Network Analysis
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Lecture – 14
Capacitor: Relationship of Voltage and Current and Initial Condition

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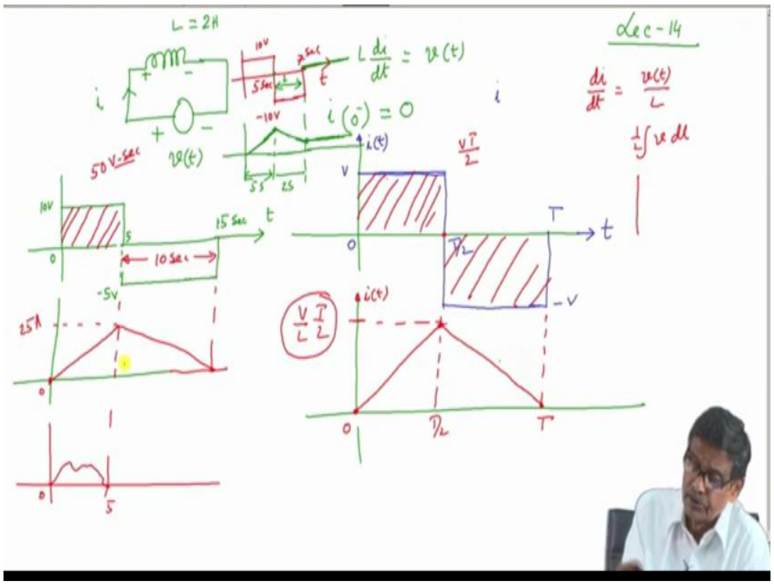

Okay, so we have been discussing with time varying voltage applied across an inductor, for example let me do another exercise. For example, adjust to highlight what I told you, I have connected across an inductor a voltage like this, this vt could be anything, L but with this polarity and I will assume the current like this i, and i is expected to be time varying because it will be a differential equation Ldi dt, this is plus minus Ldi dt as you can easily see, it is equal to supply voltage vt.

Now, suppose I say that i0 minus was equal to 0, when I connected a this vt across L previous to that there was no current in the inductor, if that be the case what will be the nature of the current, if I apply say a voltage like this, this is t, this is it, therefore suppose this is 0, this is T by 2, this is suppose T, I apply a square voltage of magnitude v and this is also minus v. Let us see what can we just tell?

Yes we can tell, see between 0 to T by 2, you have applied a constant voltage, I am not going to solve this circuit, we know that it between 0 to t to T by 2, the current of the inductor will be how much; v by L into t straight because v is constant level voltage, it will increase in the form of a straight line and where from it will start, initial current was 0, so it will start from 0, so it will start from 0 and it was increasing like this, like a straight line.

So, what is the maximum value at t equal to T by 2, it will be v by L into T by 2, put that value of t. Now, you see this axis is current, voltage applied across the inductor that is with this plus, this minus, it is indeed positive and current was increasing, you know, so during this time this is what is happening and i at t equal to T by 2 will be equal to v by L into T by 2 at this instant, fine.

Now, for t greater than T by 2 and less than t, you have applied a negative voltage, so current must rise, so at this point t equal to T by 2, inductor had some initial current of this magnitude and you have done has it some switching so that it has become a negative voltage but current cannot change instantaneously, it will maintain this current at t equal to T by 2 plus as well, a small, here from only it will start.

But applied voltage is negative and di dt as I was trying to tell you earlier, it depends upon vt by L but vt is negative, so current must have a negative slope, so current will have a negative slope and at t equal to T, it will be once again current in the inductor will be 0. Why it will be 0 at t equal to T by 2, okay mathematically, you do you will get it, okay, write down the expression of this, put t equal to T, this I leave it as an exercise to you.

Rather I will tell you physically what is happening, see to establish this much current, v by L into T by 2, how much volts second you have spent, this much volt second, vdt integral mind you, v dt, 1 by L; L is constant, so this much volt second day I have spent. In this case, it was simple; v into T by 2, so volts second spend was v T/2 to establish a current from 0 to v by L into T by 2, this level.

Therefore, to make the current back to 0, you have to apply negative volts second of same amount and you see this is also T by 2, the volts second associated with these just matches and therefore, current value once again will become 0; back to 0. So, if you apply a positive volts second across an inductor, current will go on rising and if you apply negative voltage current will start decreasing, di dt is negative here, di dt is positive.

