Network Analysis
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Lecture # 26
Circuit Analysis with Phasor - I

So, to lecture 26 I welcome you all. Now, today, what I will be telling is that after getting some idea about your various kinds of voltage sources current sources this that today I will be telling you how to find out the steady state solutions and importance of that.

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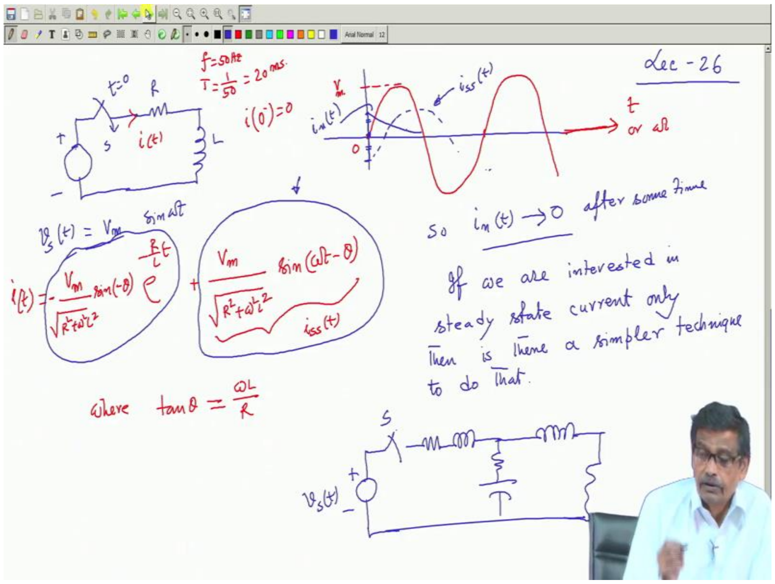

See, for example, we now know that we have seen suppose considered a simple analysis circuit series R and L, I told you that if you have connected a source which is suppose sinusoidal V st = V max sin omega t and I tell that this week is closed at t = 0 this equation. That means, if this is the supply voltage this is your V max or Vm I was writing Vm. So, if this is the supply voltage then and this axis is either time or omega t time t or omega t.

If it is t then this for our supply system in our country frequencies 50 hertz. So, time period T = 1/50 which is equal to 20 millisecond this numbers you must remember. So, this total time here is 20 millisecond. Now, if you close this week at t = 0 and I want to know what will be current, the expression of the current behind all those mathematical state forcing function natural response it will be like this Vm by the solution will have 2 components.

1 is the steady state components omega squared L squared into it was obtained earlier with the long hand calculation - theta, who this part is the steady state parts I steady state solution what is tan theta sorry tan theta where, tan theta is omega L/R reactance by resistance people call it and this part is called the impedance of the circuit. Now, there will be another part, the natural response and the natural response is, the current must start from suppose I0 was 0.

Therefore, current must start from 0. Therefore, at equal to 0 steady state current will be how much steady state current will be at t = 0. So, i t total solution steady state current at t = 0 will be this much Vm by route over R squared + omega squared L square, t = 0, it will be signup - theta is not. Therefore, a current must happier to make it, o at t = 0 a term like that, and then need it will be - R/L into t this will be the solution.

And these I can write by inspection I expect you to develop this kind of reasoning because after all current has to start from 0. So, this is the steady state current will be lagging the supply voltage by this angle theta. Now, this amount at t = 0 the steady state demands that this current flows could take her to during this steady state solution. This is the current but I am once again telling current past rise from 0 because I have closed this week at this point.

Therefore, there must appear another DC current of same length so negative of the steady state current at equal to 0 and that thing will decay exponentially. So, these + this will give you the total current in the circuit. After a few cycles only these sinusoidal portion will remain because this will exponentially decay down to 0. Is that clear? They are bored people often are interested to know what is the steady state value of the current who in a circuit like RL is excited with sinusoidal voltage sources knowing fully well.

When you close this switch, there may be some Keynesian current introduced into the circuit, but that Keynesian current is not going to last long. Because order of the time constant of this circuit maybe few milliseconds. So, after a few milliseconds or so, maybe 2 or 3 time constants, this portion will not be there. So this is the steady state part. And this is the natural response int. So, in t is the natural response will vanish to 0 after sometime means you cannot leave it like that you better say after a few time constant understood.

