Network Analysis
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Lecture – 20
General Method for Solving Differential Equation - III

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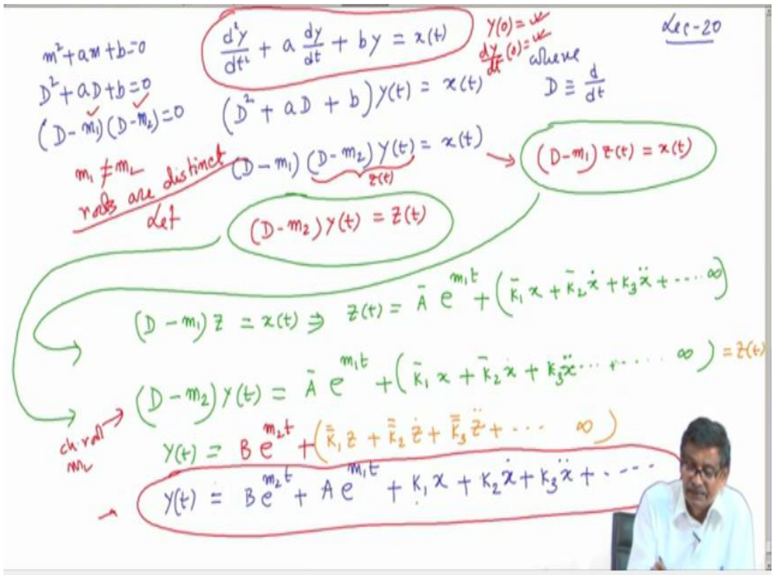

Welcome to lecture number 20 and as I told you will now see higher order linear differential equation constant coefficient and how to solve those equations, so let the equation be like this; plus ay dy dt plus dy is equal to xt, once again xt is the imported could be a any reasonable function, y is the output of the network. Now, first of all what we will be doing is this one; this left hand side I can write it like this; D square plus aD plus b into yt is equal to xt, where D is the operator; ddt, I can write like this.

And this can be factorised, is it not, I can find out the root of this equation that is characteristic equation here will be m square plus m plus b is equal to 0, characteristic root decides the natural response anyway I will write it like this, so suppose the roots of this equation is characteristic equation or this equation plus b is equal to 0. Suppose, I can factorise them after finding out the root and say that it is equal to the minus m1 into D minus m2 is equal to 0, I can write it like this.

So, this left hand side then can be written as m1 D minus m2 into y is equal to xt, okay, now this second order system can be thought of as 2 single order system, how let us see. Let, I will write it like this, let D minus m2 into yt be some zt, it is a first order differential equation and yt of course, I do not know yet but if D minus m2 operates on yt, it will give rise to some function of time and that I am telling get t, this one.

And then I will say D minus m1 into zt is equal to xt, got the point, so this second order differential equation has now being broken up into 2 first order equations, one is this and the other is this, these 2 terms, this is very interesting, zt I do not know yet but anyway, so you see it is from this equation, it is a first order differential equation, only in place of y, it is zt, it is a first order differential equation.

Therefore, we first tackle this differential equation D minus m1 into z; zt is equal to xt, this I will write, so what will be zt; zt I find out from this and then I will substitute it here to get the value of yt, my ultimate goal is to find out yt, so from this differential equation and this first order differential equation I know what will be the nature of the solution, zt; the total solution will be the characteristic root; characteristic root is m1.

So, some A into e to the power m1t plus k1 say constant into forcing function is x; x plus k2 x dot plus k3 x double dot up to infinity that is what we have learned, it is a first order differential equation and zt must be this, so zt can be obtained and if zt is known, then I will come to this equation which is another first order system, it says that D minus m2 into yt and right hand side is forcing function, which is this one, this whole thing that is A bar into e to the power m1t plus k1x plus k2 x dot plus k3 x double dot up to infinity.

So, it is also a first order differential equation, where from I will get yt and the forcing function looks like this and it is also a reasonable function, only thing infinite terms, let it be there who cares because I know, I can write down the solution for Whitey in one stroke lead over the solution of this difference 70 L is part of the solution for yt in one stroke, what will be the solution of this differential equation?

