Network Analysis

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Lecture # 25
Mesh and Nodal Analysis with Time Varying Source

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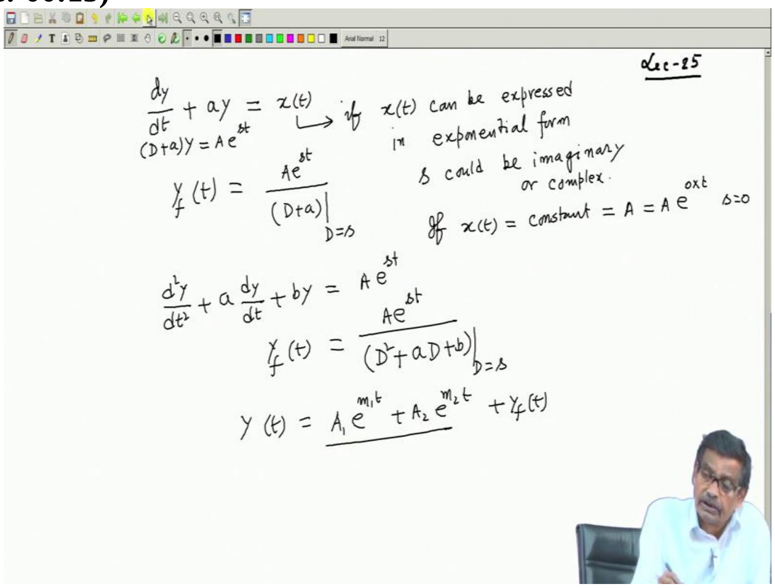

So, welcome to lecture number 25 and if you recall that in our previous lectures, what I was trying to tell that how to solve a differential equation say a y if the forcing function x d, if, x t can be expressed if x t can be expressed in exponential form then the solution due to forcing function can be straightaway written as because this is the equation. So, it is nothing but D + a into y = a call to suppose, a into a to the power x t then y f t will be simply this input e to the power st degree divided by these D + a but D = s.

Therefore, use a lot of time to find out the solution due to forcing function when it can be expressed in exponential form as is special case if it is could be complex It does not matter, it could be imaginary or complex It does not matter or complex and example of that I have shown has a special case if t = some constant A = A, which means that a into e to the power = into A S = 0 that is a strength then also it is valid.

Then whatever additive that be a faster differential equation it could be a second order differential equation for example, D 2 + a dy dt + b y = A to the power e st then solution due to forcing function will be simply the input signal into the power st divided by this will be now, you are accustomed to this ad + b, were d should be replaced by s there may be exceptions as I told you and we applied this to find out the because what happens is this in electrical engineering, the sources in general will be either semisolid varying st quantities supply voltage.

Voltage of the greed is 50 yards semisolid derivation or it could be is the source could be a battery DC. So, therefore, when such a thing happens then the solution due to forcing function can be written as it is. So, total solution of course, that natural response will depend upon this 1 e to the power m 1 t + A 2 E to the power m 2 t and then this solution due to forcing function here of course, you have to find out the characteristic rules depending upon if the roots are distinct it will be the nature and we have considered several cases earlier. Now, today what I am going to tell you.

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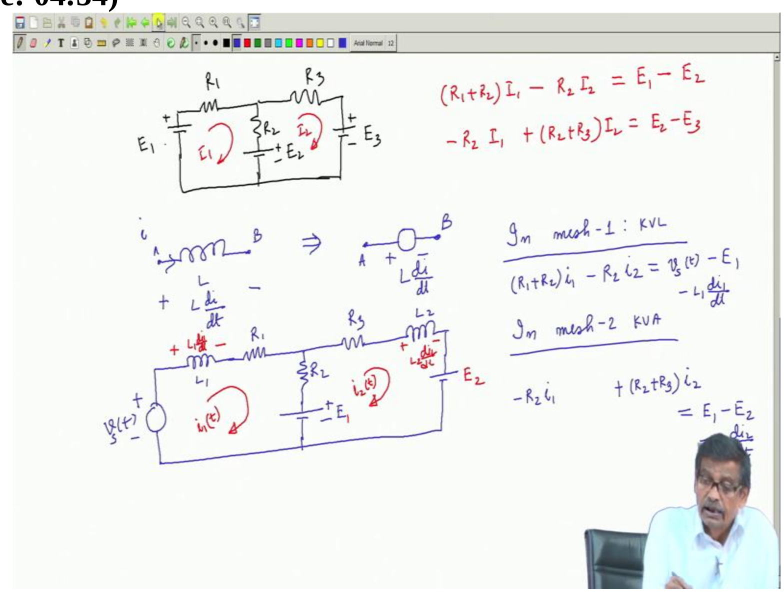

Recall that while explaining the nodal analysis and loop analysis. You recall these things that I told you these sources were DC is not just for example, if you recall this thing suppose there are only 2 measures are there mesh analysis R 1 R 2, this is the R 3, this is a E1 battery, I explained this considering the sources to be what is called constant value DC. And then these tapes were like this there are 2 messes.

