

Network Analysis
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Lecture # 11
Energy Stored in Inductor with Example

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Inductor can store energy

$i(t)$

$v(t)$

$v(t) = L \frac{di}{dt} i$

energy supplied to L over a time 'dt' ($dt \rightarrow 0$)

$dw = L \frac{di}{dt} i dt$

$dw = L i di$

Total Energy supplied

$W = L \int_0^i i di = \frac{1}{2} L i^2$

$\frac{5A}{L=2H} \rightarrow \frac{1}{2} \times 2 \times 5^2 = 25 J$

So we will see today that how inductor can store energy now, the thing is as I told you suppose you apply any voltage time varying voltage across an inductor + - V_L and then allow a current to suppose current at any time t i then what is the voltage across the inductance it will be this is + this is - $L \frac{di}{dt}$ is not. Now, if you look at the inductor, voltage across the inductor is $L \frac{di}{dt}$ with this polarity and current is entering through +.

Therefore, it will it is going to observe power. So, power absorbed instantaneous power absorbed at any time t it is absorbing a power which is the voltage magnitude $L \frac{di}{dt}$ into the current This is the amount of power at any time t the inductor is observing Now, therefore, if you consider a small amount of time dt , energy I will write it now energy supplied to L to the inductor over a time dt . dt as you know tends to 0 small perturbation over it time dt will be nothing.

But dw elemental energy that is fade to the inductor is that it will be $L \frac{di}{dt} i dt$ into the sign into dt . This will be the thing which then dt is vanishingly small, no doubt, but not equal to 0 therefore,

you can cross these two out because of that reason. So, it will be $L i di$ this will be the energy at dw . Therefore, what will be the energy at any time t . So, energy, total energy supplied to the inductor at time t energy supplied total energy supplied. To the inductor you have to integrate this into this 1.

Now, the question is what limit should I put? limit I should put depending upon the fact that I suppose the initial current was 0 and Final current is I some current I therefore, they it will be half $L I^2$ or I should write any current whenever it is getting any current suppose this is I , it does not. So, half $L I^2$ squared these the energy supply but at that energy gone it will find to your surprise they conducted since it has got not resistance of course, you should not touch with hand.

But even if you touch imagine that you are I am touching the bar conductor which is which has made this inductor possible. You will find there is no heating at that because I have assume that equal to 0 some ideal situation of course In a practical inductor there will be little it in because there will be some small resistance however small it is, but I am telling you in this ideal situation.

You will find there will be no heating at that then where that energy gone this energy will be stored in the magnetic field as I told you since you are passing some current at any given time t the current is I ni by L is age in to μ is your B and you know the magnetic field B squared by $2 \mu_0$ etc that is a measure of the energy stored per unit volume. Therefore, this energy will be stored in the magnetic lines of force.

Therefore, whatever energy I have supplied it as not meet last but it will be stored in the inductor so far as this circuit thing is concerned will not go to the level of flux lines. I know what it is, but one must know that it is told in the magnetic field this much, but it has the capacity to store the energy. The idea is we store several things in our houses. Similarly, it looks like energy can be stored in an inductor and perhaps by doing some intelligence circuit.

I will now I have got plenty of energy I will store it in the inductor and whenever I will need that energy, I will take that energy from the inductor and using my practical situations. That is the

whole idea in fact power electronics nowadays depends on inductor and capacitors their energy storing properties makes all the power electronic circuit possible sometimes stored energy in the inductor whenever at opportune time, you extract the energy from the inductor do these at a fast rate things like that.

But nonetheless, the point I want to make if you know the for example, if the value of the inductor = 2 Henry and current at any time t you find 8 is 5 ampere then what I am telling the energy stored by the inductor will be half L into i squared it will change with time is continuous value of the current square into half L will give The energy stored at that time. So, suppose at a particular time you notice it is 5 ampere 2 henry, then I will say it is 25 Jules, so much energy it is storing got the point. (FL: 08:18) I will tell you another interesting thing here.

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at $t=0$ move the switch to position "a"
 at $t=2$ move the switch from "a" to position "b"

$i(0^-) = 0 = i(0^+) = i(0)$

for $t \geq 0$

$v = L \frac{di}{dt} = 10$
 or $2 \frac{di}{dt} = 10$

for $t > 2$

$\frac{di}{dt} = 5$
 or $i = 5t + A$
 $i(0) = 0 \therefore A = 0$

Energy stored
 $W = \frac{1}{2} \times 2 \times 100 = 100 \text{ J}$

$2 \frac{di}{dt} = 0 \Rightarrow i$ must be a constant for $t > 2$

Because this problem I have already indicated to you, suppose, you have an inductor let me put some numbers so, that you feel easy with the problems suppose, I do it slightly differently suppose, it is 10 volt suppose the value of the inductances is 2 Henry and here this problem you try to understand here is a switch and here is another just to add having no resistance this switch can be moved has 3 positions one is in this position when this terminal is supposed a this terminal is b, AC is neither connected to a not to b (FL: 09:38).

