Network Analysis
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Lecture # 28
Circuit Analysis with Phasor - III

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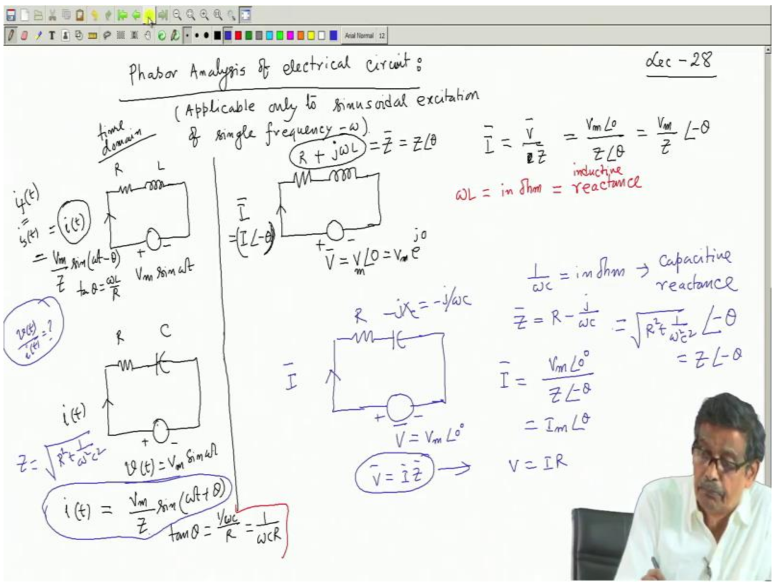

Welcome to lecture number 28. And as you know we have been discussing to analyze circuits using as a notations and in fact by further analysis, if you solve a circuit which is only applicable for excitation which is sinusoidal equation and the solution that you will get here will be the solution due to forcing function alone because transients will be born they will die after some time depending upon the time constant of the circuit how long it will last clumsy and that is decided.

But in any way for all practical purposes, they will eventually die down and only the solution due to forcing function is of concern after certain time after you have switched on this circuit and then steady state condition prevails and that steady state solution is nothing but the solution due to forcing function that we have seen earlier. So, here it is a technique which will allow you to solve for the forcing function solution rather easily by using phasor assumption.

So, you have seen for example, if it is then R l circuit I will just briefly summarize suppose this is this arcade title and This is the current I want to find out when the excitation is Vm suppose sine omega t and the corresponding current is it this is everything in time domain and I know how to solve it. If I know the initial current et cetera I will be able to solve this and this current will have I nt and I st or a Ift.

Now, this circuit you redraw when you draw the parallel analysis, our demons are here, but this inductance see you replaced by j omega L. and this voltage I will write it as time will not be present now, will not be present So, this Vt is this v bar which will have a magnitude and angle and I m I wrote it as the max it will be and similarly, the current feather who will be I bar and I will be I n - theta, these are essentially it is nothing but Vm e to the power j 0. In short instead of writing so, many times you put the angle that is good enough.

So, this is your current and this 2 together R + j omega is your z complex impedance complex numbers. It is a constant number property of the circuit only thing it takes from discipline is discipline frequency which is constant therefore, this is also so it will have a magnitude as well as an angle theta. And we have shown you that i bar is nothing but V bar by no differential equation. And this is Vm 0. And this is z theta, and it will become equal to Vm by z magnitude without the bar and angled - theta into deeper digital below it is e to the j theta to the power j 0 - theta that is I - theta.

So, this will be the current and of course, after you so this current to write the in the time domain it will be I t will be it this is the feather form I will say ohm the current value is Vm by z into sine of omega t but it is lagging - theta. So, I can switch over from time domain to frequency domain, this called frequency domain or feather approach in no time that is how this RTT solved. Therefore, wherever inductances there you write it as j omega L.

Similarly, for RC circuit for example, R and a capacitance This is in time domain E this column time domain which is where everything is real, this is R this is C and here is some voltage applied which is of course sinusoidal vt = Vm sine omega t . And this current it is steady state current we will not carry on that ift or is it because we understand with respect to the context to yet telling we are only interested in steady state current

This is same as you are is st or which is same as ift solution due to forcing function these are all things So, I will read it and sport as it. So, with respect to context you have to understand here also there will be some time domain current You have applied such a voltage and in time domain we have solved this earlier and the solution for the current was shown to be Vm by z into sign of this time it will be omega t + theta were tan theta = 1 over omega c overall that is 1 over omega CR.

