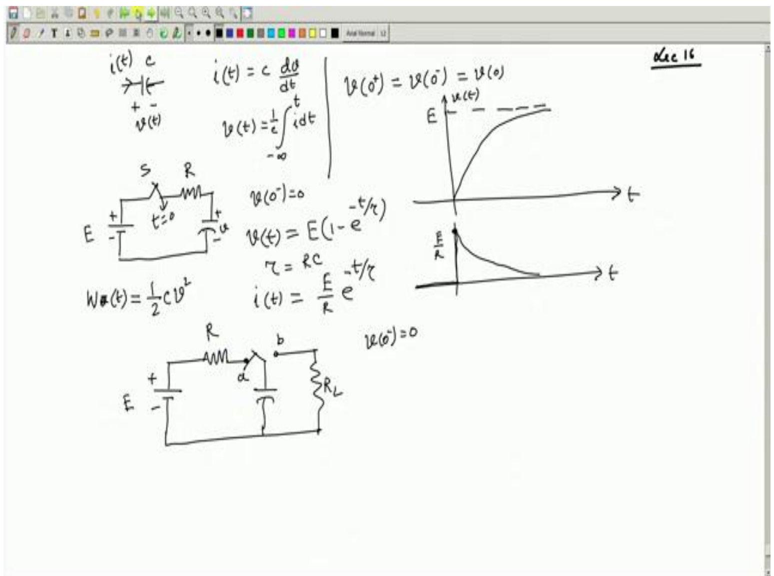
Network Analysis

Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology – Kharagpur

Lecture - 16
Discharge of a Charged Capacitor

So we were discussing about the capacitors. Each terminal relationship, voltage current relationship.

(Refer Slide Time: 00:20)



This was the thing if you indicate the voltage across the capacitor with this polarity. So the current also in this direction and if this is C then it see this the differential relationship C dv/dt is the thing and also if you exceed the capacitor with some current source from this equation only we found that Vt at any time will be - infinity to t into idt this will give you q/c. This is also the expression of the voltage.

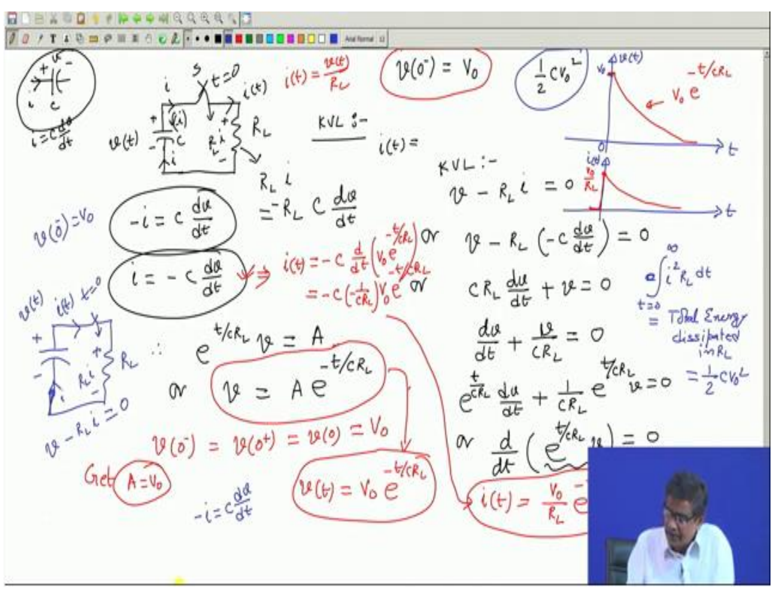
And then we have shown that if this excitation current source is reasonable function. Then voltage at t = o+ will be same as voltage at t = 0-. That is voltage at t = 0 using this then we solved this circuit in last lecture. That is if there is a voltage source battery and if you have a switch and you want to charge a capacitor. This is one of the simplest circuit you do it and if the capacitor is initially uncharged that means V0- o = 0 and you are doing this switching at t = 0.

