Network Analysis
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Lecture – 13
Retrieving Energy or Discharging of Inductor Energy

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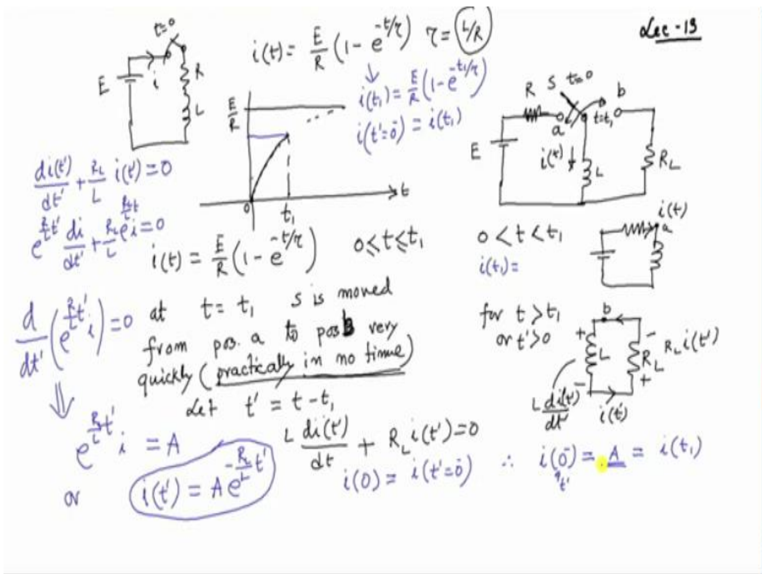

Welcome to lecture number 13 on network analysis and we have been discussing about RL circuits excited by a time varying voltage source and I told you that suppose, you have a circuit like that and you have and RL circuit here, so this which when it is in open position, their cannot exist any current, so I0 minus was 0 and then it was connected to this terminal at t equal to 0. Then, what happens this current will; equation of this current which is very popular.

And one should to remember final current will be DC current and it was like this; 1 minus e to the power minus t by tau; tau is an important thing which is called time constant and it is ratio between L and R, so it is; so and the nature of the development of the current will be exponential in nature and finally, it will reach asymptotically to this final value E by R of course, one need not wait for infinite time after few time constant it will be almost reaching E by R.

Now, similarly the question is; if suppose the same circuit, I will draw here, understand the problem first, so that it will be easier for me to talk so, this is the same circuit here but here I make an arrangement where I connect some or say I will do like this better, I will connect a similar circuit, here is a resistance R and here is a L and here is a load resistance RL, okay, switch is connected initially in position 1; position a at t equal to 0.

Therefore, at t equal to 0 this is the equation and current grows up and thereby pumping current into the inductor. An inductor when it carries current, we know it stores energy, now what I will do at some time say, at t1 time so, current started growing like this let me redraw this in this fashion, so current started growing up in this way following this curve but suppose at some time, t equal to t1 say, at t1 time, what I do; I move this switch s to position b very quickly in known time practically.

So, this switch is moved from position a to position b at t equal to t1, got the point and then I will say that the circuit is when time is between 0 to t1, okay circuit was like this, this was my it and it was growing in this way up to t1 it will go and for t greater than t1, this circuit will look like this, this is RL, this is L; RL is the load resistance. Now, this example will show you that okay, you have stored some energy in the inductor.

Now that stored energy you want to use in some load resistance for example, for heating purposes or it may be a small lamp whatever it is, whether it is possible. So, initial direction of the current; this current we were talking about it was it, so expression of the current it, I will write for this problem, I am sure about this that this will be E by R into 1 minus e to the power t minus tau, where tau is L by R.

And this is valid during this time interval, 0 to t1, you may put equal values also, no problem, so this is how current grew up in the inductor, circuit did not know that at time t equal to t1, you are going to do another switching from a to b, so it was following this curve faithfully as dictated by this equation. Now, at t equal to t1, s is moved from position a to position b; this is the position b.

