Network Analysis

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Lecture – 22
R-L Circuit with Sinusoidal Excitation

Welcome to lecture number 22nd and we have been solving on circuits, where this supply voltage will be time varying and as well as there will be energy storing elements in this circuit and we have considered RL circuit, RC circuit, first with DC sources and then the basic steps will be write down if you are interested to find out the current flowing through an inductor, first develop a differential equation involving that current.

And note down the initial conditions that is i0 minus which we will give you i0 plus or i0 which will be same in general therefore, then you write down the differential equation but after that do not integrate to get the solution. See, in our last few lectures I wanted to stress upon this point; if the differential equation is known, you should straight away write down the solution of the differential equation which will have a natural response part.

That is if the right hand side was 0, no excitation, what would be the solution; that solution depends upon the characteristic roots and exponential terms will appeared there, of course with some constants and there will be another solution corresponding to forcing function or excitation present. One important thing to be noted is this; this forcing function solution alone will satisfy the natural response.

So, we consider several problem today, I will consider another problem and show you how to apply the same technique, when the excitation is say a sinusoidal voltage, okay.

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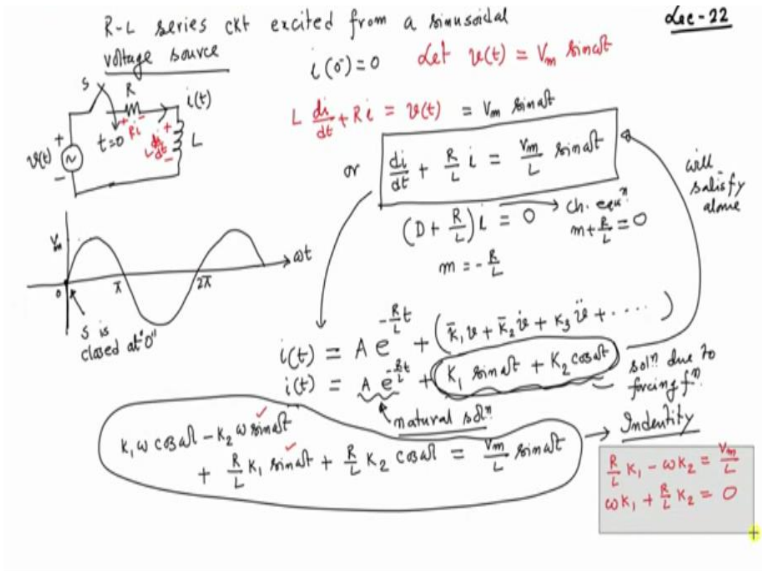

For example, consider a RL circuit; RL series circuit excited from a sinusoidal voltage source and so the thing is like this R here inductance in series L and here is a source which is time varying, vt is the voltage source and i0 minus this is the current I want to solve; i as the function of time and i0 minus is equal to 0, it is suppose given which was open for a long time and so on, now this switch s is closed at t equal to 0.

Then, how the current will grow in the circuit or will flow in the circuit as a function of time, this I want to find out, then the first step is write down the voltage drops that is Ri here with proper polarity and this is Ldi dt and we know these that the sum of this 2 voltages; Ldi dt plus Ri is equal to your input vy. Now, let vt, the applied voltage is V max into some sin omega t, suppose, so applied voltage is sin omega t.

That means I have close this switch, suppose if I sketch the supply voltage here, this the voltage I am applying, this is your V max and this axis is omega t, okay and this is 0, pi, etc., so I am closing the switch, this axis is omega t, means also t, omega is constant supply frequency, so this axis also determines the time axis in some other scale divided by omega. So, at t equal to 0, I am closing the switch, so I am closing the switch at this point; s is closed at O.

And that is why I am describing this as V max sin omega t and I want to get the solution, so I; let me be systematic rules I know, what should I do, so this is equal to Vm sin omega t, is it not.

Then, I will write it fast as divide by L; di dt plus R by L into i is equal to Vm by L into sin omega t, this is the differential equation. What is the characteristic equation? Characteristic equation is D plus R by L into i D is d dt is equal to 0 if you do, you get the characteristic equation, is it not.

And D is being an operator, so characteristic equation is m plus R by L, it is a first order system, only one characteristic equation, only one energy storing elements, so m is equal to minus R by L, therefore the solution now comes the interesting part; the solution I will write straight away, it will be equal to some constant A into e to the power mt that is minus R by L into t that will be the first part.