So, steady state current then who suppose you have an fan in your room machine accelerates there are some times initially but after that everything is in steady state current drawn from the supply is only sinusoidal. So, who bothers about that? For a few millisecond initially, that is the idea. So, the question therefore now comes that if I am interested for the steady state solution only I am interested only this part of the solution this natural part of the solution, I will not care.

Of course this term is important in case of fault analysis in power systems are key in day to day work, it may not be that important. For example, how much will be your electricity bill? You did not bother about this term. It is this steady state current and steady state voltage that will crossly decide what will be your electricity bill Keynesian are bought whenever you do some switching in presence of some energy storing elements but they will also be get down faster.

Because in every circuit there will be some resistive elements. It is not that only inductance and capacitance you are switching. Therefore R will be there. So everything so, if we are interested in steady state current only. Then is there a simpler technique to do that to, see, it looks like that circuit which may not be as simple as a simple RL circuit there may be series parallel. So, many energy storing devices will be present.

And for example, a circuit R inductance here, R capacitance, here is another R inductance and another R. And you close these switch add some t = 0. Vs = V max sin omega t, then if you close this switch there will be some Keynesian phenomena definitely taking place in this which can be leveled it can be solved from the differential equation solid beat and from that you conclude that the Keynesian part will indicate down to 0 Keynesian part of will appear where in this current and voltage across the capacitor.

But those transients will go ultimately you have excited the circuit with a sinusoidal voltage source like Vm sin omega t, everything will be only solution due to forcing function. So, that is why people were always thinking that if we adopt AC system for supplying power to solve a circuit involving several energy storing elements in association with some resistive elements, every time I have to write down the differential equations.

Then I have to find out natural response I will not do because that will not be present in this steady state it does not matter, but nonetheless to find out the solution due to forcing function, we have to replace the sinusoidal voltage in terms of e to the j omega t to the power - j omega t. Even that calculations will become tedious when a complicated circuit like this is to be solved even for steady state alone. That is what I want to say. Can there be some ways of simplifying that matter yes it can be and that is called the phasor approach.

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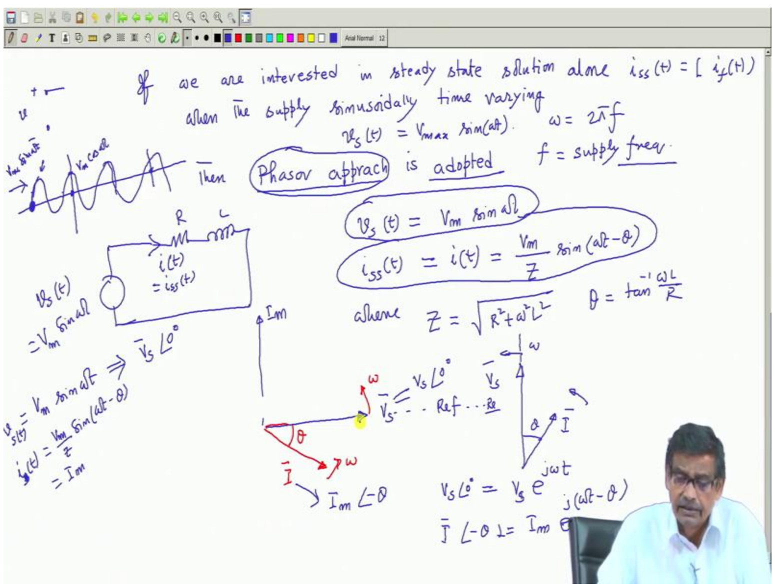

If we are interested in steady state solutions alone that is st which is ift forcing solution due to forcing function when the supply is that is a condition you cannot do for any supply for when the supplies finish sinusoidal time varying quantity like Vst is a Vmax sin omega t like that what is omega is 2 pi f, f is the supplying frequency. So, if we are interested in steady state solution alone when the supply is ready, then phasor approach is a adopted.

It can be done only person that is you need not write the differential equation then say steady state solution is like this that is the point I want to make. Now the question is what is federal approach for the first time let us consider a circuit like this is very interesting RL and here is a supply voltage we have connected like Vm sin omega t and I am interested to know i t. i t means isst only a steady state current I want to know because I know this switching Keynesian and that is the solution due to natural response is going to dry after some time.

