Network Analysis
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Lecture – 15
Charging of a Capacitor – Voltage, Current and Energy during Charging

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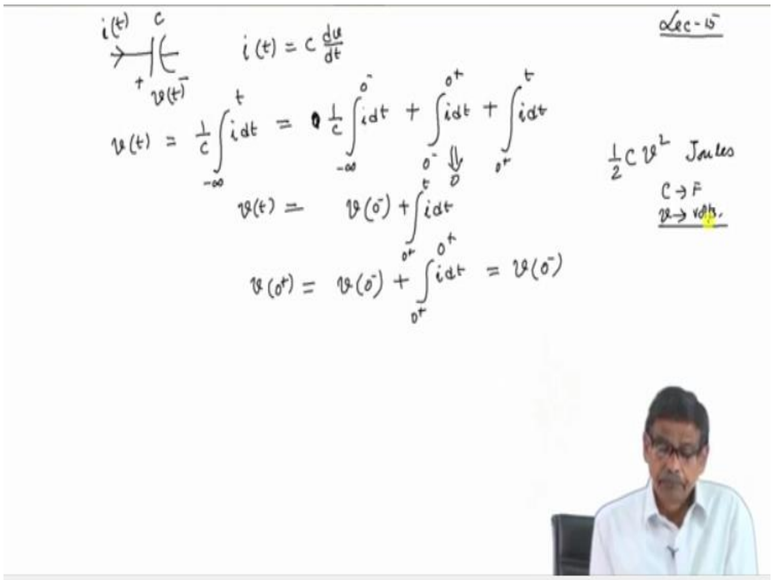

So, welcome to lecture number 15 where we are discussing about the terminal voltage current relationship of an inductor and as I told you that a capacitor you always show direction of current is your prerogatively choose that once you choose that then show the voltage across the plate of the capacitor to be like this. I am writing vt here because no other elements present generally, in a circuit if other elements are present, I will write vct to indicate capacitor voltage anyway.

So, this is the voltage across the plate of the capacitor, we will not be writing it down in terms of q and this is the capacitor, see that be the case then, what happens is this it is equal to cdv dt that will be always true and second thing is a general expression of the voltage across the capacitor is equal to 1 over c and minus infinity to t idt. So, this thing that integration of 1 over c minus infinity to 0 minus idt, how you behave to with the capacitor with currents from minus infinity to 0 minus that is important.

And then 0 minus to 0 plus for a reasonable current source that will be 0, then I will; that is 0 minus to 0 plus whatever is there that vanishes in fact, for a reasonable current waveform and then from 0 plus to t idt, this is a very famous and important equation, every bit of it you try to understand and then this being 0, this whole thing; this previous history will be encrypted in this number v0 minus and that is all.

And then, plus 0 plus to t, present current that you are giving and this 0 plus now can be treated as 0 for all practical purposes, so this is the voltage across the plate of the capacitor at any time t, initial voltage plus new current that you are injecting into the plates of the capacitor that will decide the voltage across the plate and obviously, v0 plus from this equation also as you can see, v0 plus will be the v0 minus plus 0 plus to 0 plus idt.

This eventually, has to be 0, limits being same, so this is equal to v0 minus in absence of any impulse current source, so these 2 things are important and then I discussed about the energy storing capability, if at any instead of time, the voltage across the plate of the capacitor is v, then the energy stored is this much Joules, if c is in farad and v is in volt, so this you must remember, so this is the thing.

