Network Analysis
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Lecture # 24
Solution Due to Exponential Forcing Function

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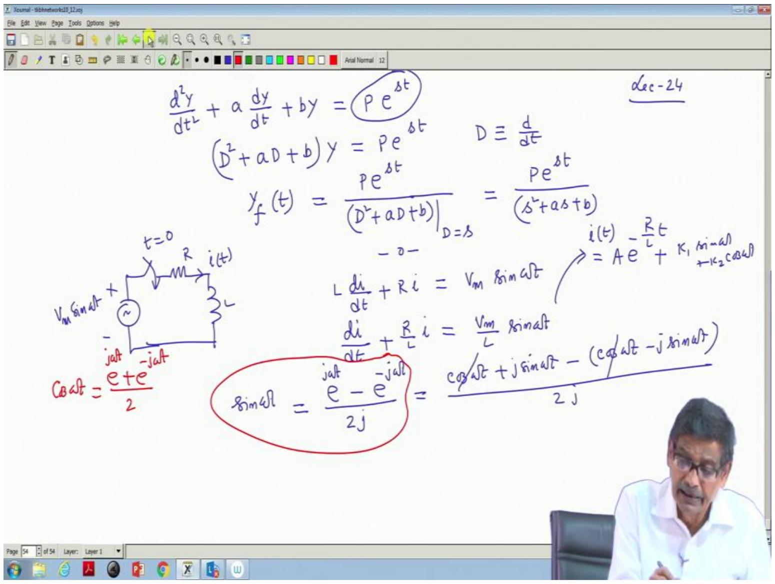

So, Lecture 24 let us continue with what we have been discussing that in a differential equation in my last lecture I told you the solution due to forcing function can be found very easily provided the right hand side you can express it in terms of e to the power st. If s = 0, it means it is equal to P constant. And I told you, this equation can be written as sometimes I am writing aD like this I hope you understand this is point is equal to P into to the power of e st where D is the operator d/dt that is fine.

Now, what it is being told is that solution due to forcing function will be P into e to the power st divided by D square plus aD plus b but this is to be evaluated at D = s whatever s is there that means, it will be equal to that the input signal divided by s square plus as plus b is it everything is known. So, solution due to forcing function can be found out very easily and of course, the other 2 terms will be there exponentially decaying terms has natural response.

Now, is there any restriction on this as such you know, s could be any number real number also it could be a complex number no problem. So, for the only thing is see you are evaluating this denominator by putting D = s, it may so, happened that this will give s to 0 then you are will be in trouble then you have to see we have to integrate or the way I told you have to solve the problem got the point. So, except that the condition.

So, in the denominator does not vanish when you put D = s that is there but in most of the cases it will not be and it is a very handy tool to find out the solution. Now, I started telling you that if the right hand side is a Sinusoidal Excitation or a constant value, then the forcing function solution due to forcing function can be easily calculated using this. Now, the question is suppose I say I have solved this problem, I will recall that problem and try to apply that for example, and RL circuit is there and you excite this with a sinusoidal voltage Vm sin omega t there is no exponential asset at t = 0 you close this switch.

This is RL this we have done classically finding out the constants K1, K2, etc and including the constant a. So, it is a first order differential equation, what was the equation it was L di/dt let us do this here plus Ri is equal to Vmax sin omega t that is what I wrote it here is not that was the theme now, and then I was dividing which is not necessary, but you always follow this R/L into i = Vm/L sin omega t. Now, what will be the solution of this is known, I wrote it earlier as it total solution will be A into e to the power - R/L into t plus some constant K1 sin omega t + K2 cos omega t etc I did this earlier in this way now, let us do it in a different way so that I can equalize this property.

So, on the right hand side as such there is no exponential time, but if you look carefully you should know from complex algebra that sin omega t can be written as e to the power j omega t - e to the power - j omega t / 2 j is not because this is true I think cos omega t plus j sin omega t. This is the first term - cos omega t - j sin omega t, if you put this is by 2j, so, on the top it will be this term will cancel 2j will cancel. So, this is the important thing. Similarly, cos omega t can be written as whenever necessary will use also that e to the power j omega t + e to the power - j omega t / 2. So, these 2 relations are very interesting.