Similarly, here it was tan theta was omega L by R this omega L is called the it say dimensioned should be own because after all current is amperes voltage is in volt. So, omega L will be in ohm you know and it is called reactance R is resistance omega L reactance not only the attempt is called inductive reactance because al is involved inductive reactance similarly here it will be like this.

So, this is known without knowing phasor diagram I knew this then once again you can draw the circuit in terms of phasor as R then capacitance everything I will now convert 2 phasor + - polarities are maintained So, R will remain R and this is actually – z theta which = - j over omega because from the sign notice what is z here z here will be equal to R square + 1 over omega squared. So, 1 over omega C is of dimension in ohm and it is called capacitive reactor capacitive reactor.

And then in this circuit from time domain I should not write no small letter V bar is the applied voltage phasor current phasor design there will be no time involved there and only thing is this z bar here will be equal to C. So, R - j by 1 over omega c, which will have a magnitude of R squared + you know this, but once again quickly revealing and this angle is - theta. So that current will then become V bar at this voltage is V m 0 degree.

So, Vm 0 degree V bar by z bar is this 1, which is equal to magnitude of z 1 - theta and z is the 1. So, z so, while dividing and multiplying phasors it is better to you express in polar form. And while you want to add complex numbers it is always better you express it in rectangular form. Addition, subtraction of presence can be easily done if it is expressed in rectangular form a + JV form and division multiplication better use it,

But in nowadays, the scientific calculators will have all the provisions you can mix up multiply, a complex number which is expressed in polar form, an another that is in rectangular form a + ab form multiply them they will give you this shortly can easily switch over from polar to rectangular, wherever you want polar from is always better in the sense that it gives you the magnitude of the feather as well as each phase relationship with respect to a reference feathers.

So anyway, so this is equal to I m and this is - theta. So this will become + theta. So you see the current will then be leading the supply voltage. It is RC circuit I am not writing any more things about it, you know a bit of it quite good in your first year circuit analysis, but the point to be emphasized is that because of this relationship that V bar = I bar into z bar, it is tremendous this relationship. It is similar to be equal to I R in your DC circuit.

Therefore, series parallel of impedances division of current into impedances all the formulas that we used for DC values is also applicable here. Only thing to remember that these are no longer real numbers do constant numbers but there may be complex numbers. That is all no solution of differential equation. Now, if the excitation is sinusoidal in nature, got the point. So, there may be for I showed you last time we cannot doing.

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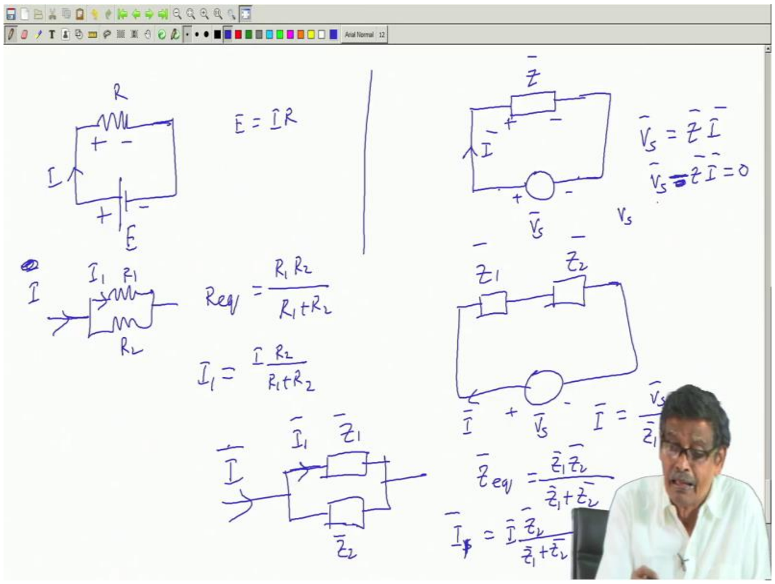

For example to impedances connected in series, just had them get the total impedance supply voltage divided by the total impedance will give you the current. Similarly to impedances are connected in parallel current in 1 branch I 1 will be total Current into get 2 divided by z 1 + z 2 and so on all the things that we learned in DC circuit will be applicable also here start Delta conversion of resistances connected can also be applied to start Delta calm conversion in impedances.