This is the switch then the expression of the voltage across the plate of the capacitor that is this voltage at any time t was shown as e to the power -t / tau. Tau is the time constant which is the product of R and C. Similarly the expression of the current at any time t will be E/R and it will exponentially decay and finally steady state current will vanish these things we have seen and the energy stored in the capacitor at any time t will be of course equal to half CV square that instantaneous value of the voltage.

In this case of course the capacitor will finally charge to this voltage that is E. This is Vt and the current if you sketch this is t current will go up suddenly to a value E/R. This level is E/R this one and then it will exponentially decay and finally no current can exist in this circuit. However, capacitor will get charged. Now today we will see how this energy can be retrieved from the capacitor.

See so for this what I will do is this, this is your initial circuit I will just redraw it in a different fashion. This was the capacitor and here you want to retrieve the energy stored in the capacitor in a load resistance R. This is plus minus E so when you connect the switch when the switch was in the neutral position neither connected to a and b. There was I will presume V0- was 0 earlier. It was 0 but if you close it here then the capacitor will finally charge with the voltage capital E. If you wait sufficiently long.

(Refer Slide Time: 04:30)



So my next problem I will without much saying I will state the statement of this problem is like this. You have a capacitor which has been and its voltage across any time I will indicate by this polarity and this one and I will say this capacitor has an initial charge is equal to some value okay. Some finite value it has been charged by some other circuit and let this value be V0 suppose I say. So capacitor is charged like this and I will make a circuit this way.

This is the discharging of a capacitor. This is my load resistance and although there is apparently no active source present like battery or current source. But mind you if you close the switch at t = 0 then this energy will be dissipated in RL. So we started with the energy half CV0 square. So it will act as a source okay. And we want to find out how the current in this circuit will flow. And what will be the how the voltage across the capacitor?

As it discharges you know it will lose voltage. How the voltage decreases? So anyway to solve this circuit now I will say that at t = 0 you have close this circuit and let at any instant of time, the current is it in this circuit. The moment I show that I should not forget to show the polarity of the voltage drop across RL. It will be plus minus like this and voltage across the plate of the capacitor is like this.

Therefore I will apply KVL in this circuit okay. So this is the current I have assume and I will then say this voltage to be plus minus like this V0 and okay that is fine. Now I will write down the KVL. What is KVL? So it is mind you here if you look let it be like this I thought I will change it suppose I have done like this. Then I have assumed the current to be like this it. Then you recall that this is correct plus minus if current enters here.

This is c and this is v then i = C dv dt. Therefore here the current is leaving the positive terminal or I can say that -i is entering through this place is not this is the one and the same thing. So I should write -i. This step is crucial -i to be consistent with this polarity of this voltage current relationship of a capacitor. So -i is really entering and these I can write it to be C dvdt. This is very important to understand.

You should be consistent with this. Since I have assumed the current to flow out like this, no problem. But then while writing i = C dvdt, you must write it -i is entering through the positive terminal of the capacitor. So that current is C dvdt okay. Now so I will write down the KVL equation. Now in this equation. what is that? This will be you start from anywhere you like. For example, this will be this drop RL into i will be nothing but RL into i is -C dvdt.

So RL into C dvdt with a negative sign like this. Then you start your journey from this point say - to + Vt KVL you write KVL will be this to this. I have defined it to be this polarity minus to plus then with this -RL into i is not plus to minus and I have reached this starting point that is equal to 0. Then what I do? I substitute the value of i this is the value of I so it will be V -RL and for i, I will write -C dvdt and that will be equal to 0 or you will get a C into RL into dvdt + V = 0.

And then dvdt + 1/CRL RL I have not written R simply to tell you that I am using the energy in a load resistance that is all. It could be R as well these equal to 0 into V. So once again a first order differential equation and the solution will be you multiply with the integrating constant CRL 1/CRL dvdt + of 1/CRL into e to the power t/CRL into V = 0 or I will say ddt of e to the power t/CRL into Vt = 0. Therefore this fellow must be a constant.