You assign the mess currents we can supposed to be the unknowns. then the writing down KVL equation in mesh 1 will be simply the coefficient of I 1 into R 1 + R 2 into I1, then coefficient of I2 will be the common resistance - R 2 into I2 and that will be equal to the sources on the right hand side and the sources where the direction of I1 is this way. So, it will be + 1 and direction of the 2 polarities such that it opposes I1 so, - I2.

Similarly, the KVL equation in mesh 2 will be the coefficient of I 2 will be some of all the resistance is that is R 2 + R 3 into I2 and this is + and coefficient of this 1 will be common resistance into negative sign and this will be equal to E 2 - E 3 then you solve these 2 equation and get the currents in various branches that we have discussed that, but now in the present scenario for example, now we consider if there are time varying sources present.

If there are inductances present capacitances president without this kind of rules will be still we can do something with these rules. The idea is that a recall that in an inductor if you recall an inductor if this is the current and this is the value of L, then voltage across the inductance with this side plus this side - is L di dt. Suppose this point is A this point is B and since these are energy storing elements, some people Say that this can be treated as some source voltage also no problem you show it as a source.

Because after all energy storing elements which are the sources, so, you can supply energy to a system therefore, you are the conventional sources or batteries or greed supply voltage that is there, but, because inductance can store energy capacitance can store energy and we have seen that even without these conventional sources like battery and greed is not present, but inductor with some initial current that is with initial energy stored will still be able to drive current in circuits.

Therefore, they can be also treated as some source and so, we can treat this as a voltage drop no doubt tell the I di dt but I can show it like that No problem. So, remembering this now, let us see how can I write down the KVL equation when inductances also present. For example, first let us take a simple case here is an inductance R 1 here is an odd 1, here these R 2, there may be worth not any other sources plus E and there is another source there another inductance they are able to and another battery like this. And here is a source which is time varying + - Vt is the applied voltage supply which could be any time varying function.

So, this circuit R2 solve and suppose I say that I have learned this maze analysis, why not apply it here. Now, first thing is in this case what you have to do is this EF to you assign the mesh currents and it is expected since the source voltage is time varying currents in the in these circuits also will be time varying and it is customary to represent time varying current with small letters generally people do not write capital I1 capital I 2 small i1 and small i2.

It is expected they will be function of time because the energy storing elements are present. Now, the question is I one to write down the KVL equation In this 1 so, this is then you know this you have assumed I1 So, voltage drop across the inductance will be L 1 di dt and voltage drop in this inductor see I2 is flowing so + - L 2 di dt once you do this and suppose this voltage is E 1 and these voltage E2.

Then I will once again apply the same technique in this way see coefficient of I 1 So, far as resistances are concerned it is fine I will write it like this that is coefficient in mesh 1 KVL equation in this 1, it will be some of the resistances R 1 + R 2 into this smaller I1 that will be there some of all the resistances do not add R 1 + R 2 + L 1 then you will be making mistake because the voltage drop across the inductances is L 1 di dt not I 1 so, with resistance this is what we have learned.

So, this will be the coefficient of I1 what will be coefficient of this is suppose R 3 coefficient of I 2 it will be - R 2 into I2. This will be the coefficient of I 2 and then this should be equal to all sources I will write a on the right hand side. So, as you can see, it will be the polarity of supply voltages, I will write VST the same way as I 1 Capital E 1 – E 1 we know this and then there is another source here because it is a L 1 di dt keep the inductor voltage drop to be source and then the polarity of the source is that write - l 1 di1 / dt.

So, by inspection I am writing now, that you must understand. So, this will be the main situation here and in the second in mesh 2 KVL will be equal to coefficient of it will be equal to add to some of all the resistances + R 3 into i 2 that is there then this is plus and coefficient of i 1 is - R 2 into i 1. So, minus R 2 i 1 and some of R 2 + R 3 into i 2 and on the right hand side all the sources will be required including the inductance voltage drop.

So, it will be equal to as you can see it is the E 1 + i 2 – E 2 and the polarity of this voltage is – L 2 di2 / dt. So, this will be the KVL equation 1 can use the same concept as that 1 only thing since things started time varying it is not all terms are i 1 there is di 1 / dt. So, there will be differential equation you have to solve and how to solve differential equations we know that so, you have to eliminate i 2 problems from 1 equation get the eliminate I to write it here and form a differential equation with I1 alone and then solve for i 1 then i 2 so this is the idea I am telling what you can do similarly.