And the second part will be as we know the; it should be since it is Vt, it should be like k1V plus k1 is a bar, k2, another constant V dot plus k3, another constant V doubled dot and up to infinity, this is a solution due to forcing function, so solution due to forcing function is a linear combination of the excitation and its higher order derivative. Now, here this you see, it is sin omega t, so it should be if you do this, so you have to go on differentiating this sin omega t, so this will give rise to which I am not going to write.

What I am telling; it will be some constant into sin omega t, if you put this plus if you differentiate this, it will be omega cos omega t, is it not, that omega will multiply with k2 bar to be another constant, then if you differentiate V dot to get V double dot, so once again sin omega t term will come, so alternatively the terms will be sin omega t, cos omega t, of course with different constants here and that is to be added up, up to infinity.

Now, therefore it looks like; there will be terms like, so this is equal to A into e to the power minus R by L into t plus it looks like, there will be sin omega t; a number of infinite number of sin omega t terms, so whatever it is I will call that thing to be some big constant into sin omega t, is it not, this I can always do because those coefficients are constant. Similarly, there will be infinite number cosine omega t terms.

And that I will group those; if you take cosine omega t out, this will be another constant term, of course C and D will have that omega terms but omega is constant, not circuit parameter decided by supply frequency that is all but this will be the solution, so you see I get the solution of this network, the form of the solution at least I do not require any time, okay, find out the characteristic root, this is the natural part, this is this part, okay.

Now, if you wish I was writing it as some big K1 and K2, okay, generally, for forcing function the constants I was writing as K1, K2, you can do that. Now, to find out K1 and K2, first I will find out K1, K2, so this is the this part, solution due to forcing function and this part is natural solution, got the point. Now, to so this forcing function alone satisfies this equation, this solution, this part will satisfy this; will satisfy alone.

Because total solution satisfies this but natural response if you substitute this will give rise to 0, so I can very well say except t alone, will also satisfy that, that is the key of this whole thing, so I will put this here, so it is Di dt, so forcing function alone will satisfy this, so it will be K1 omega cos omega t, I am differentiating this Di dt, so K1 omega cos omega t, then minus I have to differentiate this also, so K2 omega sin omega t, cos omega t if you differentiate, it gives you omega t term, is it not, that is okay.

So, this gives you Di dt plus R by L into this current due to forcing function which will be equal to K1 sin omega t plus R by L ar because R by L is common into i; R by L into K2 cos omega t and I am telling this will be equal to Bm by L sin omega t, is it not, this is the thing. So, I am determining first the constants K1, K2; K1, K2 can be determined and as I told you, so there are 2 unknown not infinite numbers of unknowns.

And these 2 unknowns can be solved for by having 2 equations, so and this is an identity, mind you, this is an identity, therefore coefficient of sin omega t on both the sides will be equal, so what is the coefficient of sin omega t; both the sides it will be sin omega t terms are here, this is the sin omega t terms, this is the sin omega t terms, therefore since it is an identity I will write R by L into K1 minus omega into K2.

And on the right hand side, there is a sin omega t, whose coefficient is Vm by L, must be equal to Vm by L, that is what I am telling, this is one equation and the second equation is coefficient of cosine omega t, equate, so it will be omega K1; this one and another cosine omega t term is there plus R by L into K2, I hope signs are all okay know, this is, this must be equal to; there is no cosine term here must be equal to 0.

These are the 2 equations I have to solve, so these 2 equations I have to solve, only thing it is Vm that I will write.

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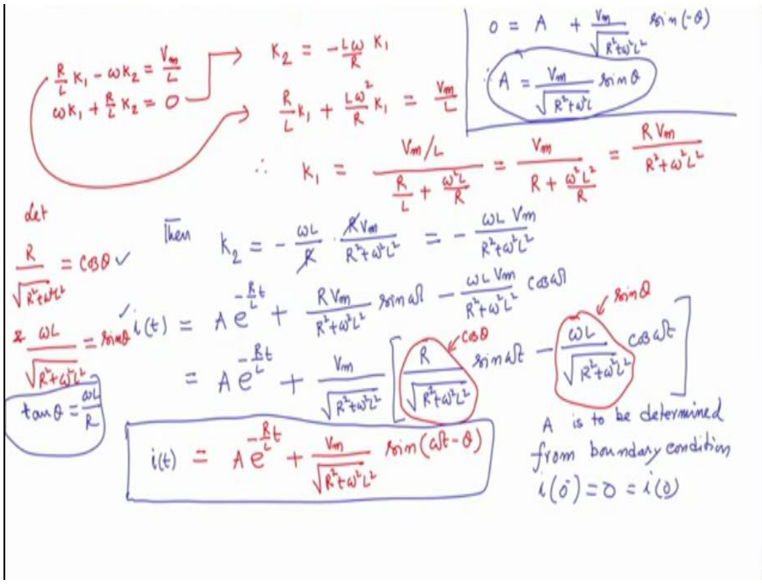

