Network Analysis
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Lecture # 27
Circuit Analysis with Phasor – II

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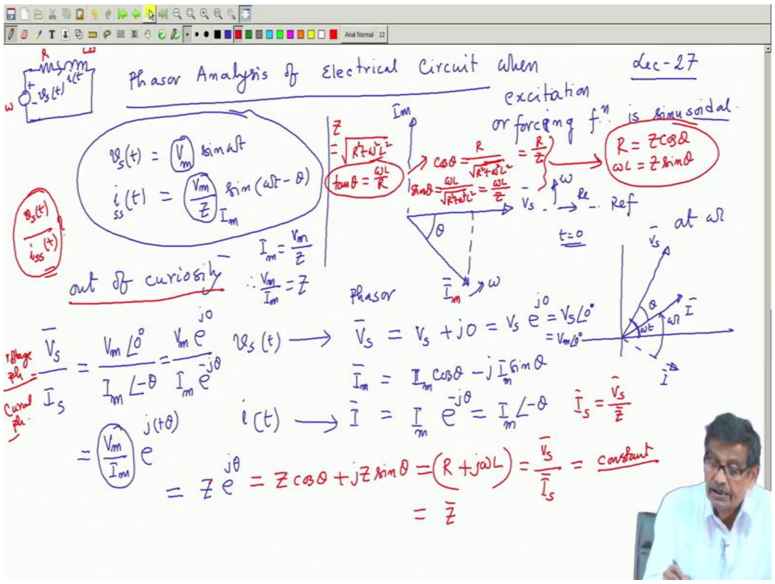

So, we have been discussing with phasor analysis of electrical circuit essentially, any circuit with energy storing elements will have a natural response and solution due to forcing function if the solution due to natural response will decay down to 0 very quickly and people are mostly interested to find out the solution due to steady state currents. If that be the case then we have found for an RL circuit the supply voltage is equal to Vmax sin omega t and the current of the RL circuit we have seen it is equal to Vm/Z get some sin omega t - theta and this is steady state current.

We are interested in steady state current only then what I am telling is this that this voltage if it is, but they must be were in sinusoidal otherwise not phase phasor analysis of electrical circuit, when excitation is sinusoidal away in excitation or forcing function excitation or forcing function, then only you can do it forcing function is sinusoidal. So, E Vst is this then I told that then this V s can be represented as a phasor moving with the speed, omega and your current phasors will be can be represented as I bar and it will lag this by the angle theta.

And suppose this is the reference for that you have drawn then in general at this plane and both of them are moving with omega so, that is fine. So, at any arbitrary position if they move that is, this is that t = 0, I have drawn the position of the phasors, but related positions of the phasor the is independent of time that is at omega t, the situation will be like this position of the voltage phasors will advance Vs and your current will come here, but the angle between them is theta and this is omega t. This angle is omega t because vs at t = 0 it was horizontal.

So, it has moved by omega t, I is also moved by omega t. That is why the angle between them is constant from these 2 omega. So, earlier I was there when v were there, I was there. So, this has also moved by omega t and it is there. But anyway So, I will always draw the phasor diagram in this way that is at omega = 0 Vs t is occupying the reference and then I can say if I think that it is a complex plane. So, I will say V as a bar the angle this is suppose the real axis and this is imaginary axis, then I will say Vs bar is only having real part.

So, Vs + j 0 is not you draw it in this way also you can relate Vs bar will be I mean theta + omega t whatever you do you do, but in this way I will just write Vs bar and then I will say that Is bar we will have that is no imaginary component present in Vs bar I bar on the other end as got a real part whose magnitude will be this length into cosine theta. And if you break it up into 2 components I cosine theta - j I sine theta sine theta. So, in polar form at this will be Vs into e the power j 0. And this current in polar form will be e to the power - j theta at omega t = 0 am writing.

