Network Analysis

Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology – Kharagpur

Lecture - 21
Problem Solving Application

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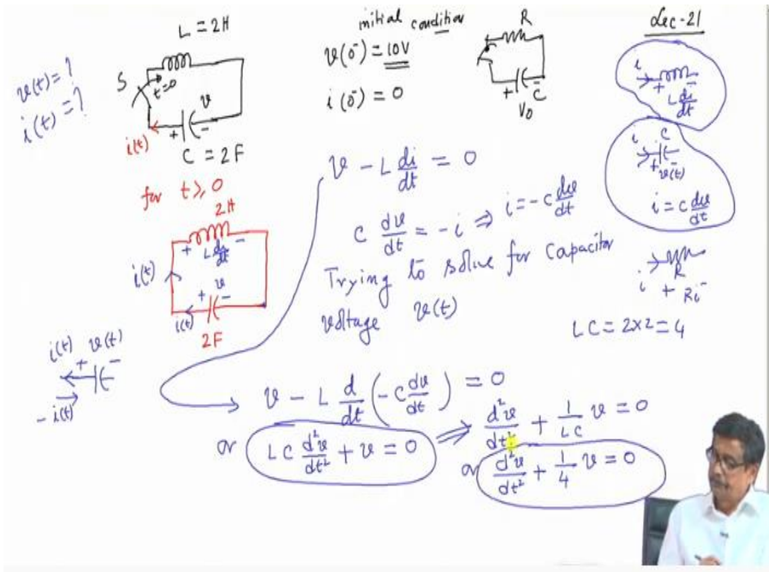

Welcome to lecture 21 and we are now in a position to do various things. After we have learnt how to tackle differential equations some interesting circuit problems we can do. Let us in the form of some examples numerical examples let us highlight how to apply those techniques that we have learned to solve differential equations direct applications.

Let us see suppose we have told about earlier that you can charge the capacitor by having RC circuit things like that. And we have so far considered in this circuit there is only one energy storing elements are present. Now let us do something interesting now for example you have got an inductor here and a capacitor okay sorry. And a capacitor and there is no source and there is this switch here now the question is if you close the switch at t=0 will there be any current?

Yes there may be a current provided this capacitor had some initial voltage already charged from some other circuit. And we have done this problem that is a charged capacitor and we dissipated the energy in a resistance. Earlier we have done RC a charged capacitor was initially charged with the voltage V0 and then we did like this. Now suppose I say that I put some number L=2 Henry and C is also 2 Farad.

Mind you these numbers 2 Henry 2 Farad are too large I mean 2 Henry inductance is very large inductance. Similarly, 2 Farad capacitor is very large to keep our computations simple and I have chosen these values. They may not be this L with 2 Henry C with 2 Farad will not be easily available. But anyway to give you the idea what is the problem? I am telling you that the voltage across the capacitor I will denote is by V.

Because we may be time varying with the condition that v0- was equal to some 10 volts initial condition. Similarly, I have to specify because the inductor is an energy storage elements. I must specify also i0- suppose it is 0. Okay now I will close this switch at t=0 from there I will start counting my time and one to find out the currents in the circuit. That is the problem statement of the problem.

So a initially charged capacitor with 10 volt with this polarity plus minus close the switch and I want to solve for the current it right. Okay therefore at t<0 circuit was open. There was no current so that was fine. Now for t>0 say equal to 0 the circuit is like this L that is 2 Henry and circuit is closed 2 Farad and let us assume the current in this circuit to be it at any time t direction of it is my choice I have assumed it like this and this was the voltage across the capacitor plate and what will be the voltage across the in the inductance.

Once you have assumed this, this will be Ldidt is it not this will be this thing then I write down the KVL equation in this loop. So start from this point from this to this +V then from this to this -Ldidt and that is equal to no other voltage that must be equal to 0 no external source is present. So I have taken everything here that is fine. Now coming to the capacitor once again you see look at the capacitor separately.