Therefore e to the power t/CRL into V must be a constant to be determined from the boundary condition or I will say voltage across the plate of the capacitor should be A into this is the integrating constant okay or constant as you can see -t/C into RL. This will be the solution so this is how capacitor voltage will decrease it will decrease exponentially. Now the question is how to determine the value of A? Value of A is to be determined from this boundary condition.

Since V0- = V0+ = V0 and that is given to be V0. Therefore by substituting these boundary condition you get A to be = V0. Because t0 this is one and V = V0. So this will be then A is V0 into e to the power -t/CRL. I am sorry I should not write it like that. So this is equal to V0 fine. So from this you get so solution then becomes Vt = V0 from this equation into e to the power -t/CRL. This will be the voltage across the plate of the capacitor at any time t.

And what will be this current it? it once I get Vt I will use this formula to get it. So it at any instant of time it should be -C into ddt of V, V I have got this. So put that value V0 into e to the power of -t/CRL is that this will be the thing. And if you see this one it will be -C was there then differentiating so -1/CRL will come out. So -1/CRL is that into V0 and e to the power -t/CRL. So the expression of the current will be then it = this e goes minus minus plus so it will be V0/RL into e to the power -t/CRL.

Of course without differentiating I could easily get it. Because you see the voltage across the resistance is same as voltage across the capacitance V. So V/RL V is V0 into e to the power -t/CRL/RL will give you the current is that, that is it is nothing but Vt/RL will also give you the same expression. Therefore the voltage across the capacitor will decrease exponentially and if you sketch that you will get this way.

If you sketch always make a happy to sketch the transient condition voltage and current in the circuit. So capacitor voltage was initially it was t0 - it was some V0, it has to start from this capacitor voltage cannot change instantaneously and then this voltage decreases exponentially whose equation is V0 into e to the power -t/time constant CRL. CRL is the time constant here.

Similarly here the below this with the same time scale if you sketch the current it in the circuit current will be, earlier there was no current t0- it was open. So current was 0 then suddenly it goes up in a capacitor current cannot have a jump whose values will be V0/RL and then this fellow also decreases and for obvious reason can you tell me what will be the value of this integration? That is i square RL dt t = 0 to infinity.

Suppose somebody integrates it. First of all, what this physically means? It means the energy total energy dissipated. This expression is nothing but total energy dissipated in RL and I am not calculating this. It must be equal to half C V0 square. You started with this energy only this much energy can be dissipated clear. Therefore, total energy dissipated in RL is this one. No matter what is the value of RL?

This much amount will be dissipated in RL. Therefore, we have considered how to charge a capacitor. Then how to retrieve that energy from this capacitor to for practical use. Of course, this is not a very good user. If you connect a lamp, the lamp will glow. It will get dimmer and dimmer as the voltage across the lamp decreases. But nonetheless some mechanism I have could be found out by which it can be done charge a capacitor to some voltage and then retrieve the energy from the capacitor when you need it.

We will see much more smarter circuit for using the stored energy in a capacitor at a later stage. For the timing this you must very clearly understand. In this case of this circuit once again I am telling this point to be repeated. How did I do it? I told this point that negativity why it comes? You remember the terminal relationship of a voltage across the capacitor at any time t and the relationship of current if you assume in this fashion then only i = C dvdt.