So, this is the voltage and current phasor this 1 instead of writing e to the power j 0 always people write it like this angle 0 it means that it is Vs into e to the power j0 but similarly this 1 people right i - theta like that got the point therefore, your supply voltage see everything is real here, but now I am some body suggested not somebody I should say is Steinmetz for the first time told that to get the solution due to forcing function, you do not have even to write the differential equation do it like this, because excitation is sinusoidal.

So, forcing function solution in this steady state will be also sinusoidal of same frequency. If that be the case, then you can represent them in this fashion Of course, this tool is also adopted classically then e is suggesting something very interesting. So, e tells that to represent Vst in terms of phasors, the current in terms of phasors like this now, the argument is like this, just out of curiosity what do you do? You divide this 2 phasors Vs bar by i, I must draw the circuits.

So, that you do not be misunderstanding this is the supply voltage time Vsd and this is the current i t. So, y as is it current It is not it is a current in the circuit i. Now, this I will just divide what is the value of this Vs it is the magnitude Vm 0, what is the value of this current i, it is this i and this is I am writing as the magnitude of the magnitude of the weather so, the prequels Vm by t Now, therefore, what I will do is this this can be written as easily magnitude and angle 0 divided by magnitude of the current that is i. This is suppose Im with maximum value in.

So, I m divided by - theta and the sin O is nothing but Vm e to the j 0 divided by I m e to the power - j theta. If you divide this to 8 will come out to be the M by M magnitude divided and the angle gets subtracted into the power j - theta, it will be just like this. -, + 0 - theta +. Now, what is the Vm by I m? I m = V m by z therefore, Vm by I m = z. So, this quantity is z into e to the power j theta, this is the thing, record recall that, this j I will write it with different color this j we established it is nothing but R squared + omega squared and squared and what is ah tan theta can take tan theta was R by omega is not.

This weak inside then you get this j into the e power theta which is z cos theta + j z sin theta and then this z cos theta you see this temp tides R by omega m. So, if you consider a right angle triangle, z cos theta tan theta is known so cos theta is known, how much it will be it will be z cos theta I am so this a road wrongly omega L by R tan theta is omega L by R so what will be cos theta is R by root over R squared + omega square l square which is equal to R by z and sine theta = omega L divided by root over r squared + omega squared n squared, which is equal to omega L by z, this will be the thing.

Therefore j cos theta is nothing but R and z sine theta = omega so, these 2 things gives you R = z cos theta and all my guys to get sine theta so, I just put j omega L. So, what I told you yesterday, I represented sinusoidal quantities in phasors the angle between phasors do not change that is comes out to be theta and then I could represent the supply voltage as a feather as Vm 0 degree peak value 0 degree current value I am it will lag this apply voltage in case of this - theta, you divide these 2 and you get this thing ratio of Vs by Is phasors.

And this is what this is the ratio of Vs further by Is further and this thing constant to be a constant of the circuit for a it depends on supply frequency that is there but no supply voltage is that So, this ratio is constant. and how did you did e established that I did all the hard work to get the solve the differential equations got this steady state voltage and steady state current then I am told that only steady state currents we are interested in and then I thought both are of same frequency and sinusoidal varying therefore, they can be expressed as some lengths whose values are peak values.

Vm and I m and this is the I s and then I represented at complex plane as Vs 0 degree and Vm I m - theta then I m telling let us see whether these 2 phasors have some relations see in case of disease are these B by i = R that is what we are always applying. Now, when it comes to a efficiently then I find that the ratio of the phasors not of the instantaneous quantities not Vst by Ist it cannot proceed part that because sine omega t by sine omega t - theta that will be also a function of time it has got no meaning, I mean no meaning means it does not help me in any way.

But we find that if you represent them in phasors then take the ratio of the phasors voltage phasors divided by current phasors. If you do, then a surprise thing happens, we discovered that the ratio is a constant complex number whose value depends on the circuit parameters are and then of course not tail alone supply frequency which is omega it is to be multiplied by omega that I can always do and this ratio is constant. So, the thing is, the current phasors in the circuit will be equal to V as bar by get complex number, this is called z bar having real and imaginary part. So, to summarize this Point is important.