Here it was connected to position a; position a to position b; small b, very quickly, very quickly means practically, in no time; no time is wasted to do that thing that is what I will have shown, practically in no time. I mean, here sometimes people feel a little uneasy because you are moving it from a to b in no time but it will take a finite time know, so there will be some time will elapse during which time it will be open circuited.

So, I mean, so with the help of solid state switches, this can be easily implemented, this you take it for granted for the timing, so I am putting a condition practically, no time is very important that is as if circuit was connected to a at till t1, then at t equal to t1, it has got connected to b, somehow, you have made some arrangement very fast time in, so that this terminal never sees an open circuit existing during that interval.

We will show you several examples, where in practice this can be done okay, so that is very important in practically no time. If that be the case, then my problem was by this 2 switching, I want to find out the current for, t greater than 0, so for t greater than 0 means, 0 to t1 is already obtain. Now, the question is how this current will behave the nature of the current variation for time greater than t1 that is what I want to find out.

Now, to avoid any confusion, what I will say that let t dashed be, t minus t1; t1 is a fixed time, t is the time which is counted from the very fast switching I started, so that is t and let me define another variable t dashed, okay which is this t, a time should be at this thing that is beyond this time. So, for t greater than t1 or t dashed greater than 0, this is the circuit; same circuit, t dashed is this and then I have to find out the current.

So, I assume the current to be it, this it is either you say or t greater than t1 or t greater than t dashed, so it dashed current flows, okay. Now, then what I have to do is this, I have to write down the KVL equation here and once you have assumed the direction of the current, so the drop here will be this is plus, this is minus and this drop will be RL into it dashed, this will be the drop with this polarity.

And since current is flowing like this, the polarity of the voltage as we have seen here will be Ld i dashed dt; is it not di dashed, it is d it dashed dt, dt dashed, so a new variable obtained I have just defined. So, your KVL equation will be from minus to plus if you go, minus to plus; L di d dashed dt plus RL it dashed and this is has to be 0 because there is no source existing. So, question should not be asked that there is no source; external source is connected.

Then, why there should be current; yes, current will be there because inductor had some initial energy stored at t equal to t1, is it not, so inductor since it is an energy storing device will stored energy and can discharge energy whenever it gets opportunity to do that, so this is how it will be, so I do not bother, so it is like this, this is the differential equation and then I will solve it, I will solve it on this page only, so that you understand what is going on.

So, like the previous one, I will divide it d it dashed dt, divide by L both sides, let me do it very fast, RL by L; this capital L indicates load resistance, nothing else, RL by L it dashed is equal to 0 here, no external source exist. So, this is the thing and once again, I will do very rapidly that you multiply this with the integrating constant R by Lt dashed di dt dashed, I am sorry, I am forgetting this t dashed plus RL by L i is equal to 0 into e to the power RL by Lt into i.

I have multiplied with this integrating constant and then I get d dt dashed is equal to the product of these 2 functions; RL t dashed into i and that is equal to 0. Since, right hand side is 0, I conclude that this must be a constant because if you differentiate a constant thing that only gives rise to 0, so R by L t dashed into I is equal to 0 here or I will say that not 0, a constant say A. So, I t dashed will be equal to A into e to the power minus RL, by L into t dashed, is it not.

This will be the solution, now the question is what is the value of this constant? The value of the constant is to be found out because you know that it1 from the previous example, it was equal to from this circuit, it is equal to it1 that is at t1, this value was equal to E by R into 1 minus e to the power minus t1 by tau, where tau is L by R this circuit and this will act as at t equal to t1 it is like this.

So, it dashed equal to 0 minus is this number, is equal to it1 is it not, it dash 0 minus, so this is the thing, so i0 is equal to this i0 minus and therefore, the value of the constant will be; I am just writing i0 minus, this 0 is new 0, t dashed 0, is equal to; equal to this is t dashed corresponding to that; t dashed 0 minus will be equal to A and which is nothing but your it1 calculated previously.