What I have done this is the polarity of the voltage I have shown, and this is the current I have shown is not what this capacitor this is it. I can say that -it this way. Therefore, that C dvdt then I should write it as -I for this problem. This point is crucial students often mistake here because you must always remember if you have assumed current this way voltage across the inductor is Ldidt okay.

If it is a capacitor C if current enters through this terminal where I have also assumed voltage polarity to be like this, then I will say i=Cdvdt but here I have assumed it like that fine Ldidt no problem. In the capacitor the current is leaving plus terminal that means -i is entering though +. So it is the -i which should be equal to Cdvdt to be consistent with this. So these two things you must always remember similarly for the resistance of course it is like this +-Ri. These are very important step now then this is the thing.

So my first goal and suppose I want to know the voltage across the capacitor at any time t, Vt is how much and also current in this circuit at any time t current will be same because they get series connected LC it cannot be different now, so these are the two equations. Therefore, my first task is to trying to solve for V trying to solve for capacitor voltage Vt anytime T what is the capacitor voltage?

Therefore, my target will be to form a differential equation involving V only. I will not form any integral differential equation. So I did this from this equation I will write V - L and ddt into i this i and the i is -cdvdt which means that i=-Cdvdt so put it here -Cdvdt and that is equal to how much 0 because no external voltage forcing function is 0. The initial energy stored in the capacitor will drive the circuit we will see.

So this is the thing, or I will say this equation is nothing but LC d2vdt2 this will become +, +v=0 and this one I have learned how to solve differential equation. I will make this coefficient of this highest derivative term that is d2vdt2 here 1. So divide by LC on both the side 1/LC into V=0 this will be the thing and the value of L, L=2 Henry so L into C is nothing but 2 into 2 =4 here or putting the values it will be d2v+dt2++1/4 into V is equal to if any mistake point out. So this will be the thing this is the differential equation.

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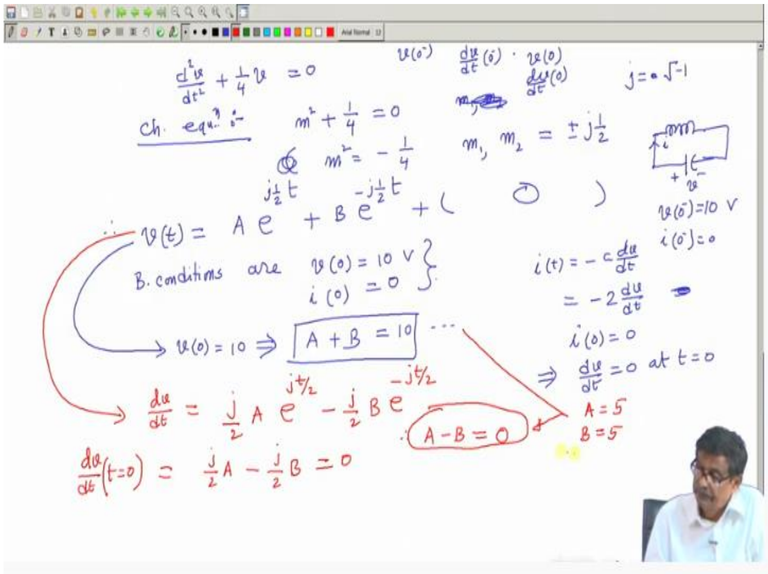

So I rewrite this differential equation as d2vdt2 +1/4 V =0 okay then what I have to do I have to form the characteristic equation, what is the characteristic equation m square+1/4=0 what will be the roots? m1, m2 will be equal to see I will write like this it is equal to I mean m square = -1/4. Therefore, if you take square root of both the sides m1, m2 will be equal to +-j/2 where j is root over -1 complex imaginary number j and this becomes up.

So once I add the roots are distinct yes, they are not equal because one is j/2 another is -j/2 they will hold the solution Vt I will write straight away. It will be equal to A into e to the power j half m1 into t+ B into e to the power-j half into t+ forcing function is 0. So this this term x, x dot these are zeros nothing is there is it not this will be the thing.