There will be a natural part of the solution and that part of the solution one will be some B into e to the power m2t because characteristic root is here, all the m2; e to the power m2t plus; plus what; plus; I will first write, zt is this one, so it will be some say k1, let me use different colour for solution due to forcing function, say this will be equal to some k1 double bar k plus because this is zt, so k2 z dot plus k3 double bar z double dot and so on up to infinity that is what we have learned, this will be the thing.

Now, I have to put for zt, this one is zt, is it not, these whole thing on the right hand side, so what it will be; it will be then the solution will be B into e to the power m2t that is fine now, this one; this whole thing you have to multiply with this k1, if you differentiate this one first equation, what you will get; this first term z dot will be A into m1 into e to the power m1t, so you differentiate this equation.

So, if you differentiate this whole lot of thing to find out z dot t, z; z is there; this one, z dot t if you differentiate, what terms you will get; another e to the power m1t you will get see, e to the power exponential functions such a nice function, you differentiate, integrate, its nature will not spoiled, it will still remain e to the power mt, only thing is its (()) (09:52) factor gets changed. So, this term will be ultimately, so for us this term is concerned, I will collect all those e to the power m1t terms from successive differentiation, x dot; x dot will be also e to the power m1t.

So, I can say it is equal to A into e to the power m1t, what else, is it not, similarly z will give x, z dot will not give you any x, whatever it is, when you differentiate these z dot, it will give you x dot, so all x I will put together and that I will treat it as a constant and it will give you xt. Similarly, z dot will give you x dot, another x double dot will come here, another x triple dot but I will collect all the terms have been x dot.

And whatever constant in the bracket, I will get; I will write it a x2 dot and these way the basic nature does not change ultimately, therefore you see the solution when the forcing function; when the system is a second order system will be exactly same okay, these steps I am not going to do, I know what are the terms will be present, so we broke the second order system in fact into 2 first order system, this and this.

Then, I solve for zt; zt becomes the forcing function of this differential equation from where yt can be obtained that is after all our goal is and therefore, I have come back to this equation forcing function looks like formidable but it is really not because characteristic root of this first order differential equation is mt; B into e to the power mt, now if you put some time, you see this is this fine but z dot will give rise to x dot x triple dot.

Similarly, z double dot will also give rise to x double dot, x triple dot, so I will now collect all the terms involving x, involving x dot and those coefficients are constants, let it be up to infinity so, basic nature of the solution remain same like this. In this derivations, I have assumed one thing that is m1 not equal to m2, roots are distinct, this is the solution, got the point. Therefore, what is the thing if the roots are distinct writing down from this differential equation it is given.

And this time 2 boundary condition must be given, y0 will be given, dy dt will be given 0, so this is the differential equation, I will only find out characteristic root and straight away come to this line, those things you do not have to repeat, no point and the rule is 1 second same, the solution due to forcing function will be linear combination of x and its higher order derivative and the natural response will be A into e to the power m1t plus B into e to the power m2t in the same way, where m1, m2 are the characteristic roots of the equation.

If you have understood this in one stroke you can at least write down the nature of the solution, after you find out the solutions, then look at the xt terms, as we have done in the previous case those problems you can try, 2 boundary conditions are given, so here also this forcing function will alone satisfy the differential equation from this forcing function alone if you put, right hand side it will be xt, so equate the keep that equation to be an identity, get the values of k1, k2, k3 etc., whatever it is.

And then, write down the total solution, then to the total solution, apply the boundary condition to get A and B that is the rule, clear.

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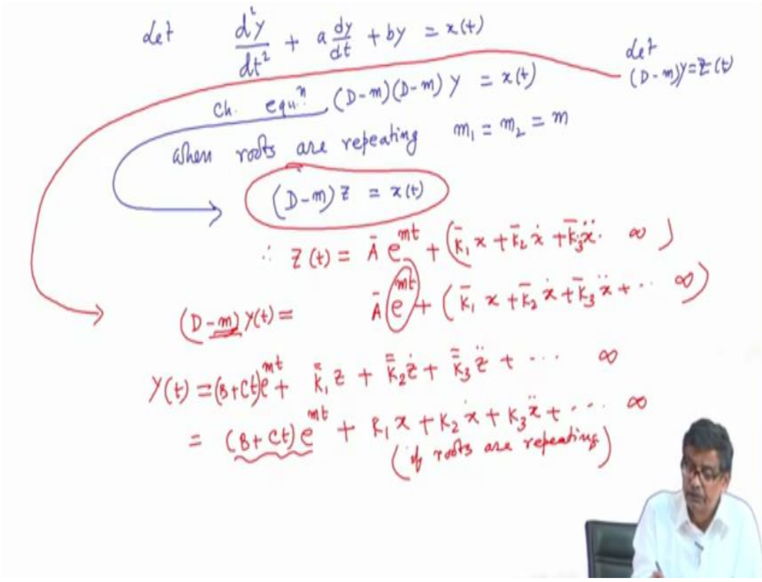