But what I am telling you should not be under the impression I am doing this thing vt by it, it has got really no sense as such at every instant It is varying what we are doing we are changing this voltages in phasor forms then playing with them, where the equations will be only algebraic nor differential equation. Then if I wish I will be always able to come back to the time domain in no time that is the idea. For example.

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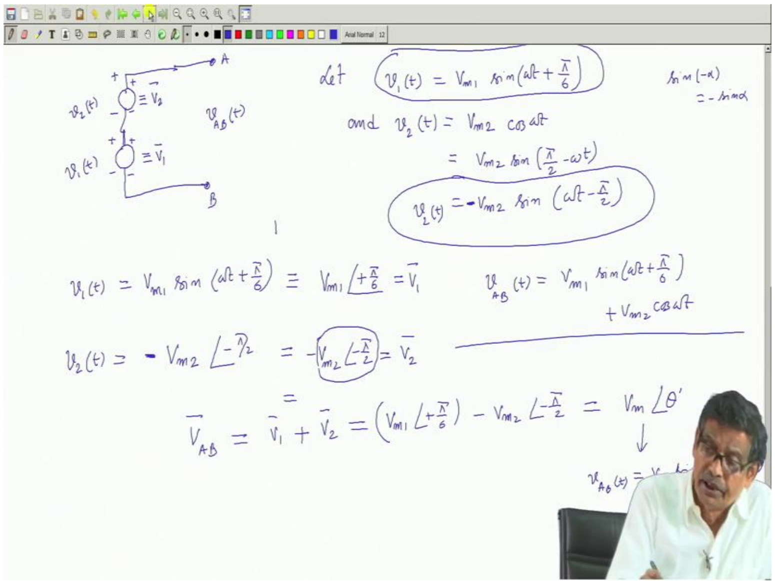

If I say giving you some example, suppose I say there are 2 voltage sources in time domain first time doing it is given as V 1t and another sources which is with the polarities are important mind you, this is V 2t and both are sinusoidal and this is A, this is B. So, suppose let V 1t V is equal to V maximum 1 sine omega t + Phi by 6 suppose, somebody says sinusoidal voltage And V 2t to somebody says V 2t is Vm to its maximum voltage into cause omega t.

Now, here you should be a bit careful, because sine cosine terms involved in phasor rotations always try to express them either in sign or cos both of them all the quantities in the same reference frame you bring. For example, I will write it this way Vm2 cosine omega t is what cosine 90 - theta this way I can write it So, you have to bring it to same quantity the design was this quantity this is saying this is also like this sin 90 - omega t and cosine is a very nice function.

So, this is nothing but sign omega no. So, cosine 90 - theta, now this can be written as sign, then omega t - 90 degree, then sign will come because sine theta = sine - theta, sine - alpha = - sine alpha, we know this. So, I will write fast transform this 1, this is V1 I have not disturbed that and V t2 I found it was given in cosine So, I brought it to sign level and I will write it like this. Got the point Vm 2 so, this is vt 2. Then you see, I will then write V 1t which was Vm 1 sine omega t + phi by 6 I was writing like this it was given. So, this I will write it as phasor its amplitude Vm 1 and associated angle.

Now, this feather is leading sine omega t by 30 degrees, so, I will right + 30 degree + means leading now, this Vt 2 V2 t I am writing as – R let me rightly this, there is a - sign. So - sign I will keep it and each feather is Vm 2 and angle is - phi by 2 means e to the power this thing. Now, if you are asked to what is the problem here suppose 2 sources which are sinusoidal in nature, they are connected in series with this polarity. We have been asked to calculate what will be V ABt. So, I am taking help of phasor rotations to calculate it rather easily in time domain also I could do it what do you have to do.

So, V ABt in time domain if you want to do it, what year what will be your sequence of steps it will be this + Vm to cosine omega t, then I I am not doing this, but what I will be doing then I will break this sine omega t cos + cos sign and then I group sign terms together cosine terms together multiply by coefficients root over coefficients of sine turns and cosine terms divided then you will be able to express in the form of some sine omega t everything I can do remaining in time domain, but the steps involved provide will be a bit more compared to the way I am doing now, using feathers same problem I am now going to solve.

