

**Network Analysis**  
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**Lecture # 29**  
**Concept Analysis and Reaction**  
**Power in A.C Circuit - I**

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The slide shows a handwritten derivation of instantaneous power in an AC circuit. At the top, it is titled "Instantaneous Power in A.C Circuit". On the left, a circuit diagram shows an AC voltage source  $v(t)$  connected in series with a resistor  $R$  and an inductor  $L$ . The current  $i(t)$  flows through the circuit. To the right of the diagram, the following equations are written:

$$v(t) = V_{max} \sin \omega t$$

$$i(t) = I_{max} \sin(\omega t - \theta)$$

$$p(t) = V_{max} I_{max} \sin \omega t \sin(\omega t - \theta)$$

Next, the power is expanded using trigonometric identities:

$$p(t) = V_{max} I_{max} \sin \omega t (\sin \omega t \cos \theta - \cos \omega t \sin \theta)$$

$$= V_{max} I_{max} \cos \theta \sin^2 \omega t - V_{max} I_{max} \sin \theta \sin \omega t \cos \omega t$$

The final expression for  $p(t)$  is shown with two terms circled in red:

$$p(t) = \frac{V_{max} I_{max} \cos \theta}{2} (1 - \cos 2\omega t) - \frac{V_{max} I_{max} \sin \theta}{2} \sin 2\omega t$$

Handwritten notes explain the terms:

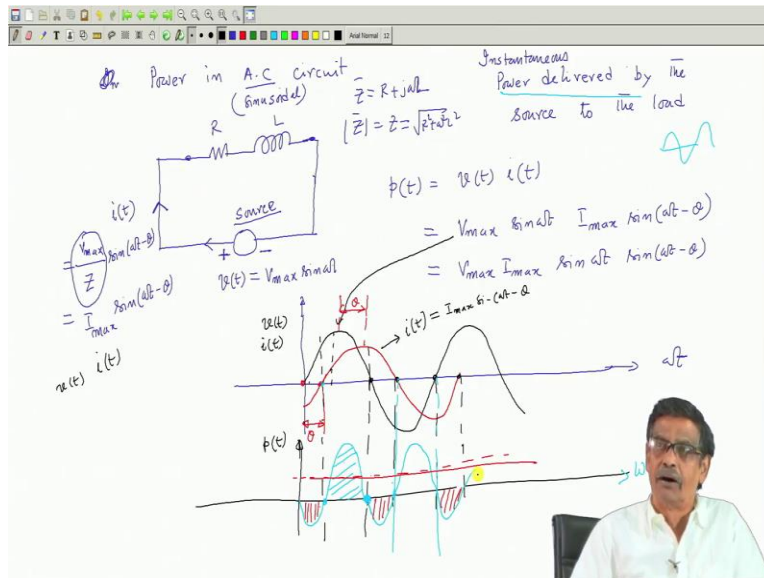
- For the first term: "This term will never become -ve"
- For the second term: "It will be equal having both +ve & -ve excursion about  $\omega t$  axis"

Additional notes on the right side of the slide include:

- lec 29
- $I_{max} = \frac{V_{max}}{Z}$
- $\tan \theta = \frac{\omega L}{R}$

So, welcome to lecture number 29 and we have been discussing about instantaneous power in an A.C circuit it may do A.C means sinusoidal excited circuit.

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And we have seen that if it is an unreal circuit that is voltage waveform black 1 current will be lagging this, then you divide several intervals following this rule wherever 0 crossing has occurred both for voltage and current, you mark them with bullets drop article lines and say during this period voltage is + current is - So, negative power but I am calculating power delivered by the source because this is how I have assumed this as it looks like it always it delivers power.

So, Vt into it mind you gives you power delivered, but in a scenario like this, I also know this one this product will sometimes you cannot + some things will become -. So, if somebody writes in language power delivered to the load is 50, 51. So, 51 correct, but if somebody calculates as some other in stands that power delivered to the load is - 10 what is indicating - 10 telling that at that time, sources observing power and load is delivering power can load delivered power yes because there is energy storing elements inductance sometimes we will come back power to the supply anyway.