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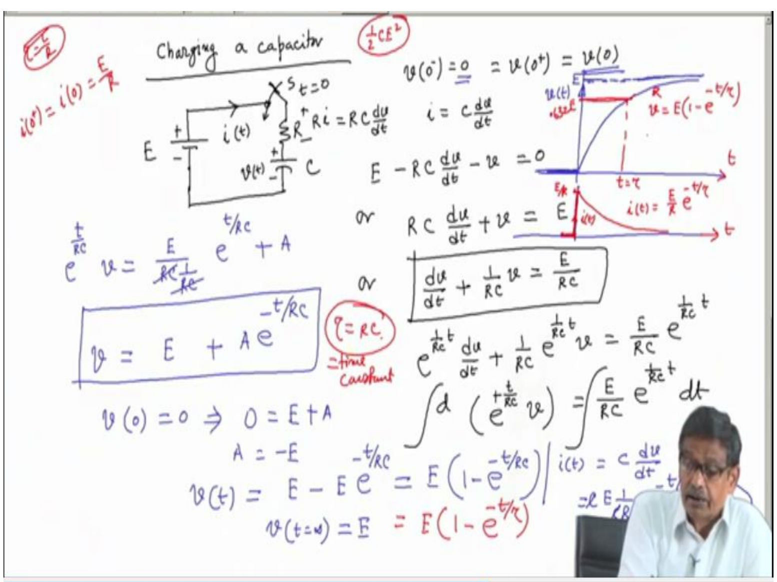

Now, we will go a bit faster, similar treatment we will do to a capacitor as we have done in case of inductive circuit. First thing; charging a capacitor, okay, to charge a capacitor means what;

that I; why should I charge a capacitor? To store energy, that should be understood, so one of the circuit is like this, this is suppose the battery, E volts plus minus ideal battery, then you have a switch here with R and C; RC charging we call it.

And the capacitor is initially it was uncharged, let us assume I never applied some current into the capacitor, never passed any current through the capacitor, so v0 minus was 0, which eventually means that this is equal to v0 plus and this is for all practical purposes, this is v0. Now, I will close this switch S at t equal to 0, that is the thing, if I do that then I expect there will be some EMF I have applied and there will be some current flowing into the circuit at any time t.

Suppose, this is it and if current flows, voltage will build up across the plate of the capacitor and these voltage, let me write it as vt capacitor voltage, then I write down the KVL, once you do that this voltage drop is add into i, i is coming like this, so Ri and I, we know it is nothing but cdv dt, so this voltage drop across the plate of the capacitor will be RC dv dt. Now, the KVL equation, I start from this point go here, there is a voltage rise, E minus to plus from this to this, there is a voltage drop minus RC dv dt Ri.

And from this to this, there is a voltage drop, plus to minus, I come to this point, then back to these, there is no drop here and this must be 0 that is the KVL, see KVL equation, KCL equations are so nice, it will always be valid, no matter whether you are dealing with constant, voltage constant current resistive circuit, RLC circuit, time varying current voltage situations, at every instant, it will satisfy, so that is the key.

So, this once again I will take it to the right hand side and write it like this, is equal to E or dv dt if you do divide by RC v, I will do it a bit faster, you also are exposed to this some point or other in your first year course, so this is the fundamental differential equation, once again it is a linear differential equation first order, so integrating factor you multiply will be that is 1 over RC into t dv dt plus 1 over RC e the power 1 over RC into t v is equal to E by RC.

This will be the thing and this is nothing but it is a product of 2 functions differentiation of that 1 by t by RC into v is equal to E by RC, is it correct? No, I have multiply both sides with 1 over

RC into t, so it will be e to the power 1 over RC into t, e to the power 1 by RC t is the integrating factor, so you multiply with this. Then, this dt you bring it to the right hand side and write it like this, then integrate both sides.

So that the integration will become on the left hand side, e to the power t by RC into v is equal to this integration will be e to the power RC over there, this is a constant by 1 over RC, e to the power mx by n, so 1 over RC into e to the power t by RC and plus a constant of integration, this is how this will be but my goal is to find out vt, so I multiply both sides by, e to the power minus t by RC to make it v on the left hand side.

And this will become E, these 2 cancels out and it will then become e to the power t by RC to the power minus t by RC 1 plus A over A into e to the power minus t by RC, so this is the solution, voltage across the plate of the capacitor at any time t. Now, you should not be under the impression that to charge a capacitor, I have to connect a current source, this somehow, you have to fit some current.

And we find okay, it flows whether it is coming from a battery or a current source, it does not change the very concepts; current fed into a capacitor will build up voltage, like that, so this is the thing. Now, how to determine this constant; to find out the constant, I will apply this thing that v0 is equal to 0, so v at t equal to 0 is equal to 0 will give you 0 is equal to E plus A, t equal to 0, this will become 1.