If this circuit is left like that, what will be the current in the inductor 0 open circuit how current can close that is I have not applied across the inductor any voltage before that now, what I will

do is this look carefully at $t = 0$, I will move the switch to position a. So, if you do that then you are circuit at $t = 0$ you have done it like this. So for t greater than equal to 0, this is the subject and these inducted is 2 Henry.

Therefore, I want to find out what will be the expression of the current in the inductor. How do I do it? I know that $i(0^-)$ - in this case $= 0$ and that must be $= i(0^+) +$ this, is the crucial thing and $i(0^+) +$ and $i(0)$ is 0. Then I will write that say, $V = L \frac{di}{dt}$ whatever way you write i and this is a constant value that is 10 volt or L is 2 Henry. So, $2 \frac{di}{dt} = 10$ or $\frac{di}{dt} = 5$ or $i = 5t$ if you integrate + some constant of integration This is another way of doing it.

So, $i = 5t + A$ and this value of A I will find out from the initial condition, here $i(0^-) - i(0^+)$ which is also $i(0)$ for all practical purposes. Therefore, I applied this condition and find out the value of A , so, $i(0)$ being 0, therefore, A must be 0 and here $i = 5t$ and we have seen that if you plot this current against time i t , it will rise. What is this slop, slop is this is the net or N slope of this current where I have drawn it wrongly like 45 degree it is not like this 5 times more.

So, for this particular problem it was doing like this is $i = 5t$ and as I told you, you should not be careful, I mean just do not clip the circuit in this position forever then your inductor will be spoiled because soon it will cross the rating of the inductor current and spoil everything. So, it is like this. Therefore, suppose at now this is this switching I have done at $t = 0$ move the switch to position. Now, what I do at $t = 2$ seconds move this switch from position a to position b.

So at $t = 2$ seconds if you just this is supposed 2 seconds 1 2, then this current will be 10 ampere. Why these 2 second was chosen most probably. It was the rated current of the inductor. I will not allow under any circumstances to pass a current which is greater than 10 ampere that is why perhaps these 2 second has been chosen. Anyway, if you did not bother about that at t equal to 2 second current through the inductor will be 10 and after that at equal to 2 second I moved this weak from position A to position B.

Therefore, for t greater than equal to 2 seconds circuit will look like this one it is shorted this current is i t , inductor current and this is $L = 2$ H is not the greater than 2 seconds this will this is

the fate of the circuit. Therefore, current at t equal to 2 - C here I am doing is switching try to understand $t = 0$ - is not a very I mean not always that thing will happen at equal to 2 you are doing some switching now. So, you have reached equal to 2 seconds. So, current at equal to 2 - is what 1.99999 more diversity is very close to 2 but not equal to that current was equal to 10 ampere is it that will be the current.

Now, the question is what will be the current at $t = 2^-$ just immediately after 2 seconds, current through the inductor cannot change instantaneously. Therefore, if in the rate card, you see a blue card, if you sketch the current, I will sketch it separately here that is nicely it can be sketched with a red color I will draw. So, it when up to 10 ampere current this time was to second this axes is time. So, $T = 2^-$ current was 10 ampere. What I am telling after you have done the switching move S to b from a then also current has to be 2 ampere immediately after this 2 ampere that it has to be this 1.

So, but what happens say at equal to 3 seconds, 4 seconds like that nothing is surprising. Now 40 greater than 2 seconds I will once again write down the KVL equation here, what is the KVL equation $+ - L di/dt$ is not this the KVL equation of the circuit applied voltage is 0. So, I will see for the circuit am writing the equation I will write $L di/dt$ is equal to 0 applied politics you know the voltage applied here, no voltage exist between these 2 points $L di/dt$ is 0 if $L di/dt$ is 0 di/dt must be a constant number I must be a constant number, which means that is a L why I am writing to di/dt to di/dt is 0.

Which implies that di/dt must be a constant this must be the constant, I must be a constant I am so. I must be constant. So, where t greater than 2 seconds time must be a constant. It will not change and that thing has happened at immediately a practical to 2 seconds and here at equal to 2 + current has 10 ampere therefore, inductor current and will remain 10 ampere the so, this is equal to 0 this is a very interesting problem are interesting in the sense that it will only enhance your handling, knowledge of handling inductors how it do be a different heating conditions.

For example, current has to be continuous when you do heating 40 less than 0 current was 0. It is a good practice you will draw also what was current equal to 0 because it was open circuit and

no current at $t = 0$ you close it then you say that current has to rise from this then you say $L \frac{di}{dt} = 10$ volt. Therefore, solve for I which is very easy the $\frac{di}{dt}$ constant I will be like this, then use this boundary condition $I(0) = 0$, which is same as $I(0) = 0$ and you get that constant A to be 0.

So, it will be $5t$ is here that is fine current was growing like this for a practical circuit, it will not happen like that if you just keep pressing this 10 volt constant across the inductor, suppose the rated. So, at $t = 2$ second suppose 10 ampere is the rated current, you decide that you make the switch from A to B switch is moved like this then from this to this, so, move the switch to position B in that case $t > 2$ seconds, this is the circuit.