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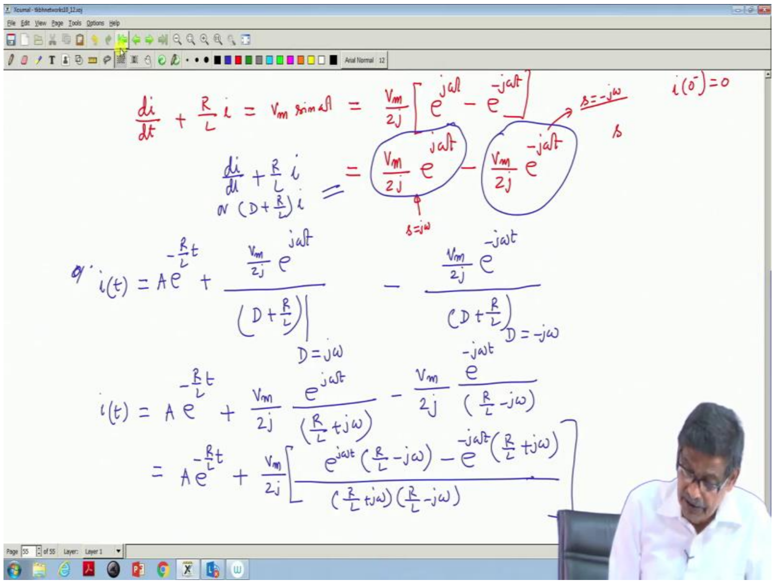

Therefore, I will get in this problem that this was the differential equation di/dt + R/L into i = Vmax sin omega t but this I will now write it as Vm/2j into e to the power j omega t - e to the power - j omega t this way I can write therefore, on the right hand side you have got Vm/2j e to the j omega t Vm/2j e to the power - j omega t. So, it is here the value of s for this 1 s is j omega and for this signal it is s - j omega.

Therefore, as if you have excited this system with 2 input signals and since I have told you RL with the initial condition 0 we have like a linear circuit therefore, I can apply superposition theorem for e to the power this signal I will find out what is the response for this signal I will find out what is the response and I will add them up when both of them will be present that is how we did it. So, that is how the things will be.

So, the solution due to this function if I want to write it in 1 stroke I will write it like this follow me i t total solution will be equal to e to the power – R/L this thing that is the most simplified way find out the route that is e to the power – R/L and then we are doing the forcing function. Now, this is the differential equation di/dt + R/L into i is equal to this is 1 input signal subtracted another input signal and I will apply super positions.

So, first we take this time what I told the solution due to so, this equation is D + R/L into i or equal to this is not D + R/L know is correct this is the thing. Then solution due to forcing function will be the input the signal I write it Vm/2j into e to the power j omega t / D + R/L you bring this down here on this side and evaluated at D is equal to j omega, what is the value of s is j omega for the signal then minus one second another exponential signal whose this - j omega. So, e to the power - j omega t that is fine divided by D + R/L into D is equal to minus j omega now, got the point.

So, I have applied superposition to these 2 signals, it was only real signal sin omega t, I converted to as a difference of 2 exponential signals. So, far as these different see everything is revealed here, so, did not worry about that, but this real signal Vm sin omega t can be thought of 2 exponential complex signals difference of that, but there was no restriction on s it could be imaginary as well. So, this is the thing you got. So, I will write the total solution i t as A into e to the power – R/L into t + Vm/2j is there and on the top it is e to the power j omega t and below it will be R/L plus j omega for D is to substitute j omega – Vm/2j into e to the power - j omega t divided by here for D is to substitute - j omega.

So, it will be R/L - j omega. So, this will then become equal to you have to do some algebraic manipulations, complex algebraic manipulations plus Vm/2j let it be there, because it is common here. Now, below it will be e to the power j omega t this one so, a plus j omega Vm/2j taken outside for both. So, A it will be R/L + j omega into R/L - j omega is it denominator and just subtracting them and then it will be here R/L - j omega and - e to the power - j omega t and R/L plus j omega is it correct that way it will be.