Now, only thing is what happens if this roots are not distinct, suppose I say let the same differential equation; dy dt2 plus a dy dt plus by is equal to xt that is the equation given but the characteristic equation is; it comes out to be suppose, A, B values are such that it comes out to be D minus m into D minus m into y is equal to xt, suppose this; so m1 when roots are repeating, that is m1 is equal to m2 is equal to m such a situation takes place.

Do not worry, we will go by this once again I will write this equation as D minus m and let; I will let D minus m into y to be some function of zt, I will write, so this into zt is equal to your xt, is it not, clear. So, I have broken this up into 2 equations, so first I concentrate on this differential equation, here is single root, no problem so, I will say the solution of this one will be is equal to some A bar; some constant, you could write A also, e to the power mt, characteristic root is m plus some k1 bar x plus k2, I am going okay know; k2 bar into x dot plus dot, dot, dot up to infinity, k3 bar x double dot, is it not, I am going up to infinity.

So, this is the differential equation but my target is to solve for y, so I will come back to this equation now, second thing I have to solve is D minus m into yt because yt is my goal, what will be the nature of yt and that I will write D minus m into yt as your zt and zt is nothing but this whole like A bar e to the power mt plus k1 bar x plus k2 bar x dot plus k3 bar x double dot up to infinity that is the thing we have learned.

Now, so if you remove this zt in between, this is the thing know, this is the differential equation, first order differential equation and it is a first order differential equation with a peculiar forcing function, okay peculiar but nonetheless it can be handled, it has got differentiation and so on, so this is the thing. Therefore, what will be yt; this part is most interesting now yt; total solution will be what; it is a first order differential equation, it has a characteristic root m.

So, B into mt that will be; this equal to 0, it will be this plus if you write it, you write it like this, k1 double bar z plus k2 double bar z dot k3 z double dot and so on up to infinity, this will be the total solution, is it not but you will see in this one, there is a forcing function on the right hand side whose exponential thing is e to the power mt characteristic root is also e to the power mt, therefore solution; natural solution of this one it should be modified, why?

It should be written as B plus some constant C into t into e to the power mt because we have seen earlier on the right hand side, if there is an excitation, e to the power mt and characteristic root is also e to the power mt, then the solution will be like this in a first order system; A plus some constant in root t, why some constant does not come here because you chose 1, it may be k into e to the power minus at, is it not.

Therefore, this is the point to be noted, so I will say the solution of this differential equation, if the roots are repeating will be B plus ct into e to the power mt plus this one but you know this e to the power mt can be clubbed with this e to the power mt and so on, so it will be finally some B plus Ct into e to the power mt plus ultimately, it will be like this up to infinity. So, if the roots are repeating, then the natural response will be like this.

That is one will be B into e to the power mt plus another will be Ct into e to the power mt because of the fact on the right hand side, you can easily identify the characteristic root is m, excitation is e to the power mt, therefore if you go to the basics fundamental e to the power minus mt, if you multiply it will become constant integrate, it will be some constant into t, got the point.

So, similarly this is the thing, therefore there may be 2 options coming, one is roots are distinct, so if 2 roots are repeating, this will be the thing, I will jut point out which I have not going to do.