So, now you see the impulse voltage has changed, it was earlier a constant voltage now, you have apply 0 voltage why I am saying 0 voltage, it is a short circuit, no matter what current flows drop here is 0. So, terminal voltage across the inductor is 0 and I want to investigate what happens to this current which was brought up to 10 ampere at $t = 2$, I want to investigate that So, I once again write down KVL equation for $t > 2$ which will be $L \frac{di}{dt} = 0$ to applied voltage is 0.

And then from these I conclude I must be constant for the $t > 2$ it has to be constant. But that $t = 2$ it is already 10 ampere. So, that 10 ampere will be preserved. It should not in any way disturb us, because nothing is violated if $t > 2$ empty air flows constant not 2 ampere at 10 ampere it reaches. So, for $t > 2$ the thing will be the shorted and this current will circulate is KVL equation violated know why, because it is a constant current voltage between these 2 point is 0 $L \frac{di}{dt}$ of 10 will be 0 and same current circulates.

Now, by this example, what is the connection between the energy storing and this one see I have now made a circuit by way, I will pump energy into the inductor and what will be the energy stored energy stored in the inductor at t rather than $t = 2$ will be $\frac{1}{2}$, because current is constant 10 ampere $\frac{1}{2} L i^2$ into 2 into i^2 that is into 100 which is 100 joules. 100 joules of energy will be stored and so, keep the inductor shorted for any time you let it will be storing that 100 joules.

So, it is as if a part where I have pumped energy into the inductor and inductor will it looks like forever it will store that energy and whenever you want to retrieve that energy you have to make a circuit for that will come to that suppose at some time I asked myself this energy is storing 100 joules of energy, let me use this energy in some fashion. So, what I want to heat up a coil like a heater it can be done nicely.

So, in any case, the point important point that I want to make is current in an inductor is continuous you do a speaking at any time you like a $t = 5$ seconds whatever it is the important point is when you do switching at a particular time say capital T you have done a switching then current at t capital T - and capital T + has to be saying that is the past then you have to write down the KVL equations in all the intervals to solve for the currents and decide yourself the nature of the current waveform as I have done here got the point.

So, this is an exercise which is very simple but at the same time it is very important to understand that every step of it and I am sure if you go through this problem, which is also simple can be easily tackled no big deal to solve this differential equation very quickly it can be done and but try to interpret the results correctly. So, it occurs stores energy. And next thing what I will do is this.

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for $t < 0$ $S \rightarrow$ opened.
 $i(0) = 0$

at $t = 0$ S is closed.

ckt for $t \geq 0$

at $t = 0^+$
 $i(0^+) = i(0^-) = 0 = i(0)$

$E - Ri - L \frac{di}{dt} = 0$

$L \frac{di}{dt} + Ri = E$

I will take the famous problem sweet you have done so, many times at least in your first year course of circuit, but let us also do it here. For example, I have a resistance and an inductor I will just indicate how to do it. And here you have a this problem I will indicate that problem since I have drawn I think I should tell it a little later. Anyway, this is the thing suppose we have a circuit like this + - E is a constant battery voltage R and they lose the inductor. What I will do is this I will close this suite at $t = 0$ for p less than 0, S was opened S in open condition is open is was opened.

Since this was open so, I draw at t less than 0 these are the circuit was just like this R and L nothing is connected. Therefore, current here 40 less than 0 must have been 0. So, $I_0 -$ this tells me that $I_0 -$ is 0. So 40 less than 0 this was the circuit it was open circuit, this battery was there connected this way, but no current can flow no close to E. Now, at $t = 0$ at $t = 0$ S is closed and if you closes at equal to 0 then I will say circuit for t greater than equal to 0 will now look like this is our this is R this is A.

So, it is a instructive always to draw the circuit topology before you are switching and after switching before switching you have to dry it because you must have a knowledge what existed for the current at the called immediately before the switching that is the thing. Suppose I say this current is i t. So, for t greater than equal to greater than 0 better rate on equal to so, at $t = 0 + I$ must conclude that $I_0 +$ since it has to be equal to $I_0 -$ that is also 0 that is the thing which is for all practical purposes I_0 is zero then have to solve the subject. How to solve the circuit have to write down the KVL equation.

So, I say we know what is the terminal relation at any time, $t + -$ this is i t. Therefore, I will say this voltage across the inductor is $L \frac{di}{dt}$ and that is the polarity and then you write down the KVL - to + E then + to - Ri , then - $L \frac{di}{dt}$ and you have come back to the same point loop is closed. And then you say that $L \frac{di}{dt} + Ri = E$ this the differential equation first order constant coefficient RLR constant E is also constant. So, we will be able to solve it I will devote some lectures on how to solve differential equations, but anyway, this is so simple. I will do that in my next lecture and also start looking at capacitors. Thank you.