So, previous 2 equations are this one, so I have to solve for them, so from this equation I will say that K2 is equal to from this equation, R by L K2 is equal to minus omega K1 and it will be L by R, this is K2 and this thing I will submit it, put it there, so I am going to find out K1, so it will be R by L into K1 was there from this equation minus omega K2, so it will become plus L omega square by R into K1 and that I am telling is equal to Vm by L.

So, this is the thing therefore, K1 will be equal to Vm by L in the numerator and below it will be R by L, plus omega square L by R, is it not, this thing I have brought it down which will be equal to Vm by this L you bring down, it will be equal to R, L cancels out plus omega square L square divided by R and these are also can be put on the top, so R into Vm, it will be equal to R square plus omega square L square.

So, this is the value of K1 and if this is K1, then K2; K2 is minus omega L by R into K1 and K1 is this one; RVm by R square plus omega square L square, so R goes and you will be left with minus omega L divided by R square plus omega square L square into Vm, this will be the thing, therefore the solution; total solution will be it will be equal to A into e to the power that minus R by L into t that is the natural response plus solution due to forcing function.

So, K1 into sin omega t and k1 is this one into sin omega t plus K2, which happens to be a negative thing like this, so omega L by R square plus omega square L square into Vm into cos omega t, so this is this total solution, only thing is this can be manipulated a bit and it can be written in this form that plus this one and here what you do; you take Vm by square root of R square plus omega square L square common from this 2 terms.

Then you will be left with R over this thing, sin omega t and minus omega L and Vm I have taken outside, 1 square root I have taken, so another square root also is left here, cos omega t, it will be like that. Now, these 2 things you see this term square and this term square gives you 1, is it not, this square plus this square, they will give 1, therefore I can define let R by; I am sorry, so R; let R I defined as R by root over R square plus omega square L square as cos theta.

And then omega L by this one, same square root has to be sin theta why because this square plus this square is 1, therefore these equation, so this is cos theta and this term is sin theta, so I have sin omega t cos theta minus cos omega t sin theta, therefore this solution is nothing but A into e to the power minus R by L into t plus Vm by this quantity and this one can be written as sin of omega t minus theta, got the point.

So, this is it, this is the solution now, in this solution, still A is to be determined, capital A has not been determined yet, mind you, after you write down the total solution, then you apply the boundary condition for example, A is to be determined from boundary conditions which is given to be i0 minus equal to 0 that is equal to i0, so I put that and find out the constant here; so here there is space I will do here that thing.

So, put t equal to 0 on both sides, so it will be equal to 0 is equal to i0 is 0 is equal to A e to the power 0 is 1 and in this expression t is equal to 0, so it will be plus Vm over root over R square plus omega square L square sin of minus theta. What is theta; theta is tan theta is equal to as you can see I have defined cos theta, sin theta, so tan theta is omega L by R. so, sin theta known in fact.

Therefore, A will be equal to Vm divided by root over R square plus omega square L square sin minus theta is minus sin theta, so this becomes plus then, if you change side, got the point, so this is the value of A.

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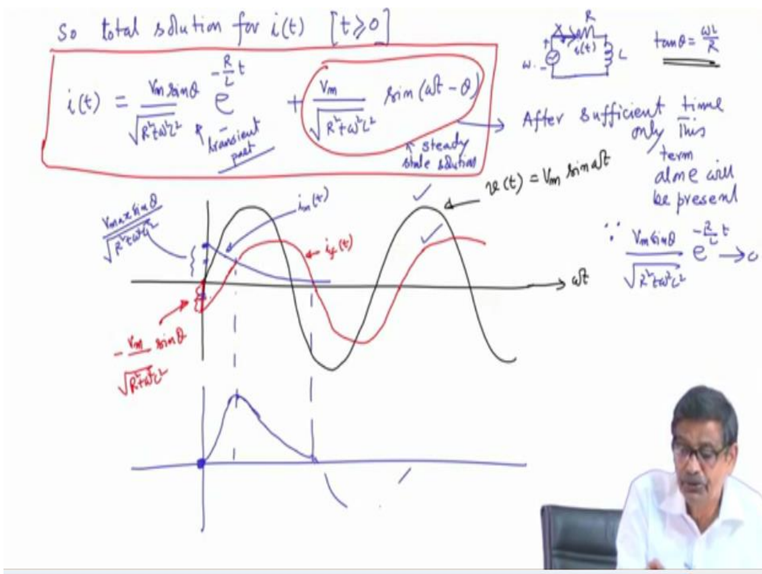