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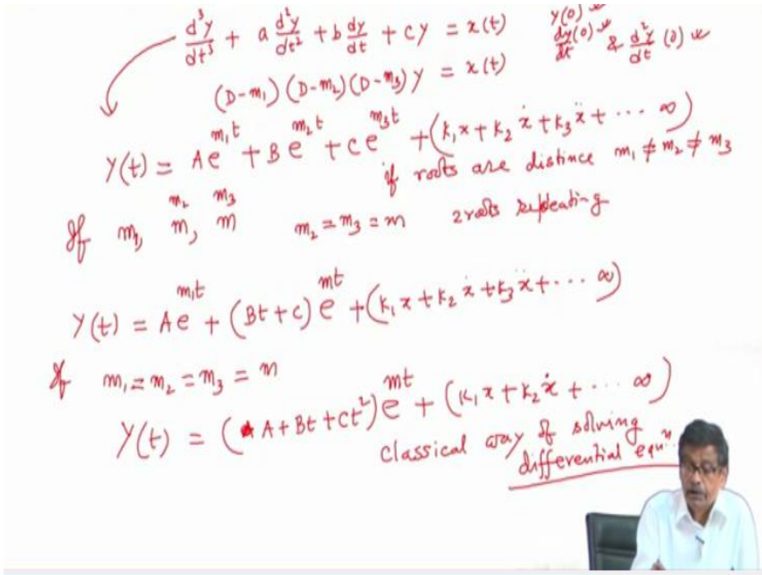

Suppose, you have a third order differential equation; d cube y dt cube plus ad2 y dt2 plus b dy dt plus cy is equal to xt. What I will do first; you first find out the write down the equation in this way; Dm1 D minus m2 D minus mt, factorise it into y si equal to xt, so this you start with z and so on, so 3 first order system you can make but whatever you do, do I am telling you, solution will be it is so nice a thing to remember, easy to remember.

It will be if roots are distinct, you will simply write okay, natural solution will be this; B into e to the power m2t plus C into e to the power m3t that is the thing and plus k1 into x plus solution due to forcing function x dot plus k3 x double dot and up to infinity, linear combination of the forcing function and so, where will you say to yourself okay, this will be the linear combination of x and its higher order derivative up to infinity.

So, knowing this differential equation, only thing you have to factorise it to know the values of m1, m2, m3, then rest of the things I know okay, this is 90% of the work done, then the second part is how to find out k1, k2, k3, forcing function alone, solution due to forcing function alone satisfies this differential equation put it there, treat it as an identity, equate the coefficients etc., of the like terms of the left hand and right hand side get k1, k2, k3, put those values.

Then, your step will be to determine A, B, C on from the total solution and what will be the 3 boundary conditions will be needed in fact, 3 will be given, dy dt 0 and d2y dt2 will be also given, this 3 if given, I can generate 3 equations and get the solution. If this is true, if roots are distinct, m1 not equal to m2 not equal to m3 suppose, if suppose let us say m1, m and m, m2 and m3, suppose if m1 and these 2 roots are repeating, 2 roots repeating.

Oh, I can easily write yt, first of all m1 is alone, so it will be total solution will be A into e to the power m1t, over, this 2 are repeating, so it will be Bt plus C into e to the power mt, mind you, 3 constants will be there but t appears, for reasons which I have explained in my previous example, why it will be different plus k1 into x plus k2 into x dot plus k3 into x double dot plus up to infinity in one stroke.

If all these 3 roots are repeating, if m1 equal to m2 is equal to m3 is equal to m say, then I will say total solution will be you can easily see it will be A, it will be say A plus Bt plus Ct square into e to the power mt plus k1x plus k2 x dot plus to infinity, got the point, therefore we will able to handle situations very efficiently. The; we will later see that solution of differential equations can be avoided by using some transformation that is using some Laplace transformation, you can solve for the currents that is solving differential equations altogether can be translated in some sort of for algebraic equations, differential equations by using Laplace transformation.

We will devote some time on that also and that is very good, nothing wrong in that but people still believe that differential equation is giving and you do not take any transformation, are you in a position to tell the nature of the solution, straight in time domain, living in time domain, no transformation nothing, yes I can do that and this gives you a better insight into the problem.

Laplace transform, okay you go to some other domain, solve it sort of mechanical things but here you see people still use them, this is called classical way of solving differential equation, clear, see while telling about the roots, I told you that there are repeating roots, there are distinct roots, it may so happen that the roots may become complex number, nothing doing, when if roots are complex number what happens they will appear in conjugate pairs and they are not equal; distinct.

