Network Analysis
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Lecture # 23
R-L Circuit with Sinusoidal Exponential

So, welcome to 23rd lecture and we have been discussing about how to find out currents in time domain by solving your classical differential equations. Essentially I am doing this but applying this to solve this market, the way I have learned and last time I told you that adults are excited from his impact it could be any source, but suicidal thoughts is one of the most popular excitation voltage.

Because we know our power system is 50 hertz and it is suicidality bearing therefore people often ask that what will be the response of current in a circuit beat are LRC whatever it is to find out the solution. Now, I will do the same thing but not completely I will leave it to you to do that.

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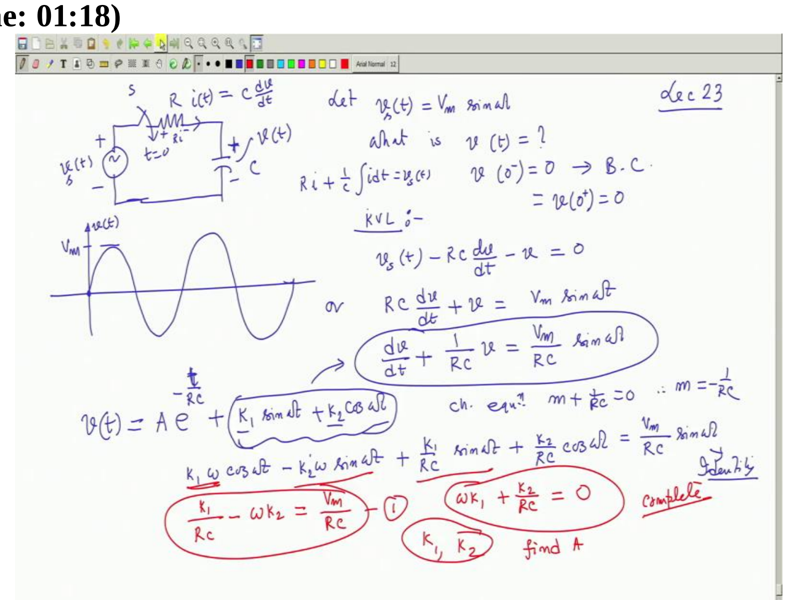

Suppose an RC circuit very quickly let me do this so, that for completeness sake it can be you can proceed further. Suppose an RC circuit is there, I will switch your need to a semisolid voltage source. So, this is Vt and I will do it rather quickly C, and the switch will be closed that t = 0 and let vt B is a call to V max BBM rating sine omega when I say I am doing this so this is my supply voltage, which is semisolid elevating.

And I am telling I will close this week at equal to 0 means at this point when voltage is going to do through 0 and towards positive, this is the instant of switching and this is the max, the max, I want to find out not only current, but voltage across the plate of the capacitor. Suppose I want to know what is BCT I want to find out which visited with this + despite us and I tell you the boundary condition is is a call to general if you just allow me to do this because otherwise I have to carry this one this I will write as supply voltage vs and here I will simply right to voltage across the capacitor is not MIDI like that.

So, Vst is V max sine omega t and this is given boundary condition which it tells you that V + 0 has to be also 0 now, so, these tapes are once again same, I have to write down the KVL here, but recall that they do the way I try assume So, this it is nothing but see divinity. So, if I have to solve for vt, I have to write down a differential equation involving beats I generally solve for vt fast then it can be found out if you have got the expression of capacitor voltage seem to DVD at you Do you get it.

Otherwise what happens if you want to write down the KVL equation this is Ri KVL equation this is also true that is this drop here Ri + 1 by C integral IDT This is the voltage drop here will be equal to VST that is perfectly. But you see this is not a differential equation it is a it is an integral sign is there, but whatever way we have learned is better form a differential equation and that differential equation can be formed easily in terms of bt what is the KVL.

We have done this in the route it will be VST - 2 + - Ri that is - RC d vdt i is this 1 and - v + 2 - back to this point and this is equal to 0. Therefore, once this differential equation is written, I will go a bit faster so that you understand what I am doing. So, this is the sine omega t. This is the thing this is the forcing function then which it is not necessary absolutely but follow 1 method first divide by ERC bring it to that form and write it like this VM by RC into sine of omega t.