So, what I have done, I have written I should not write equal to this is my V 1 bar this phasor and this 1 is V 2 phasor = - Vm 2 an angle - phi by 2 like this I have got generally phasors this - sign, it should have a magnitude polar form it is you can leave it with it or you can sometimes right this says any feathers as I told you can be shown by a line say Vm 2 magnitude this is - 5 by wherever it will be with respect to reference negative of any further is this 1 rotation of the feather by an angle of phi by 2.

Therefore you could also write it so, these are small tricks, not tricks, but you should be careful what so, you can leave it with it or you can also write it as let us first leave it there. So, this is V 2 bar let me say we to word is this further let us not complicate the issue further. So, this is the fair that this is 1 way that I can draw magnitude and angle the negativity that is what now we abt can be done like this. Now, I will do in this way, in phasors, I will say potential of A with respect to B no time I did show and this will be equal to - 2 +.

So, this is equal to your V1 bar and this is equal to V2 bar with + - remains as it is. Therefore, your potential of A with respect to B just like this is a good here - 2 + V 1 + - 2 + V 2 bar. You have to add these 2 voltage feathers and what is this one is Vm 1 + phi by 6 and what is this 1 V 2 bar is - Vm 2 - phi by 2 got the point at this using your calculator to complex numbers you get ultimately E will be getting some Vm and some angle here. I am not doing that, but some anger is easier can say detached and after you get this then he will say result in voltage across AB in time domain V ABt will be this Vm whatever it comes.

This Vm and since I have expressed everything inside So with respect to sign, E will get this angle if it is + light + data dashed, if it is - right - detached. Therefore, not only the differential equations can be substituted by algebraic equation with complex numbers, but addition subtractions of complex numbers. Also sinusoidal varying time domain functions can be very easily done transforming each of those quantities in terms of phasor and E will get the answer.

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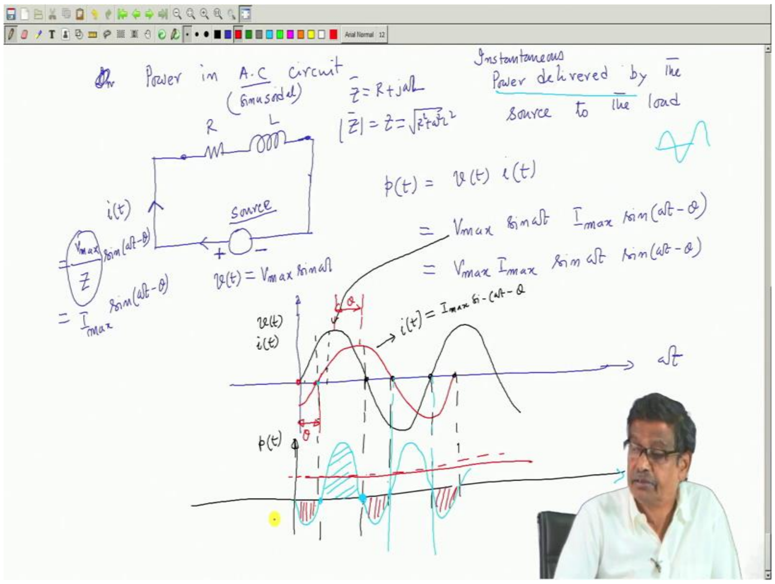

So, this is in nutshell you solve it, but one thing I will give you several problems on these to that is if the feather notation is known what is the corresponding time domain expression and so on. Now, I am going to tell you about the power in short circuits which are excited by AC source. This is important as well as very interesting to know that in case of this is not it because in this In case of DC Circuit only resistance is their power supplied by the swords will be always consumed by the resistance and nothing will be returned back to the supply whatever power goes to resistance it is last forever.

But since in AC circuit there will be RLC all are involved as circuit parameters, then the current waveform may not be in phase with the voltage waveform there will be a displacement there will be stands at we currently will be powered will be delivered to the circuit there will be instance when powered will be returned back to the circuit. So, in which way that goes on I will tell and for that I will remain in time domain.

So, instantaneous concept of power in a circuit power in People call it AC circuit underlying that AC means sinusoidal AC sinusoidal it has become sinusoidal AC means sinusoidal voltage in our country or elsewhere forward is generated as sinusoidal is the voltage waveform and that is being used. So, let us see very carefully forexample, you have a circuit like this RL and in time domain I am here this is your supply voltage Vt = say Vmax sine omega t.

