Network Analysis
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Lecture # 12
R – L Series Circuit Analysis

So, we have been telling you about inductors in my last lecture.

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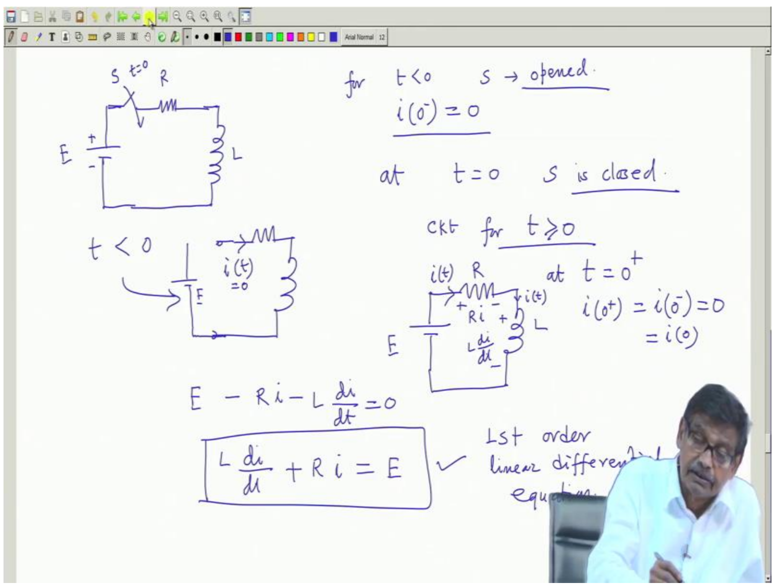

And we came up to this point that we first considered a pure inductor switched on to a fixed voltage, what will be the current response how energy can be stored and what is the expression of the energy. Then I started discussing this problem a series RLS arcade switch was open for a long time, then these are the conclusion immediately I draw i0- is 0, at t = 0 so this close. So, let draw the circuit for t<0 and t greater than equal to 0 always try to do that at least at the beginning, so that you did not make any mistake.

Therefore, here i t must be 0 because open circuit no current can flow exist here. Therefore, for circuit t>0 of course, a source is connected across 2 elements in series, 1 of which is energy storing element del and there we expect some current to flow and maybe that current is a function of time, because they least present. So, voltage drop you right meticulously and correctly right down the KVL here, you get this equation.

Know how to solve these equations. This is a 1st order, linear differential equation what happens is this if in a circuit there is only 1 energy storing elements, it will always give rise to a first order equation. In general, if 2 energy storing and elements are there it will be say inductance and capacitance both are present then it will give rise to a second order differential equation and so on. I will do a general treatment on differential equations maybe after a few lectures hence, but for the time being to solve the circuit is very simple.

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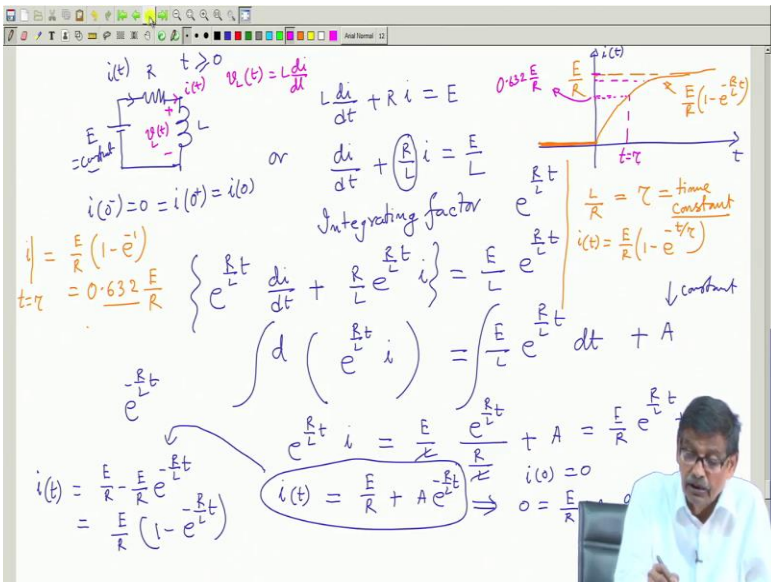

Let us see, and I will do it like this so, we got this equation. See, this was my circuit and did not forget to write that this circuit is t greater than equal to 0 and this are R this is L, this is E and this is current at any time t, this are the constant voltage constant battery. So, we got that L di/dt + Ri = E. Now, right hand side is constant so, what we do first is we divide by L. So, this is R/L*i = E/L that is 1 divide both sides by L. Then these numbers are L and D are constant.