So, here is the differential equation with a forcing function. So, once again the characteristic root characteristic equation I will write straight away it is m + 1 over RC = 0. Therefore, characteristic root is - one over RC. And then the solution before more nature of solution is very easy to write it will be only a single group no multiple Group A into e to the power - 1 over RC into T and + solution due to forcing function solution due to forcing function once again it is the sine omega t term so K 1 into sine omega t + it is higher order derivatives.

So, higher order derivatives we only is sine and cosine term, so all sine term to group together and write a big constant K 1 sine omega t, it has to + K 2 cos omega t got the point in the same way as I did in case of finding current in NRL circuit. So this is the literal solution. And I am telling you, if you have understood this expression, your only job left is K 1 K 2 and you have almost solved 70% of the problem the moment you write this from this nothing could be nicer than that.

Differential equations this is the solution you have to find out K 1 K 2 et cetera. How to find out K 1 K 2 because the solution due to forcing function will satisfy the differential equation alone. So, use that 1 in the same way as I d dt. So, put this solution to this differential equation and it will satisfy this 1. So, it will be K 1 omega d by dt cos omega t cos omega t + 1 over RC is there into K 1 omega cos omega t this will be this term.

We are still there is another differentiation you have to do that will be equal to K 1 omega cross omega t - K 2 omega sine omega t is not d by dt this part only am putting this will be there + 1 over RC into this one. So, one over RC K 1 by 1 over RC sine omega t + K 2 by RC cos omega t 4 times will be there and that will be equal to V m by RC sine omega t got the point So, this is how I will get, then the next step is it is an identity an identity it has to be satisfied for all time D.

Therefore, the coefficient of sine omega t and cosine omega t you equate so, sine omega t terms are in the same way exactly same way, this is sine term this is sine term. So, you will get K 1 by RC - omega K 2 this is the coefficient of sine omega t from this side It must be equal to V m by RC. This is equation 1 and similarly equate the coefficient of cosine omega T which will be equal to omega K 1 this term and + K 2 by RC and this has to be called to 0 because there is no cosine terms.

So, these 2 equations can be solid to give you the value of K 1 and K then put it here put the boundary condition V 0 + = 0 to detail my main discuss 20 got the point and I live this problem to you to solve for it solved for bt because it is exactly in the same line as we have done Adele circuit it will be a good exercise to try on your own and get the value of the voltage across the capacitor and then get then do CD vdt to get the current.

So, you see that the, so, there will be also capacitor voltage cannot change instantaneously, but there will be you know transit and Tapir adding in the capacitor voltage like that you please complete this and Is this because of this measure complete So K 1 K 2, then what do you have to do find out K 1 K 2 solve these 2 then fine after this we have to find A from total solution main do this now, what I will be doing says any circuit I can write like this and solve for the tangent.

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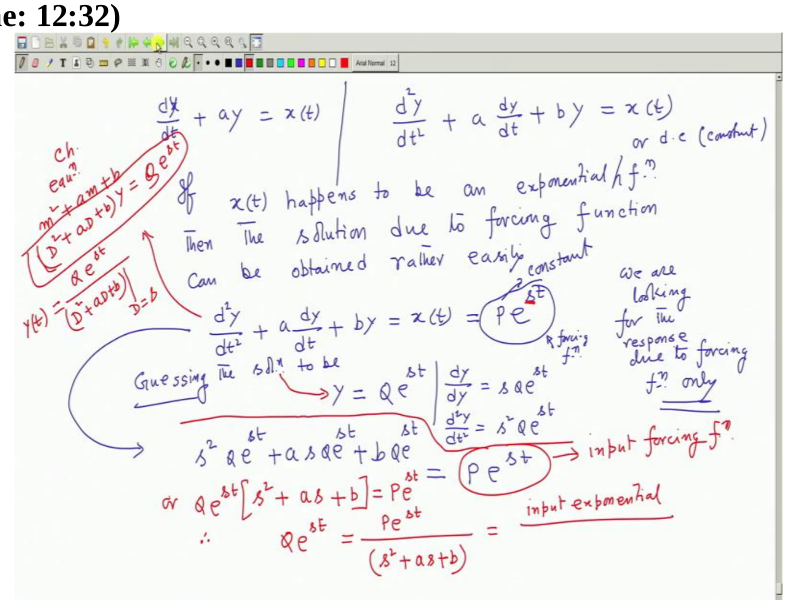

if it is semisolid, I told you the supply voltage any differential equation it will be after all a differential equation suppose we have a differential equation like this d by dt + a y = x t depending upon the problem if x t is Voltage right voltage here, y is current and so on. So, coming back to this once again any differential equation or it could be in general a higher order differential equation.