So, A is equal to minus E, therefore voltage across the plate of the capacitor will simply become E minus E into e to the power minus t by RC or people write it like this that 1 minus e to the power minus t by RC. Now, if you sketch this waveform, it will be like this, voltage across the plate of the capacitor, vt, so before that voltage across the plate of the capacitor was 0; v0 minus and suppose you never applied any current to the capacitor therefore, it was 0.

Suddenly, here you have switched on this battery, so some current started flowing, charging the capacitor and voltage across the plate of the capacitor will build up like this and at t equal to infinity, v at t equal to infinity will become E because this fellow will vanish, e to the power minus infinity tends to 0 although, it will of course asymptotically meet this final voltage; final voltage across the plate of the capacitor will be like this, here E.

Once again, the capacitor will be charged to the supply voltage E, no doubt and if you wish you can find out how current changes with time, so to find out current what you have to do is this, it will be equal to c dvdt, so differentiate this voltage now and you will see that it is equal to E and you are differentiating so, c dv dt, so c was there now, I am differentiating, so E and there is a minus here, this minus will come out, minus 1 by RC it will make it plus and it will be 1 over CR and e to the power minus t by RC, this will be.

And this goes cc, and it becomes equal to E to the power R; E over R e to the power minus t by RC, this will be the expression of the current that is how it will change, so if below these, if you sketch the current waveform, it will be like this, current was 0, earlier nothing was there, so current at t equal to 0 E by R, so there is a sudden jump in current. What is the magnitude of the current?

E by R and after that as time passes, this exponentially decays down to 0, with the same time constant as voltage builds up now, I have not defined time constant yet but I can say that voltage across the plate of the capacitor is like this, then I will say that it is equal to E 1 minus e to the power minus t by tau, where tau is equal to 1 over RC is called the time constant of the RC circuit.

At t equal to tau, the voltage to which the capacitor will charge will be that same thing; 0.632 into E at this point, may be after 2, 3, 4 time constant it will become fully charged, you need not do it for t equal to infinity for making the capacitor fully charged, okay, so this is the thing. So, finally the voltage across the capacitor plate will be capital E. Now, 2 points are to be noted here, very interesting point.

One is that at t equal to 0, current can jump, current have a jump, you see sharp job which is not allowed which is not allowed means, it should not happen in an inductor, current should not jump in no time, in that case diL dt will be infinity, very large voltage will appear all these things happens in inductor that is why current jump should be avoided in an inductor; sharp jump in current vertically.

But in a capacitor, no problem, current can jump, but voltage cannot, if voltage jumps abruptly from 0 to some finite value that means, current required will be infinitely large, sort of impulse current, we will address those problems later but here capacitor in general it will be like this. Now, you see that at t equal to 0, current at i0 is equal to E by R. I mean that is i0 plus is same as i0, it will be E by R.

As if capacitor is not there, there is a short circuit, current in the circuit is; the circuit at t equal to 0 as if behaving like a resistive circuit E by R, so some people say capacitor behaves like a short circuit, although it should not be told like that for this particular case, it happens like this, capacitor behaves like a short circuit with uncharged capacitor, it will always do like that. If there was no voltage across the plate of the capacitor at t equal to 0 minus, then you do some switching, the voltage across the plate of the capacitor still will remain 0.

Therefore, it can be treated as a short circuit but if the capacitor had some initial voltage, then that voltage will be preserved, then we should not be treated as a short circuit, are you getting, so anyway, so E by R but later it is not, capacitor will have some voltage drop and these 2 voltage drops will give you v, so that is what it is telling that is one point. So, some people say an uncharged capacitor initially, in a circuit if you do some switching, at t equal to 0, if you are interested to know the currents etc., then you do not have to solve differential equation.

Put a short circuit across the plate of the capacitor, solve it, this equation tells that very interesting thing. Achchha, second thing is and finally, you know the current in the circuit is 0, at t equal to infinity, current will be this axis I forgot to mention, always this is time axis, whenever you sketch something, mug the axis very clearly, so this is time axis at t equal to infinity, current in the circuit is 0.