Now if you look at the boundary conditions, I told you that this is the capacitor this is Vt, and this is i current flowing through the inductor and I told you V0-=0 no 0 10 volt in the previous slide most probably I told you 10 volt no I have not specified here and i0- is 0. Do not forget to write down the next 10 volt and 0 Ampere this is the situation, but you see i see it is a second order differential equation.

How many boundary conditions I will demand, I will demand that give me the value of V0- that is there. And also give me the value of dvdt at 0- and these are nothing, but I will demand V0 and dvdt 0 you have understood this point hopefully. See after you have closed the switch in no time 0- to 0 voltage across the capacitor cannot change instantaneously. So I will say boundary conditions are V0=10 volt that is one boundary condition.

Capacitor voltage cannot change instantaneously. Similarly, the current through the inductor i this current mind you this is the direction of the current i0 must remain 0. These are the two boundary conditions. Therefore, the first boundary condition will give you if you put t=0 V0=10 volt will give you A + B on the right hand side t0=10 volt. This is equation one important equation because constants are to be determined. Here is of course there is no terms involving forcing function.

So this is A and B are to be found out. Now the secondary boundary the second boundary condition is dvdt how do I get that? I have already told you the current in this circuit it =-C dvdt that is what we got. C is how much 2 Farad so C-2 dvdt this is the expression of the current. So and this current has to be 0 i0=0 implies that dvdt must be equal to 0 at t=0 these two conditions.

Therefore, to get the other equations what you do is you differentiate this dvdt see C does not come into dvdt because 0 on the right hand side. Therefore, if you differentiate both the sides now dvdt it will be differentiation of this. That is j/2 into A into e to the power j t/2 this term -j/2 this constant will come out into B was there already into the e to the power -j t/2 that will be the thing. And I am telling dvdt at t=0 is 0 so dvdt at t=0 will be put t=0 there.

So j/2 into A -j/2 into B to these are 1 and this is 0 I am telling it has to be that. Therefore, you get A-B=0 if any mistake point out so this is the thing this is the other equation. So you have got these two equations therefore add these two equations from these two equations. We will get A=5 and B= also 5 because of the equal to. So these are the two constants you have get and you have almost solved the problem.

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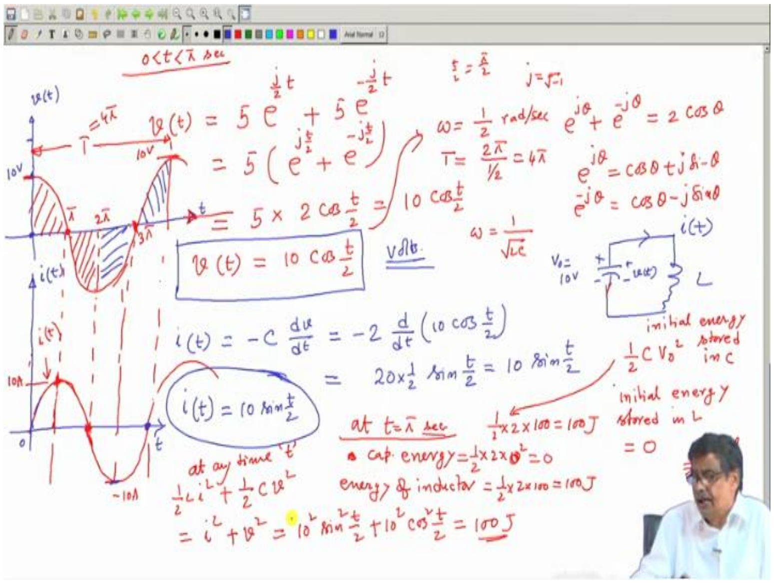

Therefore, I will say voltage across the plate of the capacitor Vt will be equal to A into e to the power j/2 into t+ 5 into t e tot the power- j/2 into t this will be the solution hopefully correct. Now this is the thing j is root over -1 imaginary field here. Now these two things can be combined together j t/2+e to the power -j t/2 you must be remembering that e to the power j theta+ e to the power – j theta will be equal to 2 cos theta is it not.