Because if there are 2 energy storing elements + A dy dt at second + b y = x t. Now, you see, we have done this for semisolid current for DC voltage, we have done that now, what I am planning to do is this another interesting way to handle the previous thing earlier it was like that when we did not know differential equation then you integrated by and integrating factor try to solve them that was a huge task.

Then we improved upon it with I told you that solution of a differential equation consists of 2 parts 1 is the natural solution depends upon the characteristic root and e exponential terms will be there. And there is another solution which depends on forcing function which can be written straight away as K 1 x + K 2 x dot + K 3 x double dot and so on. And that way applying that only I solved this RL and our society now, for a class of x t the level can be reduced.

That is what I am going to tell you how if x t happens to this point you try to understand happens to this and exponential function then the solution, due to forcing function can be obtained. Rather easily. What is this statement says? Suppose you have a solution let us take a second order system d 2 y dt 2 + a dy by dt + b y see solution due to natural response will be depending upon the characteristic equation.

Now, what I am telling if xt is the form as some say e to the power st it can have some magnitude also say been doing to the policy on the right hand side it is an exponential function and this I will just change a bit effects it happens to be an exponential or DC or DC constant or constant because with s = 0 this will appear to be a constant P is constant. Suppose, we have excited a system with this signal and want to solve for a while then how to find out the solution in this fraction let us see.

See if this is the solution, this is the function first I say that this is constant I mean exponential constant we argue like this. This is very interesting way to look at the problem the left hand side and we are looking for we are looking for the for the response or the response due to forcing function only. This you must understand forcing function only because other terms will be there as we do the b m1 + A 2 into e to the power m to T that thing We will be there. So, forcing function is these, this is your forcing function.

Now, you see, look at the left hand side and the right hand side and forcing function alone will satisfy the differential equation and I am looking for the solution. Now, the question is what should be the nature of this solution if the right hand side is exponential e to the power s t see it to the power x t. And for all time, whatever will be the nature of it for all time left hand side if you compute it has to match the right hand side that is the crucial point.

Therefore, the question is can I guess what should we the nature of y 2 the answer to this question is that yes, you have to look for such a function y whose form does not change when you differentiated then only this 3 terms will conspire and will always give you this term that is the nature of the solution of this site guess guessing it has to be also exponentially to the x power x t, then only it will differentiate it twice it still remains to be parsed into some constant dy dt you do it with the power x t into some other constant + by it is constant.

yes all these e to the power a ski towns may add up and give you 0 for all the time t it cannot be one term is d squared and that is dq and things like that which will lead up to 0 for all time t my new solution means for all time t therefore, we guess guessing the solution to be to be this very interesting point I will gaze the solution will be the sound cue intuitive the forest it has to be nothing doing peace it was the power st.

So, left hand side will also give rise to a factory to the bar st and exponential function is such a nice function differentiate hundred times it will still retain its form only thing its magnitude will get manipulated amplified or decreased that is different issue, but only this form of solution can satisfy the forcing function solution scraped. So, what I do I put this solution here on the left hand side. So, you know d y t have to calculate divided because I have to put it here.

So, dy by dt will be how much if you assume this to be solution is into Q into the power s t is not and D to I DT to also it will require if you differentiate it one small it will be a square q power st desert then I put it here on the left hand side, put this on the left hand side what do you will get. We will get S squared Q into e to the power s t that is the constant + a times dy by dt which is a cue A to the power s t + b times y which = q e to the power x t and on the right hand side I have got P e into the power st. This is the thing. This is the question that we get.

Now, on the left hand side what I will do I am going to do this thing I will take Q into a to the power s t, which is common to all the turns and inside I will get a squared this 1 + a and + B and right hand side is P e to the power s t these the input forcing function input forcing. Therefore, I will say because I want to I have guessed the nature of the solution but still t is to be determined the solution is totally not known. So, I will say as is known my Do from the input test it excitation.