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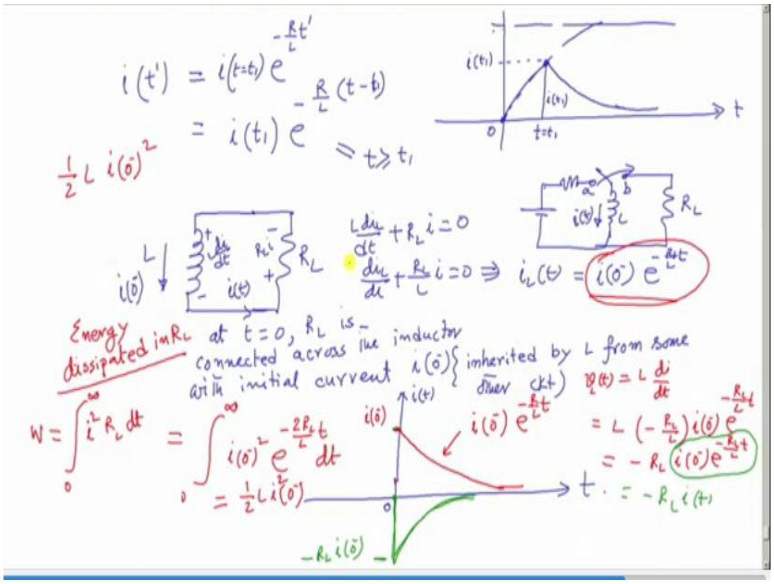

So, this number is known from which value of s known, therefore the solution of the current, I as a function of t dashed will be e to the power minus R by L into t dashed that is the thing and here this current will be current at it equal t1, got the point, so this number being known from this previous switching at t equal to t1, you put that value and get this. Now, sometimes because these t dash, t all things are existing so, you can express it as I at t1 and e to the power minus R by L.

And for t dash, you write t minus 1, so that your original t equal to 0 chosen is preserved in this equation, got the point, so but all the thing is this equation is true for t greater than equal to t1. So, if you now see what is going to happen, if I sketch it, it was growing like this, this is t axis, mind you, so at t equal to t1, it went like this, so this was the initial current, when the second switching is done.

And then that current will decay down to 0, so this is t equal to t1, got the point and this is t dashed, everything is fine, not really because if you put t equal to t1, what will be the current it will, it is indicating, how much is the current? It is 1, so it1, fine, good, so this is it1, anyway this is the mathematical thing. Now, so I allowed the current to grow up to this point it1 and then I did that switching, I better draw this circuit once again, so that you understand what is going on.

This was your R, this was your e and this is L and then this is the switch in RL and then you close it here, a to b and this current we are solving, mind you it, so equation of this current with respect to original time will be this. Now, at this problem I will just tell in this fashion instead of doing 2 switching, I will simply say that suppose, an inductor L; it was connected to some circuit and then it inherited a current, I will write i0 minus inherited from some other circuit this current.

And then this inductor is suddenly connected here across the load resistance, same problem I am telling but people will be always talking in this fashion, okay inductor had initial current i0 minus 1, now start your time t equal to 0, it is connected to this circuit at t equal to 0, RL is connected across the inductor with initial current i0 minus, in bracket you can write, inherited this i0 minus might have been inherited by this L from some other circuit, might have been inherited.

Because it was connected to some other circuit, there was current growing up okay, when it reached i0 minus, so in this case that I t equal to 0 is well defined across at equal to 0, RL is connected having an inductor current, so this problem once again instead of t dashed domain, I will simply write this is it and the differential equation will be plus minus, this is plus minus L di dt and this is RLi.

So, L diL dt plus RL into i is equal to 0 and if you solve it, divide by L, diL dt plus RL by L into i is equal to 0, means we have done this iL, now I will write t because t equal to 0 everything starts in this circuit, so iLt will be equal to this initial current which is a fixed number inherited by the inductor into e to the power minus R by L into t, not R by L, it is RL by L, okay. So, this inductor current will decay.