For example, just there to complete this discussion I ask you that look here, I have I will draw in a this portion I will delete here, suppose you see that I have suppose applied a voltage which is equal to; I will put some number now, 0 to 5 second, this is t and this is a 10 volt, L is equal to say, 2 ampere okay and I will this negative up, I will apply a voltage of minus 5 volt, this is 5 second, mind you, I will apply a minus 5 volt voltage, okay.

And I will apply it for say, and this is suppose 5; this is suppose, this interval; this is suppose 15 second, now you see and you started the game with initial current 0 in the inductor, let us try to draw the current waveform. So, here to here, it will increase like a straight line whose value will be v by L that is 10 by 2 into 5; 25 ampere, is it, it will reach 25 ampere and I am assuming inductor is capable of carrying that 25 ampere current.

So, I have applied positive voltage, current linearly grew up from 0 to 25 ampere, now behind 5 to this 15 second that is this interval is 10 second, you have applied a negative voltage, therefore inductor current has to decrease because v has become negative, di dt will decrease. Now, I will not do any mathematics, you can write down and get it but what I am telling; during this positive cycle of the voltage applied; positive up, you have spent how much volt second; 50 volt second.

In the negative voltage when you have applied, how much volt second you have applied, once again 50 but in the negative sense therefore, current will once again become 0 at this point. See, the applied volts second that integral v dt depends upon what kind of voltage waveform you have applied and no matter for example, this 50 volts second could be achieved by some positive voltage waveform like this also.

But I am telling, what I am trying to tell that at t equal to 5 second, if you have applied 50 volts second; accumulated volts second, this current will be 25 ampere, positive volts second I have applied, similarly negative volts second you have applied like this, so this is how things you must understand that is volts second applied like that and I leave it to you, also think about this later, I will discuss suppose, you have applied for this problem, for the same problem you have applied a 5 second here.

And you have applied 10 volt and in the negative up, it is once again minus 10 volt but only for 2 second, this is t equal to 7 second, I will tell physically all the how much positive volts second you have applied; 50 so, it will increase current linearly up to this point sorry, and reach some value which I can calculate, if I know L etc., I will be able to calculate. So, you have spent 50 volts second to establish this much current from 0 initial current, it will go like this.

In the negative up, how many volts second you have applied, there you applied 10 into 5, 50 volts second you have applied but this interval is 2 second, because 7 minus 5, this point is 5, how much volts second you have applied; minus 20 therefore, current after this will not reach 0 value but after that you have not applied any voltage, what does that mean? The inductor voltage between these 2 point is 0 that means the inductor is shorted beyond t equal to 7 second.

So, inductor current will circulate like this, so this is 5 second and this is 2 second and if the voltage pulse is like this, it was 0 earlier then, beyond t equal to 7 second, it is once again 0, then the current will be, it will reach some value, it will come back and I asked myself I applied some positive volts second, then I did not apply the same amount of negative volts second, so current cannot become 0.

So, whatever will be the current that will be maintained in a short circuited inductor beyond t equal to 7 second, got the point, anyway think about it, this is so nice, you know, storing inductor and solving this kind of problems.

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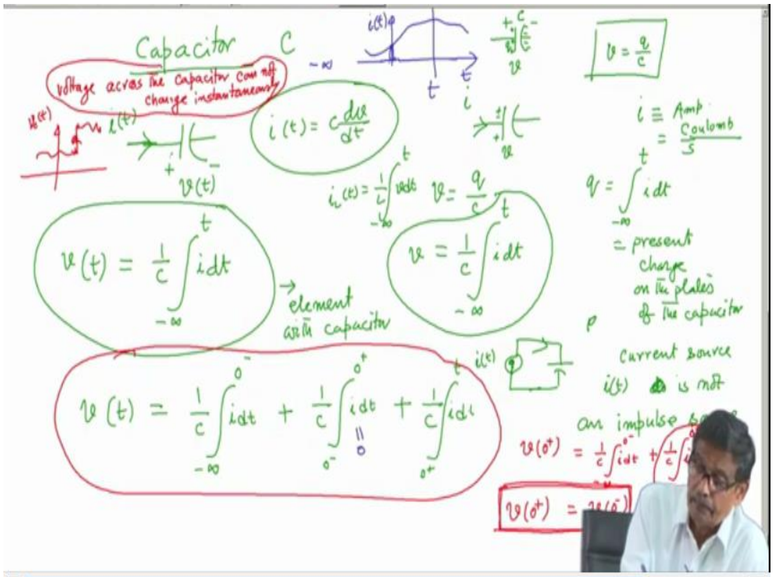

Now, today I will start similarly, another energy storing devices, capacitance or capacitor, it is a circuit element denoted by C which is just, it is called a dual element, in compared to an inductor, it is also an energy storing device and we know from our school days that voltage across the plate of a capacitor, if you denote it by v, with this plus, this minus and that this capacitance value is c, if charge on the plate of the capacitor is q plus one plate will be plus, other plate will be minus.