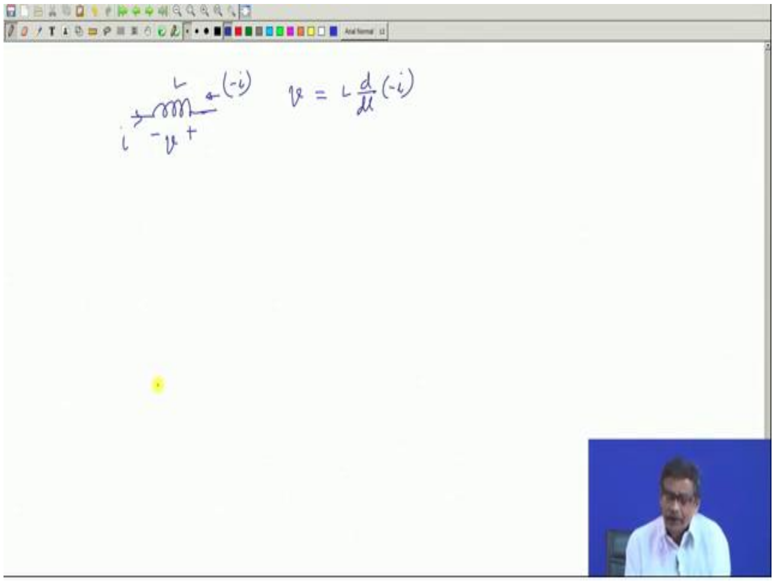
Now in this problem what I have done? I was shared with this capacitor I will show with a curved one plate. So that at no point of time you confuse it with a battery. So this is a capacitor and its initial charge and voltage across the plate of the capacitor I have decided to be like this Vt and I told you that V0- was equal to some V0 this point I am repeating. Now here is a switch S and I have connected a resistance RL and at t = 0 this is closed.

Then at any time t, I am free to choose the direction of the current do not mind. So it but once you choose this it, you cannot play with this polarities. This is fine RL into i, what is the KVL in this loop? KVL in this loop, you start from this minus to plus Vt. I have assumed with this voltage rise. Then there is a voltage drop that is -RL into i and you have come back to same point. This must be 0 this is the KVL.

Now the question is I have to form a differential equation involving VE only. Differential equation I want to form. Now the question is what is the relationship of the current and the voltage? Now here you concentred this i is like this I have assumed. So what is the current entering the positive terminal of the battery? I must say it is -i and this -i will be C dvdt. That is the point to be noted and that you put it here and then proceed.

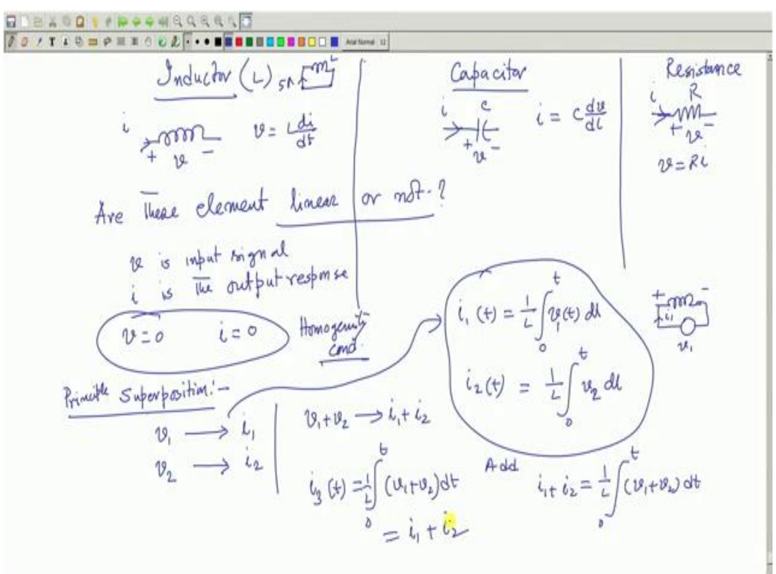
I hope you have understood this point very carefully so this is the point. Similarly I mean similar thing maybe suppose somebody has I mean an inductor.

(Refer Slide Time: 22:25)



I say this is the current okay and somebody told that I will actually be voltage across the inductor to be like this. This is L is absolutely fine. You can choose your direction of the current or voltage. But once you do that you cannot play with this. Then I will say okay through the plus terminal -i is entering. Then the terminal relationship of these should be V = L of ddt of -i. I think you have got the point.

(Refer Slide Time: 23:12)



Now anyway so capacitor charging, capacitor discharging we have considered. Similarly inductor charging and inductor discharging we have considered. And there will be as you know several problems interesting problems involving L and C can be formulated. I will highlight some problems maybe in the next lecture. But another important thing with RLC with LC particularly I would like to point out.