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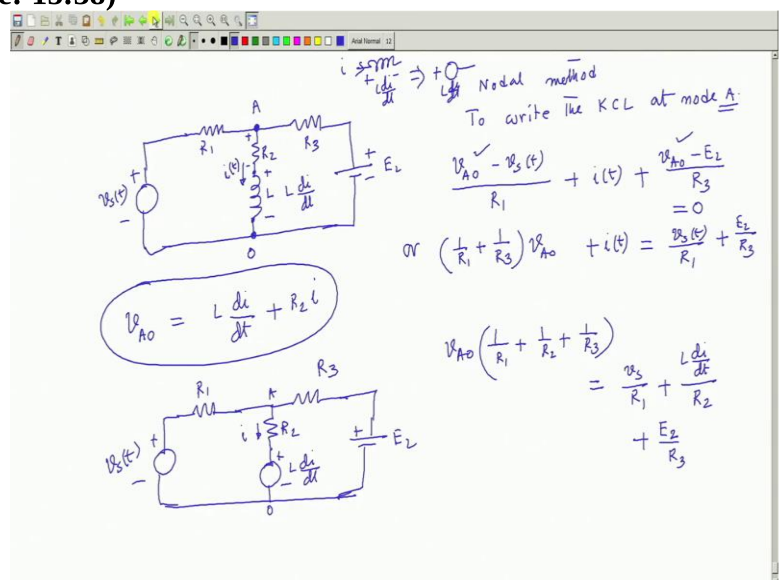

A nodal analysis also in the same way you can write for example, I will say that I have a circuit like this circuit map DC voltage AC voltage all connected together there may be time varying voltage also this is R 3 this is R 2, this is R1 1 and here is your supply voltage + - Vst Damon and I want to apply nodal method. Now, in nodal method, how many nodes are there I have to first identify these are the 2 nodes were more than 3 ends of 3 elements have been joined. This is also a node.

But this is no this is it not because current in these branch same and 1 of them you choose as the reference node and then says the other node. Now here once again and this is suppose l so what we have been doing so far that this will be the thing now I have to write down to write the KCL at no node A is it that is VAO is the unknown VAO this node voltage. So, what will be the coefficient or Vu we used to say that it will be some of reciprocal for all the resistance is connected to node A is not there is only 1 node, so, there is no problem.

So, that way we used to tell that now, but the here is now an inductor present not only R 2 there is an inductance suppose, I say that this current is I t because I am not sure whether I will write like 1 by R 1 + 1 by R 2 etc. So, KCL at node A let the VAO will be the voltage then this current flowing from right to left in this branch will be VAO – V st let us right from fundamental this divided by of course R 1. These will be the current now, this current will be plus I cannot write VAO by R 2 because there is this fellow present and if it is getting some current here some voltage.

How much is the voltage L di / dt what is I is the current flowing through the inductor, then this will be the current. So, this plus this current I t and + here no problem because VAO – E2 by R 3 and this will this must vanish at all time. So, this is the equation. Now, therefore, you see or the coefficient of this 1 VAO will be 1 by R 1 + 1 by R3 that is this term and this term into VAO then plus i t is there and on the right hand side we write the sources Vst / R 1 no problem Vst / R 1 pushing current there and E 2 / R 3 this will be the thing.

So, it looks like there are now 2 unknowns apart from this VAO. There is i must know i now if you see you know that VAO I told you several times to find out potential of point A with respect to O start your journey from O reach the point a by any path to like. Now, I will choose suppose this path then VAO will be equal to from this to this it is L di / dt + R 2 into the side this is this is the real part, but the point potential of this point this node with respect to these is start your journey from this to this you get L di / dt minus to plus then from these 2 these once again minus to plus that is R 2 into i, this will be the drop here.

So, this is the other equation. Therefore, I can substitute for VAO this quantity and get an expression for the current i t alone and i t will be as usual a second order differential equation and not second order the first order differential equation and then I will solve it by the method we have learned. So, essential thing is you have to form a differential equation correctly. For example, what here once again this circuit can be drawn R 1 this is R 2 no doubt this is R 2 handed this can be written as a source + - but this source voltage is L di dt - is not that is what I told you because of after all inductance energy storage element so L di dt.

And this you can write it as a source + - L di dt. That is what I have done here L di dt this is this was actually this was your E to this is your supply voltage, which is like this. Therefore, if you treat it as a source, but what is this i is this current. Therefore, the rules that we have learned earlier can be straight away applied. Why because I will say that you are this way I will say that coefficient of a VAO will be some of all the resistances 1 / R 1 + 1 / R 2 + 1 / R 3, this will be the coefficient of VAO.

