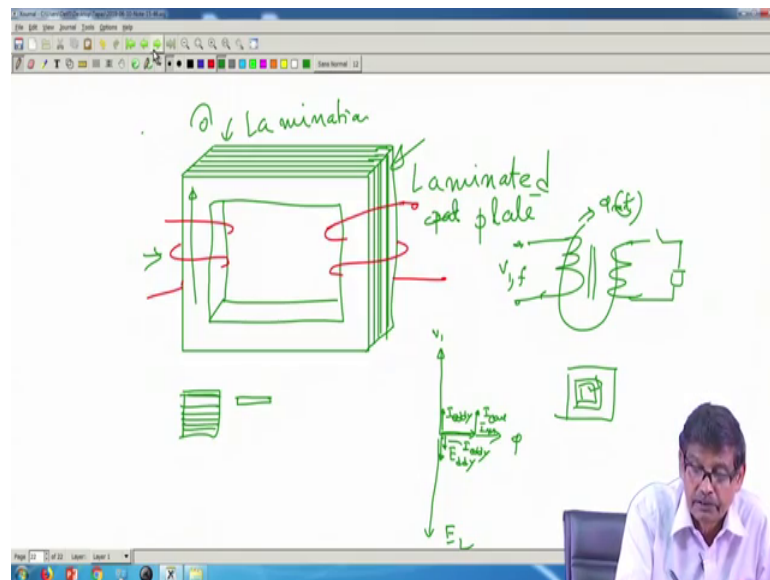
So, if the transformer rating is 50 Hertz based on that you try to make Eddy loss as small as possible. So, please remember what was a logic why the this was necessary to derive? It was necessary to derive because in the transformer if you go to this previous page as well previous page.

(Refer Slide Time: 24:41)



See in this transformer therefore, what I am telling if I spoil this page to this I will tell it like this.

(Refer Slide Time: 24:49)



It has come there this is the thing. So finally, it will look like this instead of a solid block of iron, I will use thinner plates. So, this I am showing it by lines another plate, another plate, another plate, I think you have got the ideas of this kind got the point. So, many plates like this you stack them together and try to form a core material like this. These

are also called laminated plates, use laminated plates; in short it is called laminations each plate laminations.

Then you make your winding not over a solid piece of iron, but like this another side it is like this, make your primary and secondary. So, Eddy current loss it depends on $B_{\max}^2 f^2$ and thickness of the plate square, thickness should be 0.2, 0.3 millimeter thickness and each plate is insulated from the other. And, this will be also reflected here in this toy I think this is ok. So, and this is the flux, total flux is Φ by this area you a the cross sectional area; if you look at from the top it will be now like this and your Φ is coming like that.

So, average flux density is B_{\max} divided by because each plate will contribute to the area that is there that overall area will give you the B_{\max} anyway so far so good. So, this is called Eddy current loss. Now, how to take this Eddy current loss effect into the equivalent circuit coming back to the original question. So, it looks like that even with the secondary of the transformer opened, I mean you can look at it in this interesting way as well. This is V_1 if you have switched on the supply, secondary is open nothing like that, but there is core material.

Now, when you close it there will be Φ produced alternating Φ is produced which is flowing like this and that are Eddy current losses. You can also think oh then I have applied the voltage and in the core there were turns existing they had induced voltage in the same way as your secondary coil had induced voltage and they were driving current. You can also think like that ok, that is what I am trying to tell even with this switch open it looks like as if there is a secondary coil existing within the iron core there and the circuit is closed they are carrying secondary current.

But, flux has to remain same therefore, those tiny Eddy currents two will have reflected current on the secondary side. I mean what I am trying to tell, I mean another way of looking at the thing this is your voltage ok. And, suppose only Eddy current I am showing the flux will be a around this point Φ that is induced voltage in the secondary E_2 that is there.