So, I am only interested in the long run what remains in this circuit only Vst will remain and only isst will remain and this is the supply voltage to indicate that therefore, I know I have solved differential equation and I have come to this conclusion that this is your supply voltage and then your steady state current which I will call it only although it looks like so, this is Vm/Z into sin omega t - theta.

This I have got natural response were targeted where Z is equal to route over R squared + omega squared L squared. And what is theta is equal to 10 inverse omega L by R this is the thing this is known with this thing known what is the phasor approach for the first time you have to solve it classically like this and got this one. Now, before we go to the phasor approach, we know this much suppose you have any time varying we will come back to this a little later.

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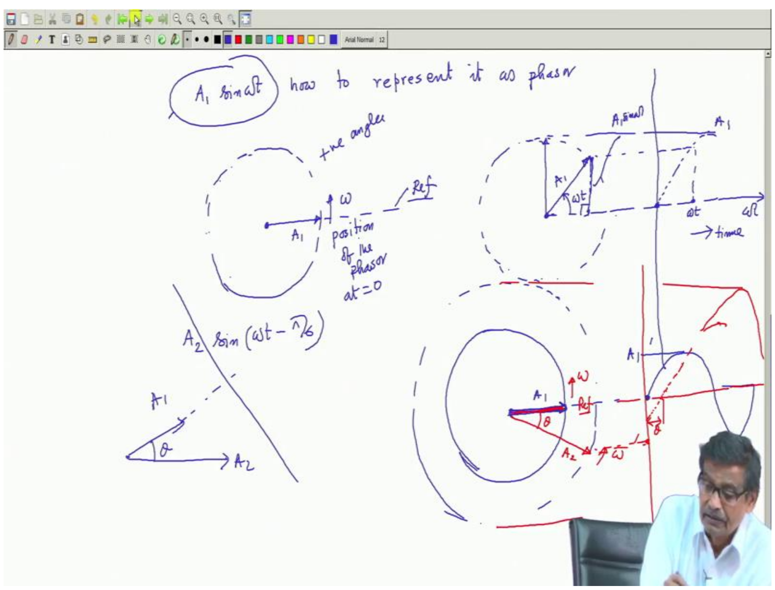

Let us go to next slide and say that suppose you have a time varying quantity say A1 sin omega t is a sinusoidal quantity. How to represent it as phasor that idea is very simple. What do you do? Draw a circle. This is the center and emerging a length A1 this is called phasor and it is moving in the anti-clockwise direction with speed the omega radian per second on the circuit, this is the position of the phasor position of the phasor at t = 0 what will be the position of the phasor at any arbitrary time t position of the phasor at any time this length is moving with speed omega and it was at t = 0 horizontal it must have moved this angle.

So, position of this phasor will come here. What is the length of the phasor I have taken it to be the amplitude of the sinusoidal quantity what is the A1, so, this will be the phasor. Now, why it is called phasors because what you do is this now draw a vertical line to the t = 0 position it has gone there what will be this intercept this length will be how much A1 sin omega t is not this length from this right angle triangle A1 sin omega t at therefore this A1 sin omega t although I have imagined that a length is moving like this I will be back in my mind.

Thinking that position A1 the projection of A1 on this vertical line will give me the instantaneous value of the sinusoidal quantity at that time at omega t = 0 it was like this what will be the projection of this vector on this vertical line omega t is known. So, this is the point at time t. So, here what I will do I will do omega t at this time it will be there. So, this is the time omega constant. So omega t so, e will generate the sin wave after some time it will come here.

Then, when it will be vertical like this, then you will get the typical value got the point. So, the relationship of the instantaneous value and representing by a phasor with constant length A1 this has got one to one correspondence got the point. Therefore, I will draw like this. Suppose, you have got another phasor A2 sin omega t - pi by 6, suppose here I have got another phasor. So, this is 1 phasor this is another phasor that what is the angle between them pi by 6.

Therefore, this is A1 omega t = 0 it was lying here at omega t equal to 0 A2 was lying. So, this is the positive direction of measuring angle anti clockwise positive angle with the reference to this reference line horizontal, therefore, this is A1 it was fine, but at t = 0, this phasor also moving with omega and depending upon each magnitude the radius of the circle will be saved. So, this is the position of these vector A1 sin omega t this phasor and at that time this was the position of these vector A2.