Because e to the power j theta=cos theta+sin theta and e to the power -j theta =cos theta – j sin theta we will be often using this. So I hope you know this complex algebra so this then will become e to the power j theta is this so if you add these 2 cos theta you will get so 5 into 2 cos t/2 which you will be equal to 10 cosine theta. So voltage across the plate of the capacitor is =10 cosine t/2.

So we have got the voltage across the plate of the capacitor and the expression of the current and what the point I want to make. See although in the expression and there is no solution due to forcing function no forcing function was there. See the capacitor was charged with a real number plus minus 10 volt. Then initial voltage this is initial voltage at any time old is Vt and this is L, and this is your it.

Now we see in this circuit therefore at the end the voltage current whatever you solve for they should be there why they should be imaginary. In fact, eh this this complex numbers will never betray you in that way. But I have followed strictly the rules roots are separate distinct although they are imaginary, but we know that it will I can write down the solution just by inspection that is what I did.

Then I applied the boundary conditions to get the constants here A, B and I followed strictly the rules and got these as my voltage what will be the current okay. One can form a differential equation involving current and solve it that I am not going to do because once I have got the time domain expression for the capacitor voltage it, I know it is equal to -C dvdt is it not that is what I got. So -C means -2 capacitance is C I have assumed and ddt of 10 cosine t/2.

Which if you differentiate cosine t negative will come. This negative will make it plus so it will be 20 into sin t/2 into half that is 10 sin t/2. So we write it=10 sin t/2. See just application of the rules I have direct I have never multiplied with multiplying factor integrating factor this that or things of the past. Now after I have learned how to solve a differential equation you can analyze this circuit and find out the characteristic routes forcing function. In this case it was like that so if I know sketch here these two things what will be the nature of the solution?

So I want to sketch capacitor voltage Vt and current in this circuit it at t=0 I did this switching t =0 I did this switching. See the capacitor voltage was 10 volt earlier. Supposedly it is a 10 volt then it changes with respect to as a cosine function. So the nature of solution of capacitor voltage will be something like this is it not 0 crossing points can also be found out too because cosine pi 2 it will be. So t/2=pi/2 so t=pi this point will be pi and so on you can mark this point.

This is time mind you not theta, so this is pi correct. Now the interesting points is, and this big value is 10 volt. So capacitor voltage will oscillate like that. What will be the frequency of this Omega of this thing? Omega t Omega is half so much radiant per second is it not sin omega t, omega is half. What is the time period? What will be the time period that is this time 2 pi/omega which is = 4 pi in this case, so this is =4 pi for this problem it is okay this is the time period that I have got.

Now let us show capacitor voltage V0- was 0 then from that it started oscillating sinusoidally co sinusoidally like this what about current? Current if you see it is a sin function sin t/2 so current will fall it will be like this if you sketch it this is it. What is this it? 10 sin t/2 this one is 10,10 ampere this one is 10 volt. It will be varying sinusoidally got the point and then it will be like this and it will go on doing like this.

Therefore, a capacitor and inductor which are ideal in nature no resistance involved. If you charge the capacitor from some other circuit to 10 volt and then switch on to an inductor which had got no initial current, then you see the voltage across the inductor or across the capacitor and the current in the circuit will be oscillating changing with time indefinitely there is no decay.

Why? Because there are two energy storing elements. Several things we will say that is why you got a second order differential equations and then voltage and current will oscillate. What is the value of omega? Omega value is 1/root/LC omega value you can easily see this omega value L and C were 2, 2 each. So this Omega is also equal to 1/root over LC. You can easily show that anyway it will do like that. So you started this circuit with no external source connected.