Current in the circuit is 0 means there is some open circuit somewhere because there is a voltage, there is a resistance, so capacitor plate will become behave like an open circuit, okay, so capacitor behaves like an open circuit when steady state is reached with a DC voltage here and it behaves like a short circuit at t equal to 0 that is one thing. Second interesting thing is that if you keep this switch on for several time seconds, you may be just assured that the voltage across the plate of the capacitor will be E and there will be no current flowing in this circuit.

So, capacitor will be fully charged to the supply voltage, hence it will store energy, what is the final energy stored; it will be 1/2c E square, is it not, now one interesting thing I would like to point out here, so capacitor finally stores energy which is equal to 1/2c square and during this charging procedure, this axis is it, this is it, so if you write down this equation, better let me write, so that this voltage, this curve is v is equal to; use Matlab and get this curve, 1 minus e to the power minus t by tau.

And this is it, how current varies with time, it is E by R into e to the power minus t by tau; tau is the time constant which is; oh! I made a mistake here, be careful, t is; tau is RC, product of RC, please correct that, product of RC is the time constant, not 1 by RC and in case of inductive circuit time constant was L by R, you recall that anyway, so this is the thing, so tau is the time constant.

Now, while charging the capacitor one interesting observation I would like to share with you, when I am going through this part of this lecture that okay, while charging the capacitor I am spending some energy in this resistance which is connected in the circuit, is it not because during the charging process, current is flowing through the resistance, therefore i square R loss taking place in the resistance.

So, to charge a capacitor to a voltage E, you are also spending some energy in some resistance, is it not. Now, let us calculate how much energy we are wasting in charge because power in a resistor is a wasted power, it cannot be recovered, it is wasted as heat to the environment. Now, the question is how much energy we are; energy; let me write it like this.

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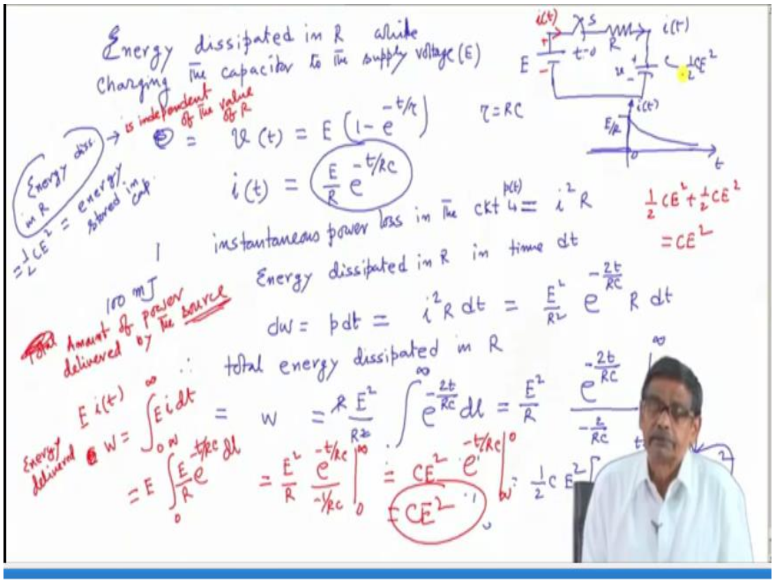

This is very interesting observation; energy dissipated in R while charging the capacitor to the supply voltage. In this case, it is E, so I redraw the circuit, this is E, so that it will be easier for me to talk, this is c, is it not, you have closed this switch at equal to 0, this is s, this is E, this is R, this is c, and we got that E is equal to sorry, v; voltage across the capacitor plate is this one, tau is RC and the current in the circuit it, we have got to be E by R into e to the power minus t by RC.

These are the 2 (()) (25:49) things similarly, in case of inductor we have found out current in the circuit and voltage across the inductor as a function of time, now anyway after we get this, now let us calculate the power; instantaneous power loss in the circuit is equal to i square; i square into R that is equal to pt, if you write pt is equal to this. Now, energy dissipated in R in time dt will be dw is equal to p into dt, we have done it several times earlier.