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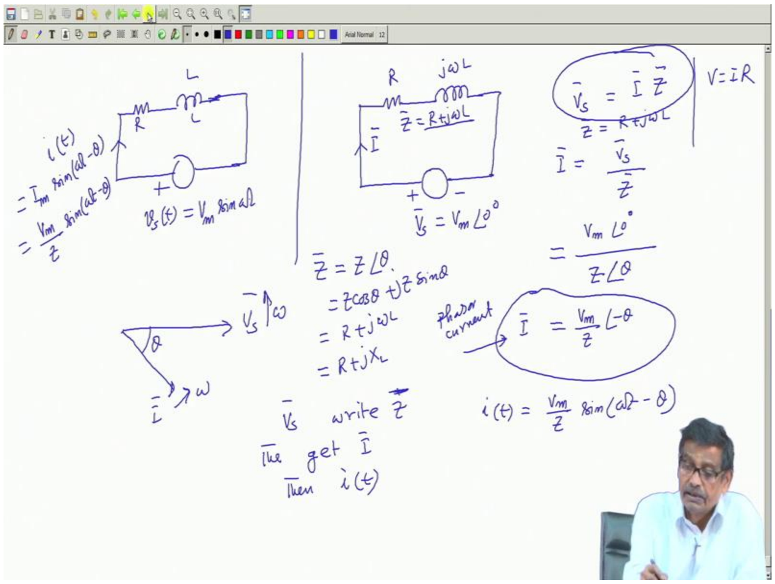

That here is a circuit R L here is your supply voltage Vst in time domain This is this archaic Vst = some Vm, sin omega t and the current will be also real why things will be imaginary no imaginary thing in real life situations applied sinusoidal voltage you will get some current the value of this current is I m sine omega t - theta and this was shown to be Vm by z m, z what is j to do over R squared + omega squared L squared sine omega t – Theta. This is all done, but now I am telling that this circuit imagine it is like this R and here you write some j omega L and represent the variables as phasors.

So, supply voltage is the angle 0 degree and then the current I bar you show it like this then we showed that Vs bar = I bar into z bar therefore, if I knew this earlier that forcing function maintain such a beautiful relationship similar to that of a disease or it vesical to I guess Nothing is better than that. What do you will simply do is the supply will descend a certain time wearing term remove that time terms represented by L and m Vs then the result in current 2 will be a feather I bar and it will lag the supply this is omega, this is also omega moving.

So, currently lag by this and if you know the supply voltage then R problem is to find out the current After establishing these I will tell henceforward I will represent the supply voltage as Vm 0, I will find out z bar like this, I will find out and say that my current in the circuit will be Vs bar by j bar What is Vs bar the am angry theater degree, what is z bar will be z angle theta z into the power j theta treatments and if you divide these 2 you will get Vm by z - theta. So, I have solved for phasor current Of course, I will be interested to know how current varies with time.

Then I will say if this is the phasor representation, i t will be Vm by z sine of omega t - theta got the point that is you do your calculation is phasor point get the current feather find out the impedance of the circuit z bar = r + j omega L it is a complex number, this is not a phasor it is neither rotating nothing like that, but the ratio of the voltage and current phasor comes out to be a complex number and so, z bar I will be always using bar over it magnitude these z and angle is theta and we have shown in my last slide.

This is nothing but z sine theta z cos theta is R + j omega L is often called lead across of this arcade and written like excel inductive reactance. So, this is a very important and crucial step that is there is another alternative way of getting the solution due to force in function. You do not have to write down the differential equation anymore, it is those days are over after knowing these the voltage phasor and current phasor this ratio is a constant complex number decided by the circuit parameters if that be the case.

If I knew this, I would if I knew this relationship I should know do like this supply voltage is known z bar is known, get I bar and I will get this Current in further form but switching from feather to time domain does not take any time just looking at the title right side omega t - theta but deployed therefore circuits with a cetacean excitation excitations. And if you are only interested in phasor sort, I mean steady state corrects then nothing is better than this phasor analysis.