And both of them are moving with the speed omega mind you phasors you can only be present for same frequency you cannot represent to sinusoidal quantities one with 50 hertz and another with 25 hertz on the same phasor plane they must move with that angle and this angle is theta and what is the idea is that you draw a particle line here. So, it will give you this will give you a sine wave like this.

For this voltage this will be the sine wave with amplitude A1 and for this quantity it is lagging these by theta so, whatever is happening to A1 now after theta it will happen there. So, similarly prediction at omega t = 0 it is not 0 it is something here negative, but for this projection is 0. This side is omega t. So, this side it is having a negative value A2 sine - theta. Therefore, at instant to instant I can also skate these and you see A2 is this one, so, it will go there and come back got the point it is lagging.

So, this angle will remain intact theta between these 2. Therefore any sinusoidal quantity with any angle theta I will be able to represent it by thinking that there are lens fixed lens corresponding to the peak amplitude of that sinusoidal quantity moving with speed omega and if you are calling it to be sine omega t that is you choose as a reference. And the other quantities with respect to these can be just drawn and it does not matter whether after understanding this I draw A1 like this and A2 like this angle theta.

A1 sin omega t then draw the vertical there. So, this picture can be rotated because the relative velocity between them is 0 and theta is the constant thing that is the whole idea. Now, we did not tell him much right now, now, let us see about the voltage and current waveform of an adult arcade have applied with sin omega t we have got the response of the current like this. So, these are sinusoidal quantities of same frequency omega.

So, I will represent the voltage for that as Vs bar a bar is put a physical quantity and capital letter supply voltage further if you do it then your current phasor will be lagging this by an angle theta. This is I bar. What is the length of this? What I am telling length is Vm what is the length of this Vm/Z? That is what I am and both of them are moving with speed omega, this is omega and this is theta got the point.

Therefore, the angle between the phasors will remain time in angle and it will remain constant. What is that constant omega L/R and this phasor, I am telling I could draw it also like that. Does it matter Vs I will draw but this fact is true I bar this is the both of them is moving on to this is the beauty then I will take this is their reference particle that component I will take their see one point must be understood clearly your supply will listen this point carefully.

Suppose you have a supply heavily available at your socket in your room plot point here there is a sinusoidal voltage existing now how this is existing here? It is doing like this supply voltage sinusoidal, it is there, there is no time attached to as it is but only thing it is attending some big value 0 negative p 0 positive p and it is going on doing now, when you so, how do what is the equation of these voltage waveform I want to write it is totally my problem supply voltage is doing like this.

If I say that I will write down I want to write down these voltage equation when voltage is 0 going towards positive then I will write this equation as we make sin omega t. Somebody may say that I will write down the equation this will be my omega t = 0 and when the voltage is maximum and reducing that instant it will start counting my time and then I will write this as Vm cos omega t. And if you choose this as out again somebody else will write it as be sin omega t, it is totally your business supply voltage is going on doing like this.

Similarly, somebody says I will take this neither 0 and on maximum this point as mighty equal to 0, then the equation of this one with respect to sine omega t will be it is leading sine omega t + theta i think you have got the point, but nonetheless whatever with the supply voltage by what angles current will lag that is fixed that is the angle theta clear therefore, now, the crucial point 1 point I just make this is the time domain expression of the supply voltage.

This I will write it as a phasor and I will attach it as an angle 0 degree because this is my reference for that then I will say my current this is Vst everything is real in our day to day life nothing complex ist i tsteady state current ist it is equal to some Vm/Z that is im into sin omega t - theta. Then I will say if you represent this as Vs angled 0 degree represent this as im angle - theta it is lagging. Now, this is one way of representing.

Now, remember this Vs 0 degree it is essentially Vs 0 is nothing but in complex domain it will be Vs into e to the power j omega t. Similarly, Is - theta is nothing but Im not Is is I, the system current I and it will go to I m and e to the power j omega t - theta in complex plane easy if you assume this to be your real axis and this to be your imaginary axis then this has got Im this is at any time t.

If you go, it will move by omega t this will also move by omega t and this way it can be it is a particular position at omega t = 0 these are the position of the phasors. So, we will continue with this in the next class and the most important conclusions then we will be able to draw. Thank you.