Therefore, all rules that we have developed can be easily applied, when we if the roots are complex I will write m1 some A plus jv like but interesting thing is roots; if one root is complex, there must be another root which will be also complex and complex conjugate of these, these are all the advantages, we will discuss and solve problems on that

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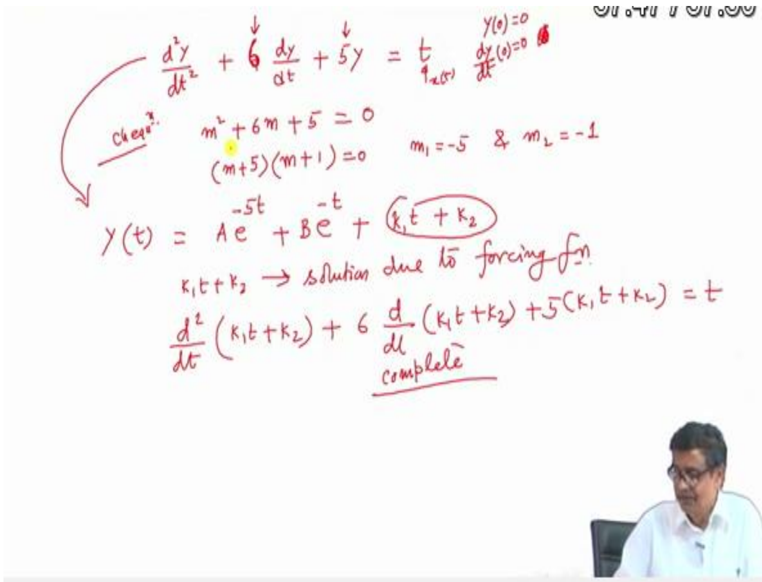

But the essential thing is to conclude these particular topic is that for example, I am telling you that suppose you have a differential equation d2y dt2, second order, let us put some number here plus 4 dy dt will be mostly concentrating on second order differential equation although, third order if any higher order system we can do. Suppose, this is 6, any number; good number and plus y is equal to suppose, some t, let us not bother, okay this is the thing, differential equation given second order.

And it is also given that y0 is 0, all initial conditions relaxed is a term which means the initial conditions that you need are all set to 0, when you want to solve, you want to get the solution for t greater than equal to 0 and these are the 2 boundary conditions given, suppose this is the thing, then the characteristic equation is m square plus 6m plus 1 is equal to 0, is it not or should I put 6; 5 here 5, let us put 5 there, so that things come.

So, you see this is the characteristic equation, on the right hand side there is the forcing function which depends on t, it is a second order system but how simple it will become let us see, this one can be written as m plus 3 into m plus 2, no, so this is 6, oh m3, m5 plus m1, let this be 5, sorry, so this you factorise, you get this and that is equal to 0. What are the roots; m1 is equal to minus 5 and m2 is equal to minus 1 they are distinct; real distinct.

Even if it is complex, they are distinct, different, complex means different and no, nothing doing, so this is the thing, then I will say the solution straight away now, no this way that way, straight away yt I will say. it will be equal to A into e to the power minus 5t plus B into e to the power minus t minus 1 plus k1 into t, this is xt, linear combination of the forcing function plus its higher order derivatives k2 into differentiate t1 and no other terms 51, here finishes, everything is finished.

Then, I will say that k1t plus k2 is solution due to forcing function, therefore this will satisfy this equation that is d2 dt2, this I will not complete I will leave it as an exercise k1t plus k2 plus 6ddt of k1t plus k2, this satisfies the forcing function plus 5 k1t plus k2, this is the term, solution due to forcing function and this must be equal to t. How many constants are there? K1, k2, so you differentiate, there is no; t square.

So, coefficient of t from this side and that side will be equal and constant; there is no constant on the right hand side but they if there is constant that should you equate to 0, get k1, k2, then put it here, then put the Bc, boundary condition. So, please complete this problem on yourselves and generate problem on your own any problem, okay but only restriction is you put constant coefficient, this term may be coefficients time varying etc., and also do not make this function on the right hand side and impulse function everything will be in place, thank you.