Supply voltages they are represented in phasor form circuit parameters unknown, right z bar your steps will be supply voltage weather, then write z bar then get I bar Then i t time domain expression that is the thing. See it has got a tremendous now I am going to make a very big statement.

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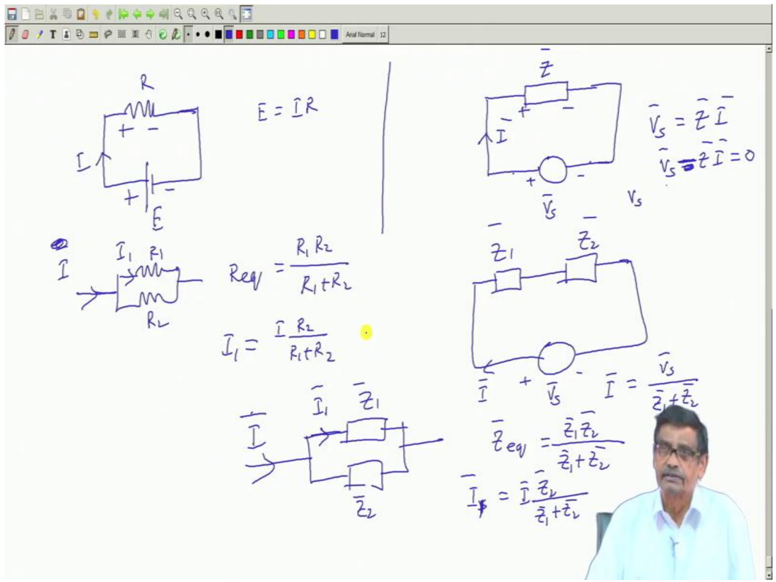

Now, in case of DC Circuit I told you this is the battery and this is the current and this is R and we know that E = IR we have seen that now in AC circuit, there will be some impedance, z bar and here is some supply voltage, which is feather I will put a bar always to indicate phasors and this current i t is saying just like this no differential equation writing, no + -. If that was the case, then I can invoke all the things I have learned with the AC circuits DC circuits that it is to resistances are connected in parallel if you volatize this R 1 R 2 by R 1 + R 2 how did I derive this based on this only wrote some KVL KCL equations here got discouraged.

For example, if the total current is I bar I what is the current I 1 will be I into R 2 by R 1 + R 2 all these things happen because E = I R similarly here only thing is these are not some real numbers you have to deal with complex numbers. But nonetheless Vs = z bar into I bar means that if 2 impedances are connected in 3 these is z 1 bar This is z 2 bar This is Vs bar current in this arcade will be I bar = Vs bar divided by z 1 + z 2, this will be the current, if the impedances are in parallel.

But this time you have to show with a bar in phasor domain will calculate everything here also z if you valet will be equal to z 1 bar z 2 bar same relationships had 1 world +. Similarly current in this branch, I 1 bar, all are phasor will be equal to total current I bar into z 2 bar divided by z 1 + z 2 bar see the power of the phasors notations lies here that immediately you get Vs = z into I hear is there any time terms no only complex numbers once you get that then all the rules of impedance manipulation resistance manipulations current divisions.

Similarly KVL here Vs bar as you can see - 2 + Vs bar + to - i bar z bar that is what Vs bar = - z bar I bar = 0. So, KVL equation KCL equations at the junctions in terms of phasors will all be valid. Anyway, we will continue with this in the next class but you see try to understand the flow of logic that brings us to phasor notations whether a relationship Vs = IR in DC a DC Circuit It was so, obvious and everything is constant here.

Similarly, there exists similar relationship in SSL, but the only thing these are not time domain equations, these are all expressed in phasors and if you know further the presentation of any of the quantities voltage or current writing down the time domain expression should not take any time at all provided you know where do your difference where there is anyway, we will continue with this in the next class. Thank you.