So, we plotted then the instantaneous expression of calculated the instantaneous expression and got this power and we found that there is a positive power average can be found out we can do right now, and we say this circuit observe so much what have power on average. But another way of looking at this one that this is much more, nicer way that is what I want to share with you. So, your thing was like this will play with those equation a little more.

So, this was my circuit. This was the voltage AC voltage, sinusoidal voltage and this was current while solving this current for this current, I have taken help of the phasor. Now I am trying to interpret about the power. So, I have come back to time and trying to indicate the things. So, what did they do? I told you let  $V_t = V_{\max} \sin \omega t$ ,  $i_t = I_{\max} \sin(\omega t - \theta)$  RL circuit. What is  $I_{\max}$ ? Did not forget  $I_{\max}$  is a call to be  $V_{\max}$  by this  $Z$ , is it a phasor? No, it is magnitude without the bar I am writing get his proof over  $r^2 + \omega^2 L^2$ .

What is  $\theta$  and  $\theta$  from this exploration I can calculate  $\theta$  is standing yet, so, I have solved this circuit I have this results before me, then I calculated the instantaneous power, which if you calculate this I wrote last time  $V_{\max} I_{\max} \sin(\omega t - \theta)$ , this is the thing I can now you see what I am doing this power instantaneous power  $V_t$  will be equal to  $V_{\max} I_{\max}$  it is there and this is  $\sin(\omega t - \theta)$  I will write it like this.

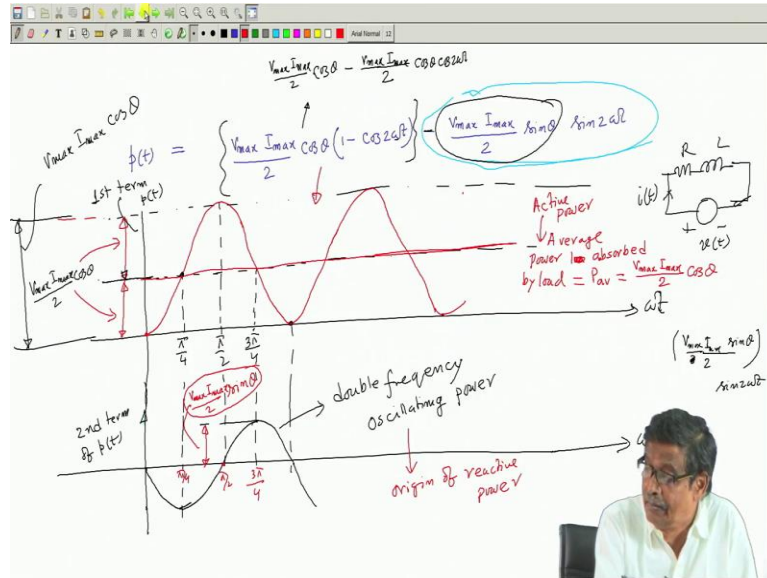
And this  $\sin(\omega t - \theta)$  I can write it as  $\sin \omega t \cos \theta - \cos \omega t \sin \theta$  mind you this is a fixed angle already changing with time  $\sin \theta$   $\cos \theta$  is fixed for a given circuit by  $L/R$ . So, this is how you right then also this one you break up in this excursion  $V_{\max} I_{\max} \cos \theta$  faster into  $\sin^2 \omega t$  this faster -  $V_{\max} I_{\max} \sin \theta$  and then  $\sin \omega t \cos \omega t$  this way can right.

Now, this can be written as  $V_{\max} I_{\max} \cos \theta$ . So, by 2 and 1  $\sin^2 \omega t$   $\sin^2 \omega t$  can be written as  $1 - \cos 2\omega t$  is the thing -, this 1 also can be written as  $V_{\max} I_{\max} \sin \theta$  and these 1 into  $\sin \omega t \cos \omega t$  is signed to a magnitude these what have done rejuvenate now, this instantaneous power it is cleverly broken up into 1 components just by simple trigonometric manipulations.