So, this constant plays a crucial role in solving the differential equations. What do you do is this you integrating factor? Later we will learn several techniques to write down the solution straight away from this straight away write down the solution that is also possible we will take up that, but for the time being, let us start from basics integrating factories e to the power R/L into T, multiply both sides with this so, it will be e to the power R/L t di dt + R/L was already there into the power of R/L into t i and this will be called to e to the power l into e to the power R/L into t is good thing.

Now, if you look at this terms on the left hand side, this is nothing but product of differentiation of product of these 2 functions e to the power R/L into t into i and this is equal to E/L into e to the power R/L into t that is the first function into differentiation of the second class differentiation of this function R/L into the power of L into t into difference function so, that is fine. Now, in the next day for do you bring this dt on the right hand side it will be like this my goal is to find out i t therefore, I will integrate both the sides now, I will integrate both the sides.

Now, if you integrate both the sides, then without any limits and putting, so, there must appear some constantly integrating constant is not now, the integration of the left hand side will be, because this itself is the differential, so, this 1 only and on the right hand side it will be E/L is constant and integration of this 1 is the to the power remix by m. So, it will be R/L into t by R/L this will be the thing plus of A of these L goes. So, this will be equal to E/R into e to the power R/L into t plus A.

This will be the thing I have to solve for i t is therefore, I multiply both sides by e to the power -R/L into t both sides, then left hand side I will be left with only it and this will be equal to E/R + A into e to the power – R/L into T this will be the thing solution. Now, here this constant comes in A this constant is to be determined from the boundary condition what was the boundary condition we know that I you know - was 0 because before switching nothing was connected and this must be equal to i0+, which is as good as i0.

So current at equal to 0 will be continuous. So, so applying that i0 = 0. From this we get 0 is equal to E/R plus A, or A = -E/R therefore, the solution will be i t = E/R – E/R, A – E/R into e to the power – R/L into t this will be the solution E/R into 1- famous equation R/L into t. So, the value of the current at t=0 if you put it is 0 because the e to the power of 0 is 1, 1-0 is 1, then it will grow exponentially.

So if I skate it is current here, it will relate this t what the circuit was doing and this circuit is i t 40 less than 0 current was 0. It is always better always draw the current waveform 40 less than 0 also. Then at equal to t at this equation obviously, if you put t equal to infinity the i as t tends to infinity, this current will be only E/R, why because this term vanishes then it e to the power –R/L into T will vanish esteem goes to infinity it will the power - R/L into t tends to 0.

So, you know you draw a horizontal line there, whose magnitude E/R and then this current will is a famous cart we know it will grow and I think critically this cart will meet this final current E/R this equation is E/R into 1-e to the –R/L into t. This is how current through the inductor will grow. Now, the question is about this t tends to infinity really I have to wait for the infinite time that remains here after here.

So, that the circuit current is here not really, this time, if you say this ratio if you define this ratio L/R by a variable called tau, which is called time constant I am going to give you some physical insight into it. Time constant and then the equation of i t can be written as E/R into one minus e to the power minus t by tau is not L/R is the time constant. Now, suppose you calculate the current at equal to tau. What is the significance of this time constant? Suppose I say I will calculate the current at equal to tau lbr will return in some number.

And each dimension will be time that inductance by R it will be time. So, anyway at t equal to tau i will say i t equal to tau after 1 time constant, the value of the current will be E/R into 1- e to the power -1 and these values some 63.2%. Now, it will be 0 point about the 0.63 to check this number about 63% into E/R. So, what happens is this, this current, we have seen that it is growing exponentially at t equal to tau E/R is this much 63.2 off E/R about 63% maybe here this point is point 632 E/R 63% this level.

And what I am telling this is t equal to a tau at t equal to tau it will be like this after this much second current assumes it is zero I think let me write it clearly. Point 6321, 0.632 or 63.2% of the final current, that much 63% will be achieved after one time constant, one can calculate in this way, after 5 time constant or 2 time constantly will find it will reach almost 90% or so, after 5 time constant it will reach 99%.

Therefore, although it asymptotically mathematically means you will did not have to wait for so long it time to see that conclude the circuit has reached steady state after a few time constants are key to election for all practical purposes E/R magnitude final current. So, that was the thing. Now I have sketched this current i t against time when an adult circuit is switched on another variable i should also skate that is with time how the voltage across the inductor vLt changes we know the vLt is equal to L di/dt. The way I have assumed the direction of the current it is it. So, this voltage potential of this point with respect to this is L di/dt I already know it is equal to E/R something.