Therefore, total solution I will write in the next page as so, total solution for it, I mean I want to get the solution t greater than equal to 0 will be equal to it is equal to some e to the power minus R by L into t plus solution due to forcing function V max over this thing into sin omega t minus theta and here the value of the constant was found out to be this thing; V max by this into sin theta, so this constant is V max by root over R square plus omega square L square sin theta.

Mind you, theta is constant, once this circuit is known, R, L, and your supply frequencies omega, then you know and we are solving for this current and we know that tan theta is equal to omega L by R, so this will be the complete solution. Let us see whether this solution is okay or not; at t equal to 0, i0 must be 0. So, at t equal to 0, this is Bm by this into sin minus theta and it is Vm sin theta by this.

So, sin minus theta is minus sin theta, so this 2 term will cancel out and will you the result, so this is the total solution, got the point, so if you sketch as I told you, it is always better you sketch the solution, so sketching for the solution, you should do it like this; first sketch vt, this one is vt which is equal to Vm sin omega t, here is omega t and then the current waveform is this kind of thing.

See, if you look at this current waveform, it has got 2 distinct part; one part is Vm by root over R square omega square L square sin omega t minus theta, so theta value is known from this, so current waveform will be lagging the voltage waveform by an angle theta, I am drawing it with red, this is the solution due to forcing function, this part is this part, got the point, only this part I have drawn.

Then, there is a part but this is the only ift but there is a natural part, you have to add these waveform to these to get the total solution. Now, what will be that part; that part is telling that this one; the magnitude at t equal 0 of the forcing function is minus V max by root over R square plus omega square L square sin theta, sin minus theta, minus you bring outside, so at t equal to 0, this is the current; negative current.

But at t equal to 0, this must give this much of positive current and in fact, it is given, so this I will use a different colour, yeah, so it will this length is equal to this length, these 2 lengths are same and this one is at t equal to 0, V max sin theta by root over R square plus omega square L squares and then it is exponentially decaying, is it not, e to the power minus R by L and this sum of this curve plus this curve will give you the total current.

Therefore, initially this sine wave after sometime of course, it will be only this part present, after sufficient time; sufficient time means talk in terms of time constant of the circuit after sufficient time only this term is present, this term alone will be present since V max sin theta, the other term by root over R square plus omega square L square into e the power minus R by Lt will tend to 0 that is what it is.

After 1, 2 or may be 3, 4 cycles it will be only black and red curve will be present, of course initially this current waveform will start has to; if you are try to draw the resultant current below, so this is I will write, it is fair to write, so this if1 and this I will write, this blue one is natural response; int. So, the total current will be some of this and it has to start from 0 and then this plus this, this plus this, this plus this, so here it will grow like this, some asymmetry will be there initially.

Then that thing, once again will fall, it will be like this, here it will be 0, I mean something like that, you get then this is vanishingly small, I mean something like that so, initially there will be asymmetry, it will be shifted, it is called decision and then of course, after long time what will remain is this voltage waveform black and this red waveform, int will vanish. So, the; this part is; this is the total solution and we find the in the total solution, there will be a transient part which will decay.

And this part is called steady state solution, this is also called solution due to forcing function is also called steady state solution and this is called the transient part, got the point. So, what is the conclusion? See, behind all this maths, the conclusion is to be understood very carefully, the conclusion is any RL circuit, you suddenly switch on with the help of a switch then, the response of the current will comprise of 2 parts.

But some of this 2 at t equal to 0 has to be 0 that is there, it has to start from 0 and there is an exponential term which will appear and that will go down to 0 and finally, it is the steady state solution which will sustain, any switching of RL circuit with some sinusoidal excitation a transient will be born but that transient will not last long, it will die down, die down by what time; well, it depends upon the time constant of the circuit may be after few cycles, that transient will be all together vanished.

That is why while analysing AC circuit, say RL, RC, this type of circuits, you energised with sinusoidal, people often neglect this transient part, I am interested because with see, one supply cycle. 50 hertz is the supply frequency, 20 milli second is 1 cycle, RL values are such that the order of time constant will be of the order of some few milli seconds maybe 3 cycles that is 60 milli second may be if it is very inducting.