Now, look at the faster very carefully this term. This term if you look at it will remain always positive, it cannot be negative. I will escape and show you and these ones in  $\sin \omega t \sin t$  is constant and this term. So, this term will never become negative. This term  $\sin \omega t$  both + - is excursion it will be having both positive and negative excursion about  $\omega$  axis equal

having both equal positive and negative x. Choose any value of omega to you put it will be always positive, or at best it can be 0 but it will never become negative now.

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This equation helps us to understand much better copy and I will paste it here. So, this is the thing we have written there and I will lead is this. So, this is the instantaneous expression of the path. Now, this one I would like to sketch this term, I will sketch then this term and it will be easily understood what I am trying to say so, this is support the thing is, now, this term is nothing but  $V_{max} I_{max} \cos \theta$ , this bracket I am opening got the point this term only am writing the  $V_{max} I_{max} \cos \theta - V_{max} I_{max} \cos \theta \cos \omega t$ . So, it will be a DC value.

This 1 is constant  $V_{max} I_{max} \cos \theta$  is constant so I have to add to this component, this is the DC component. Suppose, this level is  $V_{max} I_{max} \cos \theta$ , this constant, this is a constant number, then to this I have to add a sin to  $\omega t$  term with a negative peak values. So, it will be just like this the sum of these negative sin value which will then go like this what is at this level this will be also these 0 are equal this will be to  $V_{max} I_{max} \cos \theta$  this whole thing got the point these the only the faster I have sketched.

So this is the first term which has been sketched here, this term I have not sketched, clear and you see this quantity will never ground 0 and come negative. At best it can become 0 at some

times. So this term will basically tell you that power absorbed by dissipate always positive then I sketched this time sketched including this negative this term had sketched how we do look like it is simple signed to  $\omega t$ .

So, once again I sketched it here mind you I am skating against  $\omega t$  always whenever, whatever you are sketched show the axis very clearly. So, this is  $\omega t$  you can identify the value of this  $\omega t$  how much is the value of this  $\omega t$  there it has to be  $\pi/4$  at  $\pi/4$  if you put it is  $\pi/4$  is 0 and at this point the value is the  $V_{\max} I_{\max} \cos \theta$  only. This 1 if you put  $\pi/4$  in this expression  $\cos \pi/4$   $\cos \pi/2$  is 0 and you will be left with  $V_{\max} I_{\max} \cos \theta$  by 2 and this 1 will be how much  $\cos \pi/2$  this should be why, because at  $\pi/2$  this is  $\cos \pi - 1 - + 2$  to cancel out and this whole length is the  $V_{\max} I_{\max} \cos \theta$ .

Because these are this so, this whole team whole thing is  $V_{\max} I_{\max} \cos \theta$  that it is doing then once again from symmetry it we can easily so,  $\pi/2 + \pi/4$  it will be ordered  $3\pi/4$  and so on and it will be 0. Here you can find now, I have to scale this 1 this is z, it has got a peak value and signed to  $\omega t$  so, it will have a both + and - excursion. So, I will also stage that the second term and its value is  $V_{\max} I_{\max} \sin \theta$ .

Now, the question is, if it is possible for at this point, it will have peak value is not so, I am now skating this term  $V_{\max} I_{\max} \sin \theta$  into  $\sin \theta$  is the peak value of that  $\sin$  wave into  $\sin \omega t$  descent skating. So, at  $\omega t$  equal to  $\pi/4$  for if you put  $\sin \pi/2$ , so, it will it must have that big value occurring at  $\pi/4$ . So, it is a  $\sin$  term with this big value but anyway I have to add to this app to once again subtract.