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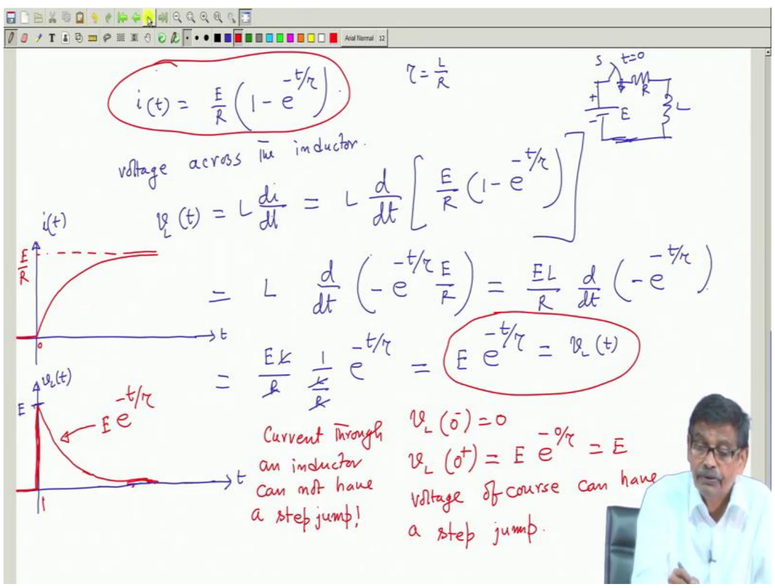

So, let me write it here. So, we have seen that i t = E/R/1- e to the power -t by tau and now is the time constant tau is L/R then voltage, how voltage across the inductor changes with time voltage across the inductor how to find it out I can easily be vLt suppose, this is nothing but there will be a L di/dt. So, L d/dt of this quantity that is 1- e to the power -t by tau this will be the thing.

Now, if you differentiate the first term is constant it goes so, it will be L into d/dt of minus e to the power minus t by tau into E/R here is there this will be the thing. So, E/R can be taken outside so it will be EL/R and d/dt let me be first minus e to the -t by tau this will be the thing, so, this will be equal to EL/R if you differentiated it will be -1/t that is plus one by tau into e to the power minus t by tau. So, but how is equal to L/R so, you put that so, here for tau =s L/R.

Then this R goes and this L goes and it will simply become E into e to the power minus t by tau and this is what they will equal to the voltage across the inductor. So, we found this is the expression for the current and this will be the expression of the voltage across the inductor. So, if you skate them one above the other that is, I skated it like this i t in the circuit it will be like this and also I would like to escape reality against time.

So, i t will have seen it will be like this final value will E/R and it will grow exponentially with a time constant tau now and before that was zero here. As I am telling this is t = 0 always try to sketch the current before t = 0 atleast t = 0- what was the current and now voltage across the inductance. So, voltage across the inductance. Before that there was no current in the circuit, open circuit. So that was 0 but at t = 0.

vL at equal to 0- it was 0. But at equal to 0+ put equal to 0 here, the voltage will be e into e to the power -0 by tau that is equal to E. Therefore, voltage across the inductor will have a sharp jump equal to 0 it will grow up and then it exponentially decays there is no DC turn plus to heat. So, it will decay exponentially also this has seem to be totally to decay and it will be like this going to 0 finally got the point and at equal to 0 this value this value easy.

So, this is the thing that is the RL circuit if you close to this which at t =0. RL the inductor current cannot change instantaneously, but look at the voltage waveform voltage across the inductor t = 0- equals 0, but at t = 0 plus it has gone up to E. So, voltage across the inductor can have a step jump, no problem it will grow up and with what is the equation of this equation of this is once again this is E into e to the power -t by tau.

Therefore, once again you can at t equal to tau you can find out the voltage by what amount it goes time constant etc but the point I want to make this finally, voltage across the inductor is 0 and it has to be because final current becomes a constant value E/R. So, d/dt of E/R is 0. So, that is what is expected, but the voltage across the inductor can have a step jump. So, conclusion is current through an inductor cannot have a step jump, we will see there is exception to this rule, when we take impulse function, impulse voltage applied etc but similarly, voltage of course, he named for an inductor can have a step jump, no problem step jump.