But, what I am telling there are some invisible secondary already present in the core. And, they had induced voltage also in the same direction why not Eddy. And, I assume

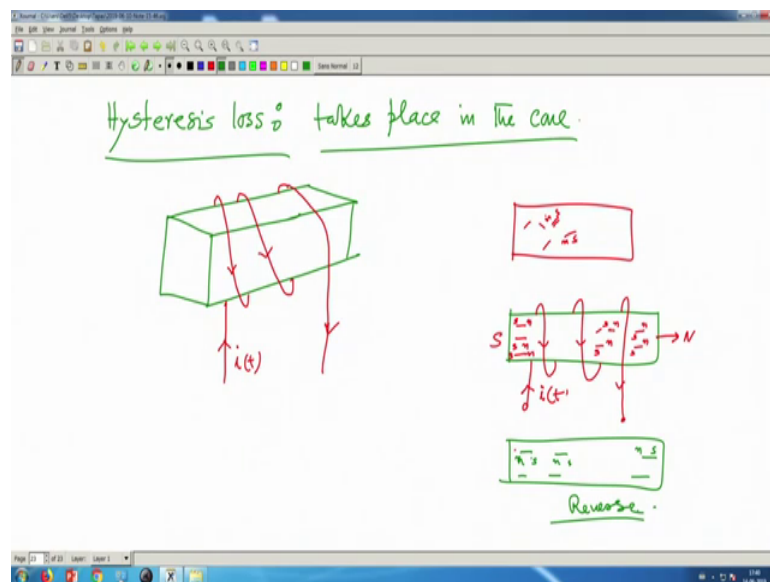
that power factor of that Eddy current path to be unity, leakage inductance therefore, with secondary open there is no I^2 here that is fine.

But, here was a secondary it has the I eddy and why not it will have its reflected component here in order that your flux in the core remain same. Therefore, apart from this magnetizing current I_m we now find there will be and current due to core loss which will be in phase with the supply as it is expected.

This is how this current will be drawn in, another way of interpreting the result; got the point. But, anyway what people say is that this Eddy loss after all the power has to come from this. Therefore, there will be another small component of current drawn from the source which must represent Eddy current that will be in phase can be physically interpreted like this.

Now, after I have done this, I will come to this complete phasor diagram slightly later. Before that I will tell you something about another component of the portion that is called hysteresis loss ok.

(Refer Slide Time: 31:43)



What is hysteresis loss? Hysteresis loss is another loss which also takes place in the core takes place in the core like Eddy current loss. Now, what is this loss? This loss is slightly I mean of the track type things for, but we know why hysteresis loss should take place.

See after all if you have a magnetic material like this over which a you wound a coil and are you getting.

So, suppose let me draw some coil say a piece of iron and here is a coil like this a coil is wound and this length is more whatever it is. And, suppose you pass current in this way some AC current connect current some AC current you will pass. If you pass current then why this core gets magnetized?

It is because of the facts will not go to the quantum level. But we can understand there are tiny magnets molecular level small magnets are there in any ferromagnetic material like this, north south, north south, but they are see north south, north south, they are tiny magnets already existing in a ferromagnetic material.

So, what you are doing the moment you have apply some I will drawn now sectional diagram, if you pass some current like this. So, inside this there are tiny magnets which are apparently oriented and we do not see any magnetism in it. But, if you pass some current, some DC current you pass to this fellow iron bar, that is how I make permanent magnets you know. So, you pass some DC current therefore, there will be you have applied h along this and each tiny magnets will try to align itself along n s all the magnets.

If you pass a little current they will slightly move and pass more current they will be further aligned n s , make it more they will school level also we study like that. So, if you pass sufficiently large current at the end what will happen is s n all tiny magnets will be moved like this, such that this will become a north pole and this will become a south pole. That is how we are taught in school level how to make a soft iron piece a permanent magnet, you pass some electromagnet north-south.

Now, suppose this current is time varying therefore, what you are asking each molecule to do if this is time varying when the current is positive each tiny magnets was trying to align along this way. Then you reverse the current, they will now start moving and finally, when the negative maximum current flows this side will become south. The other way it will become it is north, it is south, it is north, it is south I am just drawing is not.

So, each one will move and finally, they will align provided you reverse the direction of the current I am not drawing all these, these are very basic. Therefore, if it is the 50 Hertz

frequency current changing then you are asking each molecule to move this way, then that way always they will be moving. Therefore, they will encounter friction in the process and there will be heating effect not because of Eddy current, Eddy current is separate thing. But, simply because you are reversing the currents this way that way therefore, there will be always movement of the molecule.

That way you can imagine whether exact thing perhaps detail analysis of at physics level you have to do. But, we can understand this is what is going to happen; each molecular level tiny magnets they will be sometimes align this way, sometimes this way. Therefore, core will be heated up not only because of Eddy current, but because of something else here. And, this loss which will take place is called hysteresis loss and we will continue with this in the next lecture.

Thank you.