So, - of that will be like this. So, it has to start from 0 and it will reach maximum here to magnetically. So, it will be late this then at  $\pi/2$  then if you put  $\pi/2$   $\sin \pi/2$  is 0. So, it must 0 here and then once again it will reach maximum and once again it will reach you and go to negative, but this is a  $\sin$  to  $\omega t$  term it will have both positive and negative excursion. So, second term of  $p_T$  and this is this 1 is first term of the  $p_T$  that is what  $p_T$  therefore instantaneous absorbed by the RL circuit.

We are now going much deeper into it and trying to understand what is happening here. So, this is your  $v(t)$ , this is your  $i(t)$  and this first term that is this term, I understand that, it always delivers power to the circuit because this  $p(t)$  is always positive, sometimes it is becoming 0, but never coming negative. And this  $p(t)$  is the power which is oscillating between source and the load. So, a double frequency oscillating power, got the point.

And what is the peak value of this  $p(t)$  is  $V_{max} I_{max}$  by 2  $V_{max} I_{max}$  by 2 that all and it is  $\omega t$  on skating. So, this is  $\pi/4$ , this is  $\pi/2$  and so on  $3\pi/4$ . Therefore, it is this power which is which is flowing to and flow between the source and these load sometimes parties absorbed by these because inductances there are some things parties returned back to the source but this is the average power.

Which load is absorbing these the average power this all against time this much power is absorbed by the source, but on an average the load is absorbing average power absorbed by load is this this line and what is that value I will write it as  $p_{average}$  is equal to  $V_{max} I_{max}$  by 2  $\sin \theta$ . And this turn, which represents the power oscillating between source and load has a peak value whose value is  $V_{max} I_{max}$  by 2 inches into  $\sin \theta$  a forgot another term  $\sin \theta$  this is big value. It will be this.

Now, therefore, we say that power which you will be oscillating between source and the load will be called reactive power it is never consumed by the circuit. So, this is the origin of origin of reactive power and this power which is about absorbed by the load or consumed by the load is called the active power. Therefore, in A.C. circuit when things are sinusoidal excited circuit doing things are getting both voltage and current are changing with time.

No point in talking about instantaneous power, it is better you tell me how much average power I am consuming is not that A to B much more meaningful than to calculate instantaneous board at this is 10 time consuming this much power these know your electricity bill should be prepared based on the average power you are consuming. After all, it is only 20 millisecond 1 cycle of the supply is not in the previous survey. This is the this is 20 millisecond in 20 millisecond, how

much energy we are consuming what is powering through that time based on that your how much power you are consuming should be calculated based on the average power clear.

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$$I_m \text{ A.C circuit :- } \bar{I} = \frac{\bar{V}}{Z}$$

$$P_{av} = \frac{V_{max} I_{max} \cos \theta}{2} \quad (\text{Real power})$$

$$= \frac{\frac{V_{max}}{\sqrt{2}} \frac{I_{max}}{\sqrt{2}} \cos \theta}{2}$$

$$P_{av} = V_{RMS} I_{RMS} \cos \theta$$

$$Z = \frac{\bar{V}}{\bar{I}} = \frac{V_{max} L^0}{I_{max} L^{-\theta}}$$

$$= \frac{\frac{V_{max}}{\sqrt{2}} L^0}{\frac{I_{max}}{\sqrt{2}} L^{-\theta}} = \frac{V_{RMS} L^0}{I_{RMS} L^{-\theta}}$$

$$\text{Suppose } v(t) = 245 \sin \omega t \rightarrow \bar{V} = \frac{245}{\sqrt{2}} L^0$$

$$\bar{I} = \frac{\bar{V}}{Z} = I_{RMS} L^{-\theta}$$

$$\bar{Z} = R + j\omega L$$

Therefore, this is the thing now, so, this p average now we make an important statement in AC circuit therefore, in AC circuit we have seen what i bar current in the circuit is V bar by this circuit but we have seen just like your DC circuit, only thing this time the numbers are complex and p average this V max I max by 2 into cos theta that is the real power which real power that I have explained just right now.