Is the time constant same, may not be because your L by RL ratio may be different from L by R ratio during charging process, so this is called discharging of the inductor. Now, these are the mathematical so, I will say that if for this problem, I want to sketch this circuit, I will say like this, current will form. This is only t, do not bring t dashed and all this, 0 and here I am sketching it.

So, the inductor had some initial current, t less than 0, here the current was i0 minus, how it reach that i0 minus, I am least bothered because I am interested to solve the circuit and inductor started with this current, it might have grown like this in this way or some other way, I do not care, I am only interested to know at t equal to 0 minus, how much was the current, at t equal to 0, I have done some switching, so that a RL is connected across L.

Then I say, the inductor current was like this, whose equation will be i0 minus into it will exponentially decay, R by L into t, got the point and previous to t less than 0, it was this one, got the point, so all the; so i0 minus keeps all the information about how current grow, we do not care that previous history is encrypted in i0 minus 1, integral v dt, I do not care about that, if this number is there, I know what to do.

Now, naturally question comes, inductor therefore stored energy, what was the initial energy then? 1/2 L I; not minus square, this was the energy stored in the inductor, how much a total energy which will be dissipated in RL, it will be this current square energy dissipated in RL will be i squares RL is an instantaneous power energy in time dt will be dt and this I have to integrate from 0 to infinity.

Because mathematically it exists up to infinity, although for all practical purposes current will eventually become 0 after few times constant, we know so, energy dissipated will be this one. Now, I leave it as an exercise to put this value that 0 to infinity that is equal to w dissipated, 0 to infinity and this current square is nothing but i0 minus square e to the power, this is also squared minus to Rl by L into t into dt.

Now, you integrate this but I am telling you this integration has to be equal to this one; 1/2 Li 0's square because whatever was the energy stored that must have been dissipated, for example you have connected a lamp here, a small lamp, what you will observe as you disconnect the this lamp across this L, lamp will glow and its brightness will diminish as time passes and eventually, it will upwards become 0.

Because all energy stored in the inductor has to be dissipated in RL, why this information I am telling you, always behind all this mathematics, if you think physically what is happening that is much better, okay mathematical description is there, you choose some t equal to 0 but physically thinking is also I am trying to tell you about always think physically what is happening, what was the; you are doing a switching, what was the current just immediately prior to the switching.

If that current value is known, you know the energy stored and that energy you are now retrieving in some resistance you are using that energy it is, if you like to know how the voltage across the inductance changes with time, you can also do that because expression of iL is known and voltage across inductance for this t greater than 0, how to find out vLt, it will be equal to potential of this point with respect to, this will be L diL d, is it not, di dt.

And this will be equal to L and i is this one so, it will be minus RL by L into i0 minus the differentiation of this one, i0 minus is constant into e to the power minus RL by L into t, which will be equal to minus RL into i0 minus into e to the power minus RL by L into t, so this was the current how it changes with time and what will be the voltage across the inductance? Voltage across the inductance will be; it will have a sharp negative value going, then it will exponentially decay down to 0.

What is the value of this voltage? RL is known, i0 minus is known, so this value will be minus RL into i0 minus, so voltage across the inductance can change instantaneously, current remains continuous, so whatever was the voltage before t equal to 0 minus, I do not care at t equal to 0, voltage has to be like this and it will go like this. In fact, voltage across the inductance, you see these 2 are in parallel, so it will be equal to voltage across the; it has come correctly.

Because of the fact, voltage across the inductance is nothing but this is nothing but minus RL into i, this whole thing is it, why minus because apply KVL here and you get L diL dt is equal to minus RLi. So, this in short, what I told about the inductor; inductor charging is necessary to store energy and whenever you like you would like to use that energy just for fun, you do not store energy, may be in future you will retreat that energy.

So, here was some simple circuits by which you can charge an inductor to any value of the current, of course to that value of the current which inductor will be able to sustain, matalab it should not exit the rated current of the inductor, then you retreat that energy, use it for some purpose, heating a coil, lighting a lamp, not a very good example but it gives you the idea, okay energy can be stored and it can be retrieved.