Then, with this polarity marking, v is equal to q by c, you know this is the thing, from our school days we know, therefore to have voltage across the plate of the capacitor, you must have charge on the plates; equal charge; plus minus on both the plates, then only it will have some voltage across the terminal. Now, the question is how charge can be put in a capacitor? It can be; charge can be established or can be brought onto the plate of the capacitor, if you allow some current to flow in a; i is ampere which is nothing but coulomb per second.

So, this is coulomb per second; coulomb per second is the current therefore, all this is ampere, this is we know therefore, to establish some coulomb on the plates I must pass some current over sometime it, so q is nothing but if you the present charge on the plate of the capacitor will be how much current you have put on the capacitor from minus infinity to t that will be, that will give you the present charge on the plate of the capacitor, is equal to present charge on the plates of the capacitor.

So, you have to excite the capacitor with a current source, you must fit some current to the plates of the capacitor and therefore, one skew is obtained, then voltage across the plate of the capacitor will be q by c and I will write it as 1 by c minus infinity to t idt, this is the thing. While analysing the circuit, we will not go to the extent of q because circuit solution means voltage current, we do not go generally up to q.

So, this is the relationship between voltage and current across the plate of a capacitor, so if I show it in this way, I will say if you are showing this is the current deduction, this is my prerogative I can choose in anyway but once you choose that, better choose the voltage of this plate and this plate to be minus and this is the voltage, therefore the present value of the voltage across the plate of the capacitor will be q by c, so 1 by c.

And q is nothing but minus infinity to t into idt, this is I, this is dt, conversely one can write it is equal to cdv dt, this is also correct, in fact we will be using this mode but to see; you see, can we say capacitor is a memory less device, certainly not because present value of the voltage does not depend on the present value of the current, it also depends upon the previous history of currents that you are fading from minus infinity including the present value of the current.

Because this integration requires the i values to be known from minus infinity up to the current time t and like a resistance v is equal to i, voltage across the present value of the current is v by R, R is constant. So, present value of the voltage only decides the present value of the current, no previous history is needed but in case of inductor, we have seen iLt, inductor current; it depends upon the previous voltage you have applied, you recall, same equation in different form.

So, this is a memory element with memory circuit element with memory capacitor, got the point so, this is important, the direction of the current, polarity of the voltage and their differential form of relations and integration form of relation that is the thing. Achchha, then I will do exactly what I did for the inductor. For example, the voltage across this capacitor, I will write it into 3 pieces like 1 minus c into minus infinity to 0 minus into idt.

So, what is this circuit here; circuit means here is the capacitor and we are feeding it from a current source, time varying current source, got the point. I am pumping current, charges are being stored hence voltage develops across the capacitor, that way if you think it will be better, so 0 to minus, then next term I will write; 1 by c 0 minus to 0 plus idt and then I will write 1 over c 0 plus to t, I can always write like this, this whole integral minus infinity to t can be broken up in this way.

And then I will say that this current whatever this current source is a reasonable current source, it is not, it does not include impulse, it current source, it does not an impulse source, it is a reasonable function, may be triangular, may be sinusoidal, whatnot so, i has a reasonable good function. For example, of course of any arbitration for example, it could be like this, suppose you feed the capacitor with this current, what time; minus infinity to up to this time t.

And in the same way, 0 minus is a time here and 0 plus is the time there and at t equal to 0, I am doing some switching in the circuit that is what I want to mean. Now, the question is; if you exclude it will not have any impulse, then this integral as to be 0, because this integral 0 minus to 0 plus as you make that 0 plus as small as you please 0 minus as small as you please, this area vanishes, this is nothing but the area under the curve.

So, this area will be 0, this integral and so this is the general expression therefore, I will say that what will be the voltage across the capacitor, so this is the general expression, fine, put any value of time, t equal to 5 second, you will get the voltage across at t equal to 5 second, so I will now say, v at 0 plus, so what should I do? t will be 0 plus, so this will have only this 2 pieces; 1 by c minus infinity to 0 minus idt plus 1 by c 0 minus to 0 plus idt, you know t equal to 0 plus, so this term will vanish I have reached 0 plus I will stop here .