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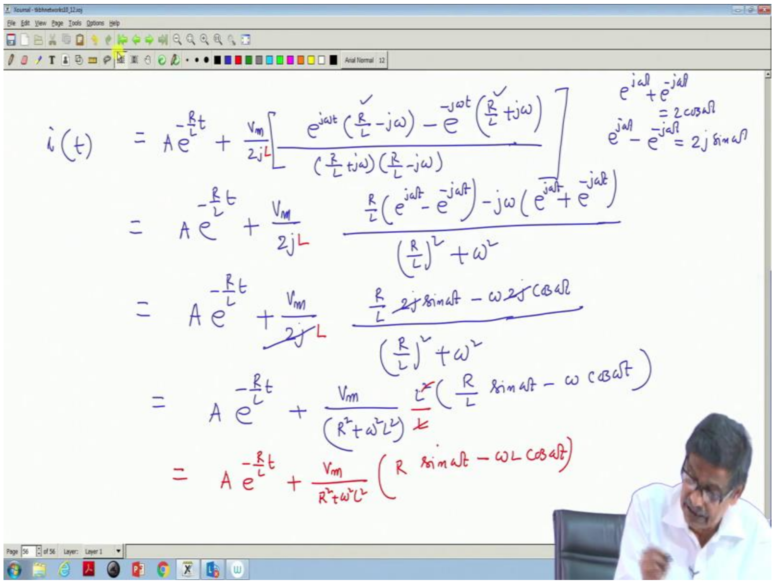

So, the solution then will be I will copy this thing I start from there. So, it was some comes from the top then this was - j omega t and there was bracket is it this is hopefully. So, this is the thing. So, I just simplify this. So, this is equal to i t total solution A into e to the power – R/L into t + Vm/2j and then below it will be R/L whole square plus omega square A + jv into a - jv is A squared minus b squared if you do this will become plus.

Then on the top what you will be getting is this R/L if you take common it will give you e to the j omega t and from this R/L will give you - e to the power - j omega t. These 2 terms have taken and then you see - j omega. If you take common it will be e to the j omega t from this and it is also minus. So, this will become plus e to the power - j omega t is that correct. So, this will be the theme. Now, this let simplify this – R/L into t + Vm/2j.

So, this will be equal to R/L whole squared plus omega squared. Now, this 1 as we have right now told you this is equal to RL into 2j sin omega t right now, I have to do and this will be equal to - 2 cos omega t - omega 2j cos omega. Always remember that e to the j omega t plus e to the power minus j omega t is equal to 2 cos omega t and e to the power d omega t will be using this very often in circuit analysis 2j sin omega t. That is what I have done here clear now, this 2j goes and your solution is A into e to the power – R/L t + Vm by here this can be written as R squared plus this denominator, this square and on the top there will be an L squared coming in and it will be equal to sin omega t - omega cos omega t.

Is that correct? Something I missed? Hopefully it is correct. One thing I missed here, see I divided by L got the point this was the differential equation I started should we give di/dt R/L. So, here was the term was in the VM/L. So, everywhere it was l got the point. So, you should be careful I let me put in different color this L is missed, because in the excitation it is there coefficient Vm/L this is L, this is also L, this is also clear, hopefully have incorporated everywhere. So, this one was L, so this was L.

Now, this was also L now, here was 1/L square and there was an L here these L square I pushed it up. So, you have only one L left here. So, your solution will be A into e to the power - R/L into t + Vm/R squared + omega squared it will be push it inside it will be r sine omega t - omega L cos omega t. And this one, I have done just 2 lectures back that this you know, you can write this as this thing, but let me do that so that you do not miss any point I want to make.