So, in this example also we have been able to store energy in the circuit is not E/R is the final current, but, you know if you want to sustain this current in the circuit in this way, there will be additional power laws always taking place in our got the point. So, I want to tell you you can do one thing, the same circuit to charge the what people say is that I have to pump current into the into inductor why I have to pump current the inductor because I wish I want to store the energy in the inductor because the expression of the energy stored is here. So, somehow inject some current into the inductor you have to apply some voltage etc.

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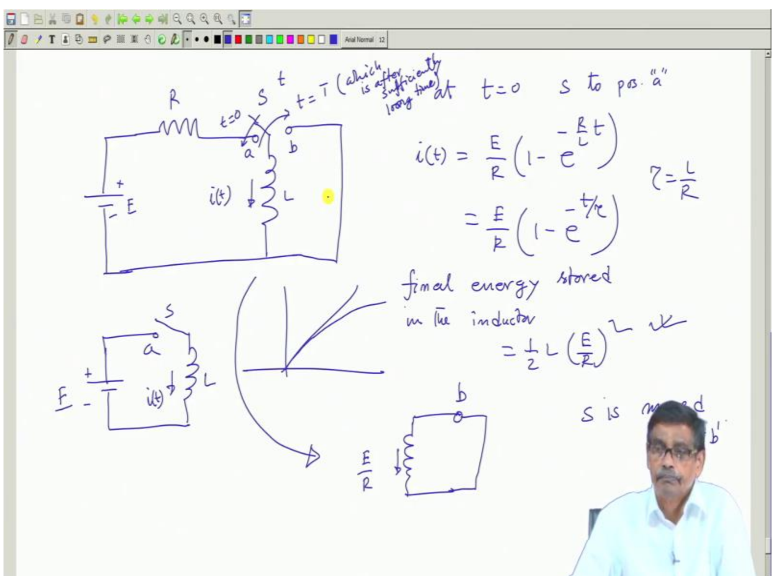

Now, the same circuit I will do like this that I will take it existence and this circuit I will make like this got the point and here I will keep it short it. So, here you connect to this is R this is L and suppose initially the switch was in the opposition there was i0- was 0 etc. Then what I do we connect this week at position A first at t = 0 S to position a then this is a series that will circuit i t, this is E and then current expression we know it will grow up like that we have seen this that is E/R into 1- e to the power -t by tau also good tau is L/R time constant of the subject.

Now, one good thing about charging an inductor with a resistances in series what will be the final energy stood final if you keep the switch just like that for sufficiently long time final energy stored in the inductor will be 1/2 L (E/R) square is not this much of energy will be stored. Now, one good thing of this circuit compared to charging an inductor like that will I want to make several points when I was charging an inductor just connecting with a battery.

The response of the current when AC connected to A was just growing like this, but here, this current will reach a finite value, even if you connect it here, there, how do you select your R such that E/R is the rated current of the inductor and you did not run the risk of exceeding the rated current of the coil. Of course, that current also should be the rated current of the resistance that is fine. But here it was growing, but here it is going like this E/R anyway, this will be the stored energy in the circuit.

But if you keep a AC position A because after you wait for several time constants, because we will see that in our practical circuits, the time constants typical values of the time constant constants will be in milliseconds For view milliseconds then you can say okay current in the inductor easy by then what you do here for the circuit I am telling here what you do at t equal to some time you move this way to position b is moved to position b of the circuit, this circuit I am not talking we have discussed at length. So, if you move to positioning it will be like this.

And this current we have seen it is equal to E/R when the next weekend was done from A to B So, at whatever time of course, you have I presume that you have waited sufficiently long so that the current has reached T/R. E/R was the current then you suddenly moved a to b. Then at whatever time you do that current through the inductor discontinuous immediately after that current has to be E/R. Not only that, then the applied voltage is 0 and that current has to remain constant to E/R level.

That is the same thing is achieved now, with these LR circuit in depth that does not care how you pumped current in the inductively knows that, if I am carrying this much of current I am storing this much of energy, what is the extra circuitry that you have made to pump current into the inductor it does not matter is how much energy it is flow. And that energy then if you want to avoid the unnecessary power loss in the resistance, you can make a circuit like this, here is the inductor, then move this to these and when suppose you do this next week you do. So this meeting is done at equal to 0.

Then this next reaching a to b you do at t = t, which is after sufficiently long time. Then the current through the inductor will be once again E/R and circulating here without defining any of the common rules okay. Therefore, this is the charging of an inductor and I thought I will tell you about capacitor in this lecture it is not possible. Then in the next class I will talk about discharging of an inductor and then about deeper luke into the another energy storing element that is capacitor. Thank you.