And this will be equal to i square R into dt, now this i square will be E square by R square, I am putting this thing here, i square E square by R square into e to the power square, so it will be minus 2t by RC in a i square into R into dt, therefore total energy dissipated in R will be equal to w, will be equal to E square by R square, this R will also come also outside and you have to integrate this e to the power minus 2t by RC into dt.

And this integration is to be carried out from t equal to 0 to infinity, recall this current was like this, this is E by R and this is it and this is your time, this is 0, so this integration, this one are goes here, it will be E square by R and this integration is e to the power minus 2t by RC, so e to the power mx by m, so this will be minus 2 by RC, is it not and the limits of integration is 0 to infinity for t, limits are 40.

So, this will be the thing and this negative sign you can do away with by changing the limits, so it will be E square by R was there here and this will be RC by 2 and this will be e to the power minus; you do on your own, I am doing it in my own way, this thing I have taken into account, so nothing is there, here 1, so this negative sign I have removed, so I will write it is as 0 to infinity like this, okay, limits I have changed, negative sign goes.

So, this will be this R goes, it will be equal to 1/2c E square and what is this integration, I mean this limit if you put it is equal to 1 minus 0 because at t equal to infinity, e to the power minus infinity that is 0 and t is equal to 0, it is 1, so this result eventually it is 1/2c square, got the point. Therefore, to charge a capacitor to a voltage capital E you have made a circuit like this, it will finally store how much energy; 1/2c square that is known.

But we also observe that the energy dissipated in R is also 1/2c square, got the point for example, if I say in language see it is how interesting it is, I am telling that okay, this is here is a capacitor store 100 milli Joule into the capacitor, this result tells you that okay, you make a circuit like this, if you want to store 100 milli Joule; 100 milli Joule should be also dissipated in R that is you must be ready with 200 milli Joule.

And another interesting thing is the amount of energy dissipated in R, this quantity, so 1/2c square, this energy I write it like this; energy dissipated in R is equal to 1/2c square which is equal to energy stored in R, energy stored in 1/2c square is independent of the value of R that is very important, is independent of the value of R, no matter what value of R you are connecting to charge a capacitor to this battery voltage E, this much of energy will be dissipated in R.

It is independent of the value of R that is why we say that if you want to store 100 milli Joule in a capacitor, 100 milli Joule will be also dissipated in the resistance and the amount of power delivered by the battery, amount; this can be easily corroborated by this one, amount of power we know, who delivers power battery, this is plus this is minus, it is like this, so it is delivering power into the circuit, amount of power delivered by the battery, by the source is how much; voltage across it into it.

And a total amount of power is this, so amount of energy delivered; energy delivered will be w will be nothing but Eidt, energy delivered by the battery source and from 0 to infinity now, this integration if you carry out, E it will be 0 to infinity, here no i square, i is what; i is E by R into e to the power minus t by RC dt and the this one will then become; this I can rub know, it is there.

So, this one will be equal to how much; it will be equal to E square by R and this integration will be e to the power minus t by RC by minus 1 over RC and integration limit is 0 to infinity, so this will be equal to you know, these R, R goes c square and this is e to the power the same thing minus RC, limits you interchanged to take care of this negative sign and it will be equal to c square and that is true.

Because energy stored is 1/2c square must have been supplied by the source and energy dissipated in R is also 1/2c square must have been supplied by the source therefore, energy supplied by the source should be 1/2c square plus another 1/2c square that is equal to c square that is what we have got. Therefore, you see these are the interesting observations one must been that is you to charge a capacitor with certain amount of energy to store some certain amount of energy, x joule; you must be ready that you supply; should supply to x joule.

And that joule if you have think that I will vary R and may be energy dissipated in R will be less; no, that amount of energy will be dissipated in R, independent on the value of R, whatever final energy stored that amount has to be dissipated in R, so these are the observation; interesting observations you should make whenever you do some circuit analysis, energy point power point, is very interesting and important tool.

It will make your life easier to understand and analyse the results obtained in circuit analysis, thank you very much go through it carefully, next class we will do discharging of capacitor, thank you.