Now, this is there and coefficient and that that is all this is the thing all leave 1 node, this will be the thing and this will be equal to the sources connected. So Vst / R 1 current drives there + this voltage, here is a voltage source here also L di / dt, this voltage divided by R 2 will be the current going there. And plus another sources there which also pump current the polarity circuit is trying to pump car into so that is y + e 2 by in or this is 1.

And the same thing, see the same equation so, sometimes what you do, he will replace the inductance by showing it as a voltage source L di / dt but this equation has 2 unknowns Vs VAO and I then the other equation is this 1. Anyway, you will gradually be habituated with what I am telling so normal once the current is known similarly with the capacitors, if capacitors are present you can find replace the capacitor for example.

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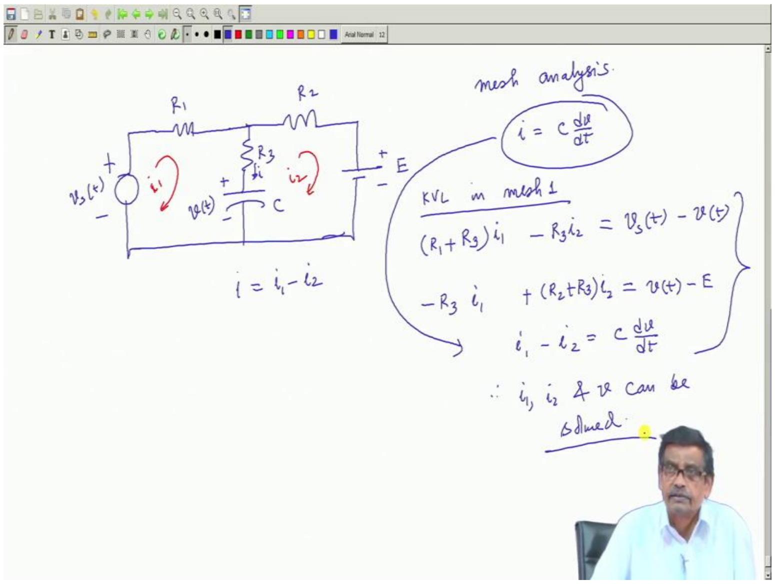

Loop equations, I will just tell like this. Suppose we have a capacitor I will just give 1 example. And here is a voltage source which is supposed time varying Vst. Therefore, I want to solve the circuit R1 R2 R3 and there is a capacitor I want to solve this arcade by what is called the mesh analysis so, for capacitor you generally choose the unknown as the capacitor voltage that is what you choose. But once you choose that this current from top to bottom, this current i will be equal to c dv / dt I am ready with this equation before I start solving.

You have assumed this to be voltage then the current flowing into the positive plate of the capacitor is c dv / dt that is there. Now, there are 2 messes so, you choose this mess current to be I 1 this mesh current to be I 2 small letters because things may be time getting so I went at it I choose as a function of i time then I will apply the rules mesh analysis so KVL the mesh 1 will be equal to some of all the resistances that is R 1 + R 3 that will be the coefficient of I 1 coefficient of I 2 will be - R 3 into I 2 to that will be there.

And on the right hand side all the source voltage is what that source voltage is Vst plus because i 1 is flowing like this Vst and there is another source voltage Vt minus Vt. Similarly, for the second mess it will be equal to coefficient of i 2 will be R 2 + R 3 into i 2 am so these are all small letters. So, this is i 1 this is i 2 and this is R 2 + R 3 into i 2 this is the mesh current elevation plus and then this is - R 3 into I 1 and that is fine and this on the right hand side the sources will be vt - that will be the thing and these 2 equations you have to solve, but there is a third equation needed here the source voltage this vt depends on i.

So, i is nothing but this i is nothing but i 1 - i. So, this i is nothing but i 1 - i 2. So, you have got you can translate this into this 1 wherever i will be there and you can have it essentially 3 equations 1 2 and this 3 equation with smaller i = i 1 difference i therefore, that is what I want to say this equation is is 1 - i 2 = C dv dt this is the third equation, how many unknowns are there i 1 and i 2 and V. How many equations are there 3 equations, so, I will be able to solve for either i 1 R 2 R 3.

So, i 1 i 2 and small v can be solved can be solved therefore, the distinction we put for applying nodal and mesh analysis can be removed I after all what is there because the interesting point is KVL and KCL will be valid for all time to come in a circuit that is the sum of the currents at every instant must 1 vanish at a junction and seemed similarly, some of all the voltages in any closed loop will also vanish to 0. Anyway there will be several problems will be given and I will conduct a tutorial class also based on this. So, hope you have understood this part of this circuit analysis and we will proceed further in the next class. Thank you.