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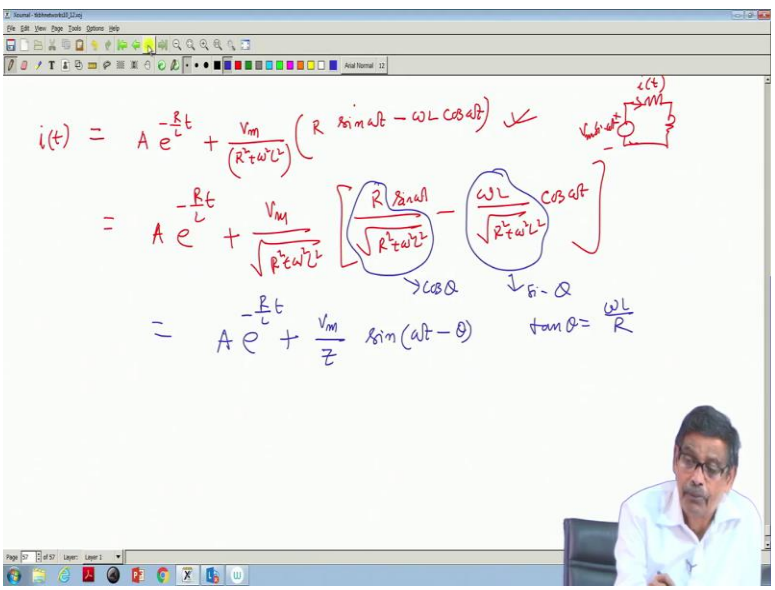

So, so, this is your i t total I got this all the things that they see first thing you should note is this that there was a circuit RL which was excited by a sinusoidal voltage and everything was real their whole response in the circuit ultimately although in between some complex number came into the per j omega t, but ultimately I expect it you do have to have real number. So, things are correct why response will be complex it will not be they did these terms conspired in such a fashion the imaginary terms vanish and everything is the real you have applied real signal you getting real signal for getting some advantage in computation, I converted that real signal sin omega t as some of e to the power j omega t - e to the power - j omega t by 2 j omega that is there by 2j.

So, it is like this then we stop the things I know how to handle so, everything is real. So, this is Vm by this is only right R squared + omega squared L square and this will be you know as usual route over R squared plus omega squared L square minus omega by route over R squared plus omega square L square. Here is sin omega t and there is cos omega then you define this term R by these as some cos theta then this has to be sine theta and then you get the same answer as I have got earlier by using K1, K2 etc.

So, Vm by this if you call it as z impedance you must be knowing and to this will be then written as sine omega t - theta, where theta is equal to omega you have got the bind you therefore, I think the number of computations Things have become less. In fact with numbers if the circuit is given it will be much faster because I am carrying on all the variables are L in some numbers is given you can do it rather fast that we will see the second thing I want to tell you about.

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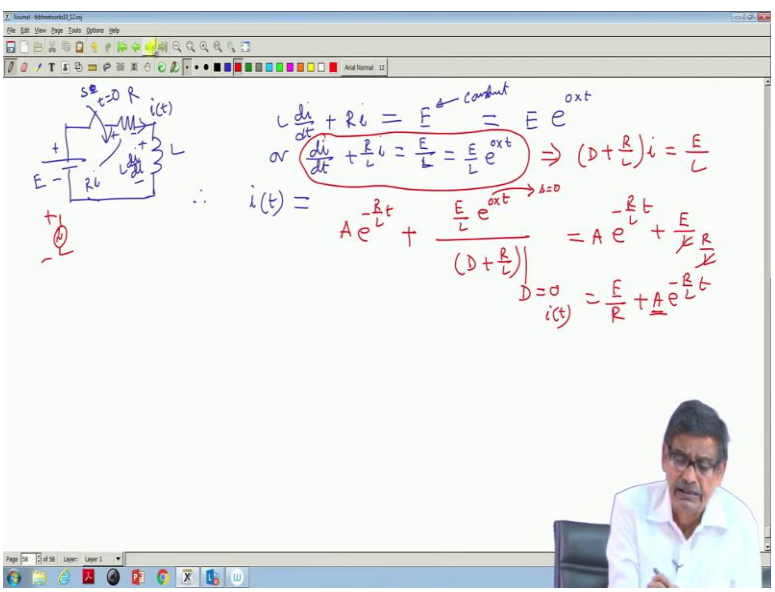

Another problem which I have already solved, for example, an RL circuit it is there and you have a DC source. See I am trying to do same problem in different ways then choose your method. So, this was E and I want to find out current but this time I am applying a DC voltage. So, what will be this drop is L di dt. So, L di dt P Ri do not write down these differential equation without showing the direction of the current and the polarity of the voltage always draw the direction direction of the current indicate very clearly, then polarity of the voltage correctly.