Before that so I have got inductor suppose I write it here inductor L and on this side I will write capacitor. See this is what the terminal condition of an inductor is +-V and V = L didt while writing the forming the differential equation. We will write differential equation not integral differential equation better because we will be better solving a pure differential equation. That is why this notation.

For capacitor it was like this plus minus so the current like this i, C and voltage the relationship i will be equal to. This is how i and V are connected in a capacitor in an inductor. Now we ask ourselves and considered a resistance also resistance that is R, this is i, this is voltage and V = Ri. Here no differentiation that is why it is memory less this has got a memory it requires previous knowledge of voltage and this has also got a memory that is fine.

Now I will use a term called whether I would like to investigate whether these 3 elements are linear or not are these elements linear or not okay? To test the linearity of a particular element what we have to do? We have to see that whether super position principle is satisfied. If it is satisfied then I will say it is linear. Now how to apply the superposition theorem? Superposition theorem tells that if you do not apply any signal to this element.

By the term any signal means I have not applied any voltage. Suppose for an inductor V is input and i is output I say. V is input signal and i is the response is the output or response. This point you listen carefully. The question is if you do not apply any input signal then this output i must be 0 okay. That is the first thing for superposition V = 0 i must be 0 then what you do is this.

This is called principle of homogeneity for any linear system for example in a resistance applied voltage is 0 i is 0. So obvious i = V/R so V = 0 i = 0 but here if V = 0 i may not be 0 why?

Because it is differentiation an inductor can sustain current when it gets short circuited without loss of any principle is that. If i is 5 ampere constant in an inductor, how it can be? If I say this is L and somebody draw the circuit like that 5 ampere is it possible? Yes very much it is possible. Most probably you have charge the inductor from some source and kept it shorted. This we explained earlier therefore even so what is the voltage applied between these 2 points of the inductor? 0 but still there is current.

Therefore an inductor with initial current cannot be linear okay. Because it will fail this test V = 0, i may be present in an inductor. This you must think about and this equation also tells you V0 does not mean i is 0 i may be a constant ddt of a constant thing is 0 including that constant value to be 0.

Therefore this is homogeneity condition. This must be satisfied. And the next condition is that so this is required. If you test this and this is not following then you say no it is not an linear element no other test is necessary. But if it satisfies this then also you test for superposition. Principle of superposition what does it say? It says that if you apply a voltage V1 across the inductor and it gives a current of i1.

Then if you apply a voltage of V2 and separately it gives a current of i2 V1, V2 are different. Then superposition principle demands that when you apply V1 + V2 the current in the circuit should be i1 + i2. If this is true then I will say inductor is a linear device. Of course inductor will fail this test, the first test if it has got initial current. Therefore for testing an inductor for linearity you must ensure there is no initial current in the inductor got the point. The proof of this is very simple.

When you apply V1 voltage see let us follow this principle. I really write it like this, response in an inductor current i1 t you apply V1 only this one V1 alone. So I will also draw the circuit so that no confusion remains. You have applied the voltage V1 this current is i1 so i1 will be equal to 1/L -infinity to t or say -infinity to 0- is 0. It has to be, so I will write it then 0 to t is not Vdt. Because I have assumed previous to that no voltage has been applied.

So this is the thing I will write or you can write -infinity also let me see. So when you have applied V1 voltage this will be the current in the inductor. When you apply a voltage V2 the current in the inductor will be V2 dt/L 0 to t say and this will be equal to i2t. Now you just add these 2 equations add and you get i1 + i2 will be equal to 1/L 0 to t and these 2 will added up. But this current i1 + i2 will be the current when you have applied V1 + V2.

So when you apply V1 current is i1, when you apply V2 current is i2, when you apply V1 + V2 to where I do not know I write current will be i3 t and I will write it like this 0 to t V1 + V2 into dt. If you separate these 2 integration, it will give you i1 + i2. Therefore the conclusion is an inductor without any initial current will be a linear device got the point. Similarly we will see that a capacitor without any initial current will behave like an linear element. We will consider this in the next class. Thank you.