You had a capacitor where energy was stored. How much was that energy? Half C into V0 square initial energy stored in C. What was the initial energy stored in inductor 0? It started with that initial energy stored in L was 0 because i was 0 at that time. Initial means t=0 implies t=0 like that. Now you see as we have connected let us now try to see the physical thing. What is happening? Okay equations are fine you have got the solution.

Initially you started with this much energy. How much it was half this one half C was 2 and this was 100, 10 square that is 100 joules. You started with 100 joules there is no energy storing, energy dissipating element no resistance in this circuit. So capacitors start discharging energy losing voltage means capacitor energy is decreasing a time will come at t=pi second capacitor voltage will collapse to 0 at that instant.

And at that instant, the current through the inductor will be 10 ampere maximum at t=pi second capacitor stored energy in capacitor energy is half C 0 square that is 0,0 square 0 energy stored, energy of inductor at t=pi second will be equal to =half L into i square, i is also 10 ampere into 100 =100 joules. That means capacitors started losing energy and inductor was gaining energy when capacitor energy was 0 inductor. That is why zero crossing of the capacitor voltage will give always tell you that current in the circuit is maximum. That is here also here this maximum current +10 -10 squared if you make it is plus so this is also -10 ampere it will be like this.

So this is how things will continue forever it is a very interesting circuit used called tank circuit. It will start oscillating. You start with that what about at any time t energy stored in inductor is half Li square energy stored in the capacitor is half C V square L and C are 2. So this is equal to i square + V square for this problems C=2, L=2 what is i square? i square is 10 square sin square t/2 +10 square cosine square t/2 and that will be equal to always 100 joule.

That is this you started with 100 joule, 100 joule no one is there to dissipate that energy. So energy capacitor will charge the inductor to its maximum current then inductor will start discharging when the current becomes negative and capacitor will charge out. That is one thing and another thing I would like to tell you that you see between 0 to pi if you are really following me, I will appreciate if you also try to understand the point.

See voltage across the capacitor during 0 to t pi 0 to t this pi is second 0 to this second 0 to pi second voltage applied across the inductor was positive and current must go on rise that is what exactly happened. You spent this much volts again to charge the inductor here and the current grows from 0 to 10 ampere positive volts again we have spent this is also pi so 2 pi you will get.

During this next pi second you have applied negative voltage so the, did not has to be negative. So current has started decreasing voltage applied is this negative voltage across the inductor. Therefore, current wherever it rose to 10 ampere from that it must start decreasing. And during this pi interval you have applied in negative volt second and these two are same. These if you integrate these volts second then that volt second this is same.

That is why it rose to 10 ampere but once again came to 0 ampere from this to this interval. This is 3 pi from this pi interval what you are doing we are still applying negative voltage. Therefore, didt must be negative it and it will continue like this. So negative volts again you are applying during this interval so that current becomes -10 ampere and eventually these two areas are same and current will once again come to 0.

So this is a rather simple circuit but nonetheless it will always think behind all this mathematical steps that I have done physically try to understand what is happening. All the concepts are volts again similarly you see the capacitor voltage it is decreasing here. Why it is decreasing capacitor voltage because you are draining out current. This it mind you is drained out is it not from the positive plate.

So capacitor voltage you have essentially applied a negative current to -it will be like that. So think in those terms and we will consider several. We have now made enough background to analyze any circuit. Now I am telling I will not put any restriction to the voltage let it be DC voltage only. No I will take anytime varying voltage I will also not use only resistance in this circuit. In fact, in practical circuits there will be inductance capacitance is present.

So most general and with initial conditions also. So your excited him maybe time-varying your response maybe time varying all the restrictions are gone. And this will be the most general differential equation. He will write down for this system in the form of KVL, KCL here and then try to solve the differential equation. And at this stage solve it classically the way I told you.

Then at later stage we will also tell you the use of Laplace transform which is also a very elegant method. But this approach of solving the differential equations classically gives you more insight into the problem. At every step you can check oh this is how it looks like. So make habit of sketching waveforms of voltage current in a time varying environment and try to interpret the results. Thank you.