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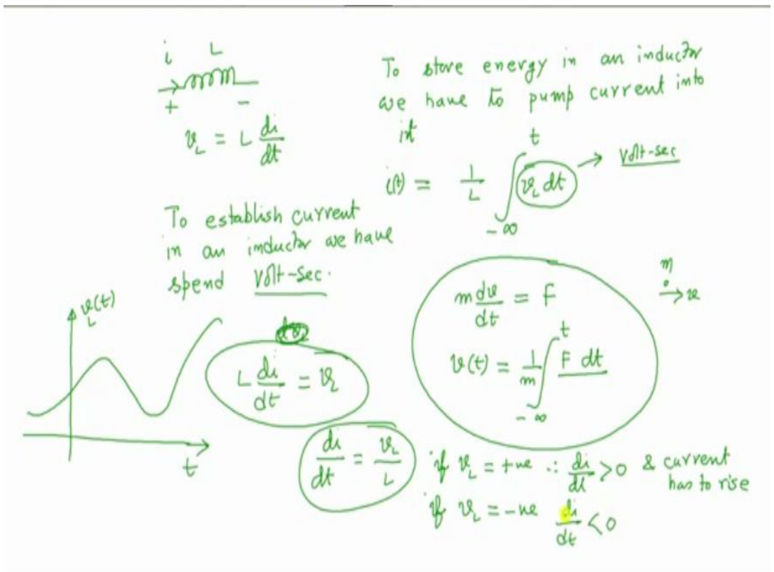

And mind you the last word about the inductor I must tell and then I will stop talking, remember few things about inductor that is suppose, you have an inductor L and if this is the current you are showing as I told you voltage across the inductor is Ldi dt, is it not. Now to store energy in an inductor what you have to do; you have to pump current in the inductor, I mean talk in this terms, let us talk, to store energy in an inductor, we have to pump current into it.

I mean pump current into it and must push some current flowing through the inductor okay, therefore if current establishes, then only it stores energy, now you see I wrote several times, i = 1/L, then vL dt, this is the value of the current in general at any time t from minus infinity. So, to store energy in an inductor, we have to pump current into it and to pump current into it this quantity, see dimensionally it is volts second is it not, whatever will be the result, it will be multiplication of voltage into current you are doing.

So, to pump current into inductor, to pump; to better write this time another words, to establish current in an inductor, we have to spend volts second at the cost of what; if you apply some voltage over certain time, then only current will be established, L is constant, so you have to spend some volts again in order to establish energy and it has got a striking similarity with your Newton's laws of motion.

For example, m dv dt, a mass, this is the applied force and this is m dv dt is the applied force, if you want to find out what is the current velocity, then you must have to do this integration, similar equation, 1 by m and you have to integrate from infinity to t, see do not lose sight of this fact as well oh, in mechanics applied force is equal to m dv dt, v is the velocity of the mass at any time t.

So, to establish velocity in a mass, you have to spend Newton's second, then only and divide it by constant mass it will be, so sometimes if is you can correlate with voltage applied, inductance can; inductors; it looks like has got inertia like mass, so inductance is often compared with mass as well, therefore to establish velocity, you have to spend some this one and at this equation, we will also sometimes very useful that is L di dt is equal to vL.

It is also can be very effectively used with common sense suppose, I say in an inductor applied voltage is like this, something very arbitrary voltage, it changes with time like that now, you see di dt is equal to vL by L, vL is the voltage across the inductance. This equation tells you that at any time if the voltage applied across the inductor is positive that if this is really positive this is minus, then current must di dt will be a positive number.

If applied voltage happens to be positive, then di dt is must be greater than 0 and current will rise, I am not telling any numbers, current has to rise, on the other hand if instantaneous value of the applied voltage is negative, this tells me this equation; di dt will be negative, then current will decrease. So, given a input voltage waveform, I will be able to tell many things without doing any maths, okay and we will continue with this in the next class and starts about capacitance, thank you.