And from this I will get this, but this time this is what constant but constant can be written as E into e to the power 0 into t this is what therefore, I will say this time the total solution i t will be equal to beta you the way I am always doing so, this is this is phi then di dt + R by L into i = E by L these exactly that a L I t squared in my earlier thing.

So, E by L not to and this is equal to E by L this constant number E L or constant e to the power 0 into. So, this is the differential equation in the form you try to express then you say I t will be equal to A into e to the power - RVL into the characteristic route + E by L into which is equal to E by El e to the power 0 divided by D + T is equal to the value and for this value D you put 0 that is what. So, this equation is nothing but d + R by L into i = E by L. So, this can be expressed in terms of e to the power st where st equal to 0 am so, D equal to S S is 0 here S is 0 in this particular problem, so, difficult to get to you.

So, if you do that, you will be getting a into e to the power - R by L into t + e by L and R by L this Ll goes that is all So, solution due to forcing function can be very easily found out and then rest up these steps you know this is your i t then apply boundary conditions to get a determined. So, what is the thing that if an excitation is given and that excitation can be expressed exponentially as some of 2 more signals, then the solution due to forcing function where those K1 K2 K3 were involved in that sense your labor is reduced to a great extent.

Fortunately, in network analysis 1 of the very important signals are sinusoidal signals or constant well signals either a battery or sinusoidal voltage source. So, it is what studying then if your input signal happens to be either constant or a suicide le wearing, so, you cannot apply this mind new for any arbitrarily voltage source or current source you cannot do that. But, then an important signal in network analysis in signals and networks is sinusoidal signals.

Therefore, it is what studying why it is like that later we will tell you something more about these are called characteristic function of this system and so on. That will take later now but now as you see.

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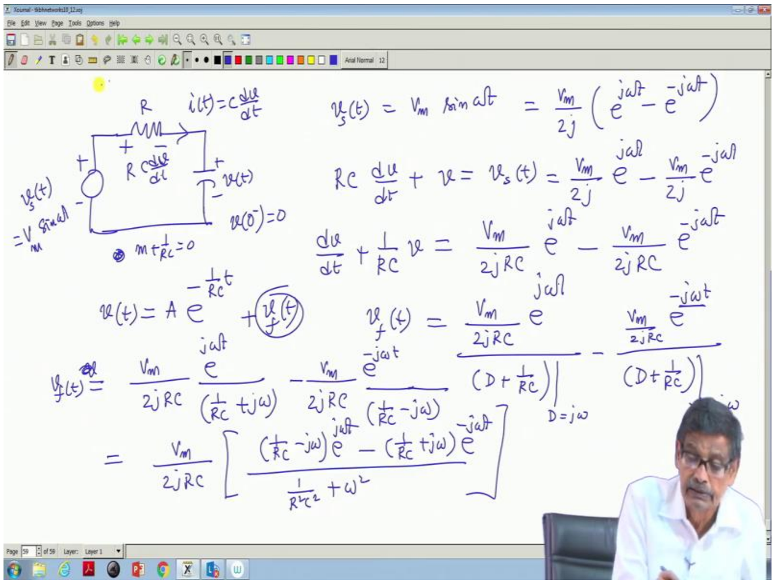

I did not really solve this RC problem I told you to solve, but let us also solve it in this way I will give you indication to this suppose a sinusoidal voltage you have applied this problem I told you what few Stapes using those constants K1 K2 et cetera but later seen the light of this 1 let us try to approach that problem sine omega.

So, first of all I will write vt is a call to you are applying a sinusoidal equation i omega t, I think Vst this is the supply voltage Vst and this then I will write it as Vm by 2 j and e to the power g omega t - e to the power - j omega. This is my input voltage and what is the current in the circuit in terms of the capacitor Voltage this is vt. So, the current in the circuit i t =3 dv dt. Therefore KVL equation + - R i that is divinity and + v.