But after that 60 milli second, only this fellow and this fellow will be present, who bothers that transient was there when you switched on the circuit of course, these initial current which is produced which is some of transient and the steady state current, natural and force response, these are very important piece of studies in power system, when you energise a power circuit line have been lot of inductance, then initial current search may be more.

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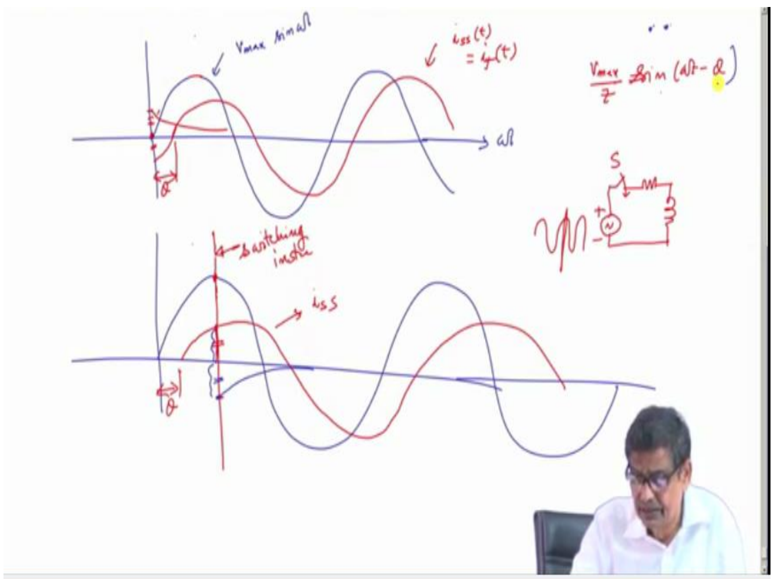

And last point I want to make it behind all this maths, you should not forget this for example, let me tell you which am not doing but I must point this out, so to create interest, for example you have a supply voltage like this, this point you listen carefully, this is omega t, no max here, V max sin omega t is the voltage I am counting time here. Now, the point is that this supply voltage is present like that between to say your supply or to supply, it is supply is present and it is doing like this.

Should I call it sin omega t or else depends upon me, if I start counting my time from this instant I will say, okay it may be called cosine omega t and so on but what I know for certain is that this steady state current will be doing like this, this I can draw, red one is the steady state component; isst, which is equal to i forcing function t, I just did last time, so circuit; the steady state current is very simple you know that; V max by z root over R square plus omega square L square is called impotence into sin of omega t minus theta data and this is theta, is it not.

Now, what I am telling; I am not going to do any maths but I am telling you one very interesting observation, this if you understand, I will be happy, I mean, you can do several things, for example I say that I have a RL other circuit here, I am going to energise it with this switch and I will close, what is this supply voltage; supply voltage is doing like this, I have not yet called where is t is equal to 0.

But I will tell that when I close this switch, I am not really sure because I cannot see the supply when I switch on say your fan in your home, you switch on, it is after all some sort of RL circuit anyway, I may switch on when this supply voltage was maximum, when the supply voltage was 0, it is left to chances, I do not know really. Now, suppose I say in the previous problem I told you, I have close the switch, when the supply voltage was 0 and going towards positive.

And then I will tell but current through the inductor cannot change instantaneously, so steady state demands this much negative current is flowing, so this much of positive current must appear to make it 0 and that will be the transient part that way I also I can now imagine how things are going. Suppose, I say that with this I will say, this is your supply voltage, I am not writing omega t equal to 0 etc., okay this is your supply voltage.

But steady state current will be displaced from A it by an angle theta; this angle, is it not, this is the steady state current, red one; iss, now suppose I say that I will close this switch at this instant, when the supply voltage was around this point okay, here also there will be we expect some transient to come and that will die down but what will be the value of the transient current, right at the (()) (41:28), this is the switching instant, you close this switch, when it was doing like this at that point.

But current through the inductor cannot be 0, steady state demands that the current is this much, positive, do your max, the way I did but I am telling you believe me, your; there will appear a negative current of same amount, here lies the catch, these and these 2 will be equal and this current will decay down to 0, this will be the nature of the transient. So, behind all this mathematical states getting the results, one can easily write down the steady state part of the solution, which is very easy to write; sin omega t minus theta.

From that decide at whatever time you switch on the circuit, you make up what is, we will continue with this in the next class, thank you.