Now, the question is this quantity is how much; if it is not an impulse, it will be 0, the area under this curve, this will be 0, so you will be left with v0 plus and what is this quantity; minus infinity to 0 minus idt, vt is minus infinity to t idt by c, so this is nothing but v0 minus, is it not, therefore I will say that unlike an inductor voltage across the capacitor plates cannot change instantaneously, it must be a continuous thing.

I am assuming, presuming that at t equal to 0, I am going to change something in the circuit, I am applying different current to the capacitor or things like that, therefore a and in an inductor capacitive circuit, it is the voltage which will be continuous whenever you do some sewing (()) (27:29) that is the important thing, okay. Therefore, we write in language also, so that you remember it.

Voltage across the capacitor cannot change instantaneously, it may capacitor voltage vct; if I write voltage across the capacitor, this vct; it will never have, it was doing like this, it cannot do like this, a sudden jump cannot happen, t equal to 0 minus some finite minus 5 volt at t equal to 0 plus, plus 5 volt, say 2 volt then plus 5 volt, no, a sharp jump vertical line will not be present in a capacitor, as sharp rise in current in an inductor will not be present.

Therefore, we say that voltage across the plate of the capacitor cannot change instantaneously, okay.

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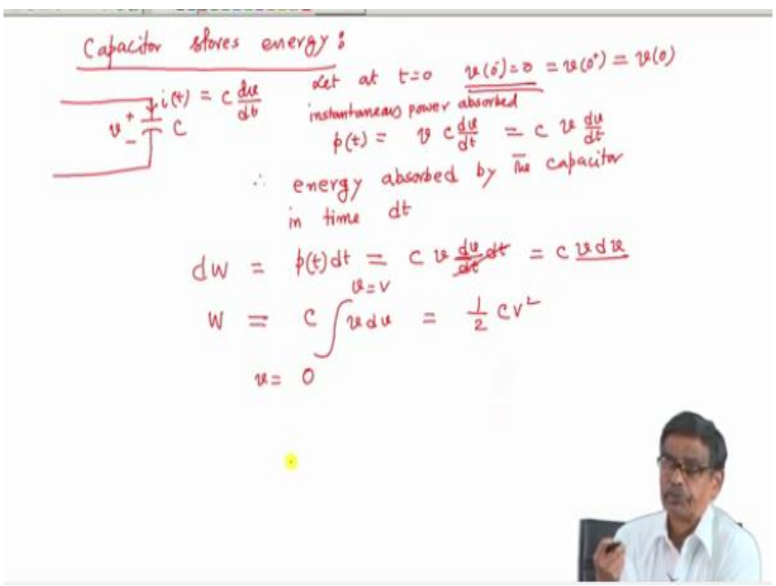

Now, the second thing is that capacitor stores energy and obviously, it is an energy story element like inductor and whenever you like; you would like to retreat that energy for practical uses, so how to find out that energy stored in the capacitor, it can be very easily done, suppose you have a capacitor like this, see and here, you have connected a; suppose current flowing through the capacitor is it and voltage across the polarity convention must be followed it.

And this is suppose the voltage across the capacitor is v, so it is equal to cdv dt, is it not, current through the capacitor is cdv dt, let at t equal to 0, is the v0 minus was 0, there was no voltage across the capacitor, so v0 minus is equal to v0 plus t that is fine is the same as for all practical purposes v0, voltage at t equal to 0 is this. Now, therefore if that be the case, then how to calculate the energy stored.

So, instantaneous power at any time t will be voltage across the plate of the capacitor into current, cdv dt, is it not, this is the instantaneous power and this becomes equal to cv dv dt, instantaneous power, therefore power and see the direction of the current and polarity of the voltage is such that it is absorbing power, so power; instantaneous power absorbed, instantaneous power absorbed is equal to this.

Therefore, energy absorbed by the capacitor in time dt, very small time interval dt will be pt into dt; power into time gives you the energy that is equal to dw and this will be equal to cv dv dt into dt, so dt goes, so this is equal to cv dv, therefore the total energy supplied to the capacitor, so it is now an integration to be done with respect to voltage across the plate of the capacitor. Suppose, the voltage across the plate of the capacitor change from 0 volt; v equal to 0 volt to v equal to some volt capital V.

So, this will be v dv and this will become equal to 1/2 cv square, in general if you know the instantaneous value of the voltage across the plate of the capacitor at any given time t, then at that time, the energy stored simply will be 1/2 c into that instantaneous voltage square. Therefore, a capacitor is the energy storage device; energy storing device just like an inductor and what we have concluded is that voltage across the plate of the capacitor cannot change instantaneously and voltage will be continuous whenever you do some switching. I stop here, we will continue in next time.