So, R c dv dt + vt = Vs t supply voltage, which happens to be equal to Vm by 2 j 2 sources - Vm by 2 j e to the power - j omega t. An initial condition given is v 0 - = 0 so that I can apply superposition to them and so on. So, this is the thing. Therefore, the first rate is dv dt + 1 over RC into v = Vm divide by this thing 2 j RC e to the power j omega t - V m to J by RC I divide both sides RC and this is putting into the pod to make it then what will be the solution voltage across the capacitor at any time t will be the characteristic root that is the natural response will be to the power - 1 by are seen to be that part you have to do what is the characteristic equation d m + 1 by RC = 0 that is all from DC find out - 1 by that party is over.

Then the next part, the voltage across the plate of the capacitor and here is an exponential input. So, I say that Vft i i separately that is the solution due to forcing function the empty will be here is an exponential. So, that voltage Vm by 2 j RC into e to the j omega t that is the input divided by D + 1 over RC because, you know, so, D + 1 by RC but you calculate these for D = 0 to give my then - the Vm, this is the input signal e to the power - j omega t divided by D + 1 over RC into this to be evaluated at D equal to what is the value of s - g omega - j and let us proceed.

So, solution due to this time the Vf t will be equal to put that number Vm by 2 j RC into e to the power j omega t. And if you put this number here it will be 1 over RC + j omega this term faster and then - Vm by 2 J RC it is there then you have to put the D = - g omega e to the power - j omega t and this divided by 1 over RC - j omega you have to just manipulate this so what it will become be m by J RC come outside this factor, take it outside and then below this into this a + JB into a - JB is a squared + b squared.

So, here is this 1 by r squared c squared + omega square no imaginary thing vanishes in the denominator, then you are adding these 2 terms. So, it will go up on over our C - g omega into e to the j omega t. The theme to this - 1 over RC + j omega into e to the power - j omega t. That is the thing irrigate once again this can be written as Let us complete this so that at least to some extent, let me complete. So, this is the thing copy go to next page and paste it.

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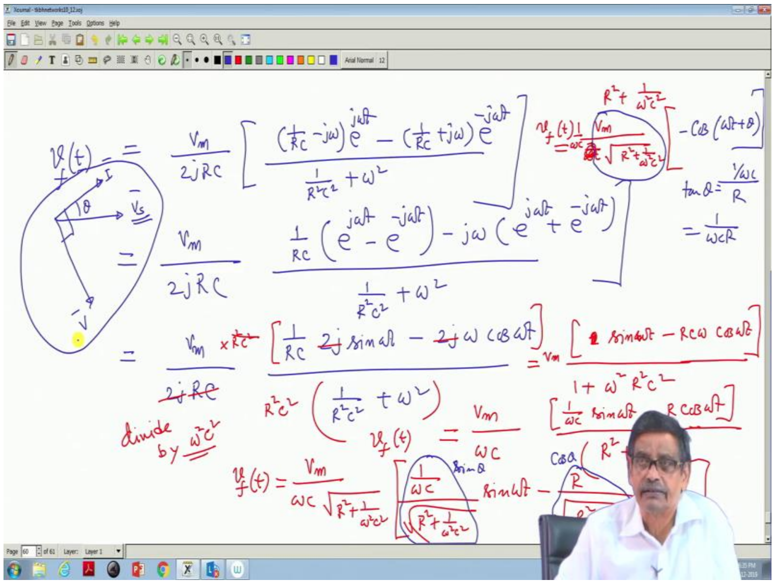

So, this is the thing for you have got. So, this is equal to all the diff solution due to forcing function it will be like this. I think this came from previous Tina and here was the bracket close this was the thing anything missed no. So, this thing is Vm by 2 J RC that is fine and below it is 1 over R squared c squared + or - square that is also good. Then you see it is 1 by ERC and you will be left it to the power omega t and 1 by RC - e to the power - j omega t it will be there then - j omega this term and disturb and both are minus.