And the current is i t and I am interested only in steady state current as I told you. So, I will go to complex plane solved for current further then I have come back to i t and this current will be Vmax we know that divided by z into sine omega t - theta, this Vmax is only the magnitude of this z what is this z complex number is r + j omega and magnitude of z by which I will simply write z is this 1 R squared + omega squared.

So, here a here nothing is further so, in time domain I have got now, look at the direction of the current and polarity of the voltage that indicated here. And we know that this source this is the supply is the current is flowing out from the positive of the supply. Therefore, power delivered. So, with this polarity I will only write in language as power delivered by the source. Instantaneous power delivered to the source no power delivered by the source to the load this 12 ohm R and L this is load to the load.

This is the thing or to the circuit it is delivering power, how much it is instantaneous power. So, it will be Vt into i t if this product comes out to be negative and you are telling instantaneous power delivered to the by the source to the load that will be actually in the opposite way current is coming got the point this is very important Vt. Now, let us put this magnitude here and this is ah Vmax by z I will write it as Imax So, I will multiply Vmax I will put these values sine omega t into Imax sign off omega t - t, which is equal to Vmax Imax into sine omega t into sine omega t -.

You see, you are multiplying 2 sine term sine can vary, it can have magnitude between - 1 2 + 1 both this term and that term and that will not occur simultaneously. So, sometimes this product will be positive, indicating that at that instant power is absorbed by the circuit. Sometimes this quantity may become negative that means it will be it will be pumping power back to the source. So, this is source. (FL: 32:18) Let us just this much I will tell right now, suppose I state it in time doing this is omega t, omega is constant and I state here I would like to state here, Pt now how can I escape it I will first state the supply voltage.

This is you are this Vmax I know sign omega and then I will skate it Id RL circuit it will lag by some angle theta value of theta may be 0 to 90 degree only. So, curent will be lagging like this steady state current it was there here also it is dead anyway and this angle is theta, this angle is theta whatever happening to the voltage waveform that is 0 going towards positive similarly, this point current 0 going towards positive will happen after theta.

Similarly, this data will be between peak to peak also you know this so, this is the thing now you look then this is voltage this is your voltage and this is the current i t is not so, this power instantaneous power, so, this side is corrected vt and i t I have sketched this is it this is bt. What = Imax sine omega t -theta. Now, you see below this I will escape the instantaneous power what is that product of at any instant I will multiply voltage and the corresponding current at every instant I we were not doing like that and stated here.

Now, you see, you first break up this thing wherever voltage has got a 0 crossing and current has got a 0 crossing changing from + to - like that and let us divide this interval got the point. So, what I have done voltage waveform was there 0 crossing marked with bullets, similarly currents wherever 0 crossing I have marked up with the bullets. Now, during this interval voltage is + and i t is negative. So, accurately current is flowing like this and this is +.

So, supply the absorbing power got the point and this product will be negative use a different color for that. So, this product will be this during these to these voltages + and current is - something of this sort will happen at this point 1 second it will be 0. Now, during this interval, this to this is both voltage and current is positive that means, it will be something like this. At this point once again it will be 0, but power will be positive really voltage is positive in real terms at that during this instant anytime you choose voltage you will be positive means this is really + this is my is current is really positive.

So, it is flowing this way only Therefore, I will conclude it will it is delivering power here power delivered and during this Joule power delivered have calculated and which has come out to be negative. So, I will say it is absorbing power sources absorbing power during this Joule during this you see once again bold current is + but voltage is - therefore polarity will be such that it is pumping back power back to the supply and here in this joule both voltage and current in this time interval, both voltage and current has reversed.

So, once again it will deliver power productivity +. So, it will be we like this and so on. Therefore, you see as a function of time the instantaneous power delivered by the source to the load looks like this we the positive side, more average value of this is not 0, it is not a sine wave like this equal excursion on the + and - sign it is the positive side will be more interval more values like that. Therefore, during this time power is delivered and during this time power is observed power deserves R got the point and this is gives you a fair because after all, there is a resistance.

So, some average power must exist. Therefore, people say that the average this waveform P t, this average power can have this waveform, blue colored way form which shows the instantaneous power will be positive indicating that this arcade on average absorbs power because of the presence of the resistance there because their power will be lost there. So, we will continue with this in my next lecture, please go through it carefully. Thank you.