So, this is the thing. So only being in DC circuit current in DC I just got down here in DC current is V/R power is V into I, now here is in the expression of the average forward we find peak value of that sinusoidal voltage into peak. Well the current, but there is a factor to appearing it is nothing wrong one can leave with this apart from this this turn of course cos theta will be always there. So, this expression is written like this V max by root 2 into I max by root 2 into cos theta and you know V max by root 2 is nothing but RMS value of the supply sinusoidal pairing quantity with time be V max sin omega t.

What is the value V max sin omega t, similarly current is also sinusoidal what will be sinusoidal value, it is I RMS into cos theta sometimes people just did not understanding, once you write the

average power consumed by the circuit is  $V_i \cos \theta$ , it means these elements  $i$  and  $r$  got the point. Now, after knowing this, so, it is quite then similar to that of DC circuit only thing 1 factor comes  $\sin \theta$  that you of course, you cannot have it cannot be productive voltage and current RMS value of voltage and current except for this case when  $\theta = 0$ .

But anyway then we will leave with this average is voltage and current and last one thing I will add, why dividing the phasor I told you suppose I told you that  $Z$  bar is a equal to voltage bar for that divided by current for that and this voltage phasors I defined in terms of maximum values they be executed and current failure also I wrote like this  $I_{\max} \cos \theta$  for RL circuit is not this is what I wrote. But that is phasor represented by the peak values there will be absolutely nothing will be wrong.

If you divide these by  $\sqrt{2}$ , and  $I_{\max}$  by  $\sqrt{2}$  to both numerator and denominator you divide by  $\sqrt{2}$  and rightly this and say that while you presenting phasors, I will represent in terms of RMS quantities, that is I will write be  $V_{\text{RMS}} \angle 0^\circ \cos \theta$ . So, you get burn, demand sank age. Therefore, at the end what we will be doing what people will do is that if any sinusoidal quantities they are suppose I say that the  $v(t)$  is equal to some voltage  $\sin \omega t$ .

Then while telling in terms of phasor what I will do  $V$  bar I will hence take these values that  $245$  divided by  $\sqrt{2}$  angled  $0$ . It does not change  $Z$  bar it is equal to  $r + j \omega t$ . So, I will calculate  $Z$  bar but then if you calculate  $I$  bar from these  $V$  bar by  $Z$  bar,  $e$  will get obviously  $I_{\text{RMS}} \cos \theta$ . Therefore, representing in phasor rotations because, to get similar expression for power as indices are similar, not identical  $\cos \theta$  is there, these  $\cos \theta$  it is called the power factor of the circuit.

So, how nice it is therefore, in a circuit, sinusoidal current and they will have in general some phase displacement between them. So, sometimes the instantaneous product of voltage and current will become  $+$  sometimes  $-$ . If you are using some power in this circuit there must be some average value, so this thing to summarize, once again I am telling this is the expression of the average power just from this is the expression of the instantaneous power.



Then we intelligently broke up this 2 into 2 terms, 1 term is always positive, it will never have negative this term never becomes negative it may come to 0 that is there and there is another term which is an also a double frequency term and this will have identical positive and negative excursion. So average value of this quantity is 0. No doubt about it from these, if you sketched this way form, as expected, there was no negative value of this faster it will be oscillating.

So, it will have an average value and that average value will  $V_{\max} I_{\max} \cos \theta$ , because average of  $\sin 2\omega t$  is 0 all the time. If you calculate average over a cycle,  $\sin \cos$  in terms average value is 0. So this will give you 0 value. So this is the average be  $V_{\max} I_{\max} \cos \theta$ . And the second term which is only I have been signed to  $\omega t$  know DC term as they added to it.

So, it will have identical + - excursion and it will continue like that. So, sometimes the circuit will absorb power from the suits and sometimes some other interval, it will return back the same power to the source that power will never be consumed by the circuit. Here electricity bill is consumed on the basis of this average power you are consuming who will in the next class tell you something more about this power factor and its implication. Thank you.