So, it will be inside the bracket it will be it to the power omega t + e to the power - j omega. So, this is the theme, then at rest of the things, just manipulation with complex numbers you should be very careful. So, this is equal to 1 over R squared c squared + omega square below it is there, do not touch that. And here it is 1 over RC this thing I have learned it is equal to 2 j sine omega t and this thing I learned it is cos omega t so - 2 j omega cos omega t that is the thing.

Now, this today goes everything should ultimately become real, because we are dealing with real circuit everything is real. So, it will be like that, but the point so, this will be the thing. Now, what you do is this, hopefully everything is fine. You can multiply with R square in the numerator and denominator so, if you multiply with R square. So, what should I do omega squared let me died. So, this can be written as 1 over RC.

So, you write it like this below 1 you divide fast by square c squared r squared c squared on the top and bottom if I multiply and divide by r squared c squared you multiply this 1. So this then will become below it will become 1 + omega squared r square c squared it will become and on the top on RC goes here on as he will go and these are C will make it to sine omega t hmm to DJ gone, so r squared c squared multiplied it is like this 1 RC was here I cut it these are cm entering here so then it will be 2 only 2 left RC goes this actually goes and 2 sine omega t 2 will not be there, light and then - RC omega cos t.

It will be like this here and outside it is Vm this will be the Vm. Now, we will write down another way because you are see impedance, we know r square + 1 over omega squared c squared. So in that fashion, I want to bring it and I will be able to manipulate how so I divide by divide numerator and denominator divided by omega squared c squared. I think you can do better than what I am doing. But essentially, so I am dividing by omega squared c squared in the numerator and denominator.

And so what I will get it here is R square + 1 over omega square c squared. I will get it and on the top what I will get omega square c squared if you divide it will be Vm 1 by omega c you keep it outside and another 1 you enter - cosine omega here and that is the thing you do in your own way, but the point I want to make it will be like this. Then in the same way you say that, this is the voltage across the plate of the capacitor due to forcing function only forcing function.

So, that is that will be eventually the steady state voltage so, this can be then written as next day that I will not proceed further, but I can I now know what is happening be by omega 3 is there. So you take out a square + 1 over omega square the same way I did for this 1, and this 1 can be written as 1 over omega C route over r square + 1 over omega squared c squared sine omega t and - r by root over r squared + 1 over omega square c squared hopefully if done correctly, you take it as nonetheless.

Then you define this to be sine theta and this to be cos theta. Then, sign a sine B – cos C because p. So, if you do like that then Vf t can be simply written as the Vm divided on 1 over omega c divided by r squared + 1 over here dimensionally also discovering correctly into you define this term to be sine theta and this term to be cos theta if it is costs today it has to be scientific If that is the case of sine B - cos A cos B is what - cos a + b.

So, it will be some - cosine a + b theta were 10 theta is 1 over omega c divided by r is equal to one over omega c and that is reactance by desistance etc. So, this will be the voltage what is happening. So, this will be the voltage everything is defined where 10 today is this 1 if you like you then calculate the current C dv dt will be the current. Do not get upset by this negative sign because it is not that this is the voltage you see Vm by 0 is the current into reactance gives you the peak value of the voltage that is fine.

But in a capacitive circuit this is your supply voltage, we know that this corrected supply voltage is we as I win although I have not done severe diagram, but I am trying to see whether things have gone getting correctly. So, current will be leading a current will be leading and capacitor voltage will be voltage across the capacitor will be lagging this, is the supply voltage current ground in the circuit leads it by theta that is fine, but voltage across the capacitor will be lagging the current by 90 degrees.

So, these things contribute to this negative sign anyway do this, but the emphasis I want to put on these particular lectures and the last 2 lectures is that in case the excitation function happens to be DC or sinusoidal things can be expressed in terms of your some of 2 complex exponentials then you were through very quickly calculate and as I told you if these are numbers here I was struggling a bit here but anyway you can handle these situations This is hopefully correctly check that and informed me through mesh or whatever it is a very anything is connor most surely everything is fine.

Then you get although the phasor notation I have not you yet introduced but former own benefit I wanted to check whether I got the correct results hopefully it is correct. So I will continue with this things in the next class. Thank you.