So, I will say to you into the Qe power s t is nothing but P into the power s t divided by A squared + A + B is not. Therefore, you see, if the right hand side happens to B a exponential function of the form e to the power is t and this is the solution because this is the solution due to the power st then becomes the input exponential in language if I right input exponential divided by is the characteristic equation of this one of this differential equation.

Characteristic equation is how much m squared + m + b these the characteristic equation is not all you can say in terms of differential equation you think in your own way a D + b into y = Q into e to the power st Q into e to the power st is not this differential equation in short can be written like this D squared is the operator plus a D + b into y. So, it looks like that y t solution due to voting function will be this one we have got from this equation I will say it is q you a to the power s t divided by D squared + aD + B as if some algebraic thing.

Why is this divided by this I bring it, but, this is not correct to write the operator in this way it is difficult to manipulate, but the result is these characteristic equation for visions calm and I will say O this is d call to s. So, the solution due to forcing function when it can be expressed in the form of an exponential the solution will be simply that input divided by this characteristic equation evaluated at D equal to s. So, to summarize what I am trying to tell.

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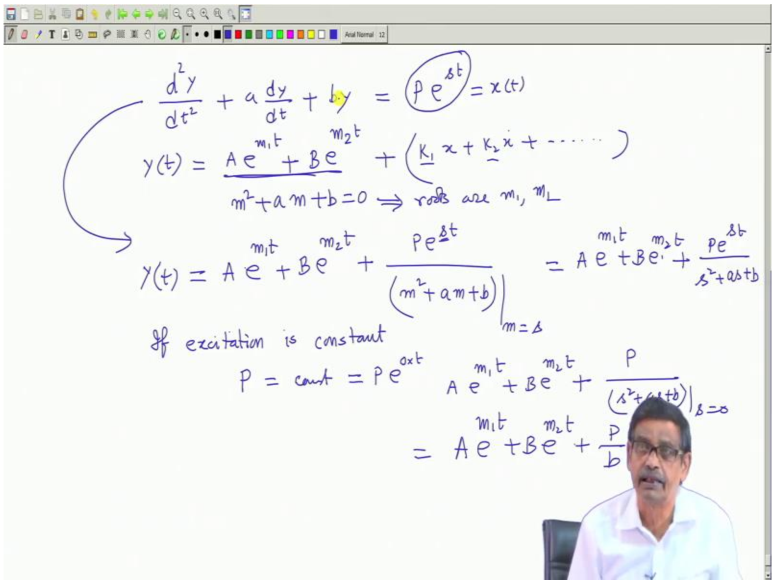

The next thing is that if you have got a differential equation d 2 y dt 2 + a dy dt + b y = suppose, Q into e to the power st so, this is p what was the solution I was telling you earlier solution will Be from earlier thing total solution will be A into e to the power m 1 t that natural response will be there P e into the power m 2 t + then we learned it would be like this if you say x t, it will be K 1 x + K 2 x + and up to infinity like that. But I am now telling something new that is fine, but we found out from the last slide.

What is this m 1 m 2 is the root of this characteristic equation m + b = 0 root m 1 m 2. So, this natural response will be there whether there is an some consolation we get to calculate the solution due to forcing function. Yes, it is Now, I am after learning that I will say total solution will be this A e into to the power m 1 t + B e into the power m 2 t if the roots are distinct like that + what I am telling if the right hand side is of this special form.

I will say it will be simply p into a to the power st solution due to forcing function input divided by the characteristic equation. that is m squared + A + B but put for m S that = A into e to the power m on t + b into e to the power into t + off P into a to the power s t by square root for m patois this coefficient is square + s + and you are ready with the solution got deeper determination of constant those things. Now become that K 1 K 2 finding out was slightly bigger.

What happens if, excitation is D c is constant say right and that is, right hand side is p = constant. Anyway this can be written as p into the power 0 So, the solution then will be A into e to the power m on t + b into e to the power m 2 t + p into the pod 0 t that is p divided by a sequel to 0 that is a square + A + B = 0. That is, it will B = m 2 it to the power m on T + b into e to the power m 2 t + p by b. If the right hand side is constant, then the solution due to forcing function will be simply supplies these 2 terms p by b. Anyway, we will continue with this in the next class as well. Thank you.