

Network Analysis
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Lecture # 30
Concept Analysis and Reaction
Power in A.C Circuit - II

So, welcome to lecture number 30. And we have been discussing about active reactive power in A.C circuit.

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Active & Reactive Power in A.C Circuit :-

average power absorbed by load

$$P = \overbrace{VI \cos \theta}^{\text{RMS}} = \frac{1}{2\pi} \int_0^{2\pi} v(t) i(t) d(\omega t) = \frac{1}{2\pi} \int_0^{2\pi} V_{\max} I_{\max} \sin(\omega t) \sin(\omega t - \theta) d(\omega t)$$

$$= \frac{V_{\max} I_{\max}}{2\pi \times 2} \int_0^{2\pi} 2 \sin \alpha \sin(\alpha - \theta) d(\omega t)$$

$$= \frac{V_{\max} I_{\max}}{4\pi} \int_0^{2\pi} [\cos \theta - \cos(2\alpha - \theta)] d(\omega t)$$

Reactive power inductive

$$Q = \frac{V_{\max} I_{\max} \sin \theta}{2}$$

It is peak value of the oscillating power between the source and the load

$$= \frac{V_{\max} I_{\max}}{4\pi} \int_0^{2\pi} \cos \theta d(\omega t) - 0$$

$$= \frac{V_{\max} I_{\max} \cos \theta \times 2\pi}{4\pi} = \frac{V_{\max} I_{\max}}{2} \cos \theta = VI \cos \theta$$

So, active power is denoted by generally capital which is average power average power absorbed by the load by load. So, this average is generally it is average I even if I did not write it means average capital P and it was shown to be equal to be V RMS into I RMS into cosine theta is called average power then reactive power that is this really this is nothing but essentially over a full cycle vt it d omega t 0 to 2 phi. This is the average over a cycle.

If you put this one you can easily show as 0 to 2 phi This is V max I max sign omega t into sin suppose RL circuit - theta, then omega t it was this which just one line derivation, which of course, we have done in a different way but average means this -, the V max I max by 2 phi is there and then you have 0 to 2 phi. Then you multiply the denominator by 2 an numerator by 2, 2

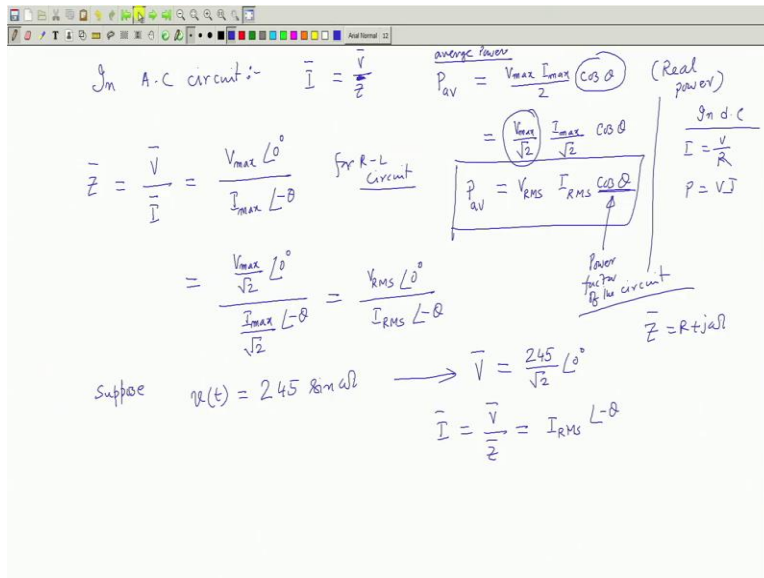
$\sin A \sin B$. So this will be signed waiting $2 \sin \omega t$ into $\sin \omega t - \theta$ into $d \omega t$ formality average over the full cycle.

0 to 2π integrator and this - will be $V_{\max} I_{\max} \sin 4\phi$ here and this one can be broken up into 2 terms $\cos A - B$ will give you this - $\cos A + B$. Now, $\cos \theta$ is constant therefore, this will simply become $V_{\max} I_{\max} \sin 4\phi$, get you to $\phi \cos \theta d \omega t$. And, whatever will be left over it is cosine ωt you are averaging over a full cycle that must be 0. So, this will then be $V_{\max} I_{\max} \sin 4\phi$ into $\cos \theta$ is constant and integration 0 to 2π is 2π .

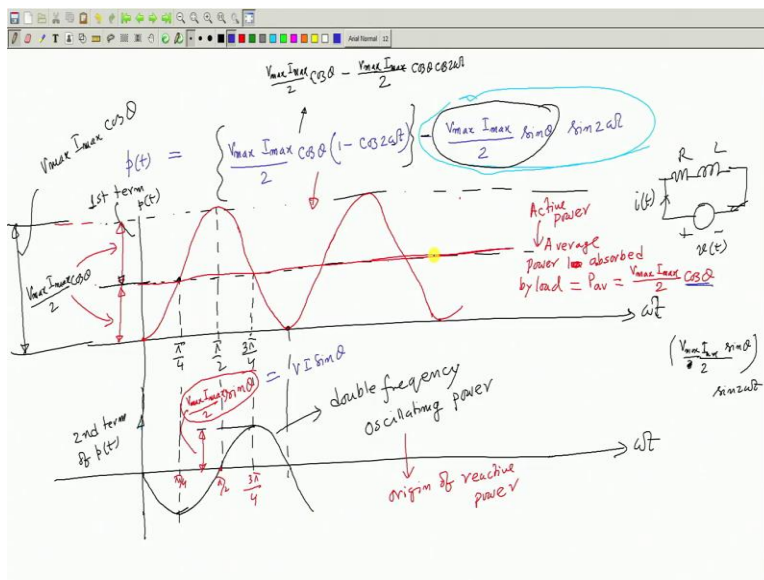
So, this will be the $V_{\max} I_{\max} \sin 2\theta$ which is equal to $V_{\text{RMS}} I_{\text{RMS}} \sin 2\theta$ into $\cos \theta$ this to do to route to you so, RMS voltage into cosine θ is this -. So, these are our RMS values are the phasor said no, just magnitude of the voltage whatever reactive power is denoted by Q and it is written as the max that is what we got last time $I_{\max} \sin 2\theta$ which is equal to be $V_{\text{RMS}} I_{\text{RMS}} \sin 2\theta$.

But mind you this reactive power $V_{\max} I_{\max} \sin 2\theta$ for example, real power $VI \cos \theta$. If I am asked to tell you what it is, I will tell take the product of this 0 to 2π average value of the power very clearly understood. Now, what is $VI \sin \theta$? Our answer should be $VI \sin \theta$ is not the average value of anything, it is the peak value equal to of the oscillating power not average between the source and the load source and the load.

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That is what you must understand because you see, you recall that average power origin of reactive power $V_{max} I_{max} \sin \theta$ by 2 quickly now after knowing RMS value I know it is $VI \sin \theta$ and what it is it is the peak value of the double frequency oscillating. So, the physical significance of $VI \sin \theta$ is also about the oscillating power and the peak of that what is the real power like the power it is the average value of p high you must understand that.

Therefore, this is how things goes on. So, what is the signal we can stop this AC power I will now tell a few words about that. So, that that is why the AC circuit and interpretation of power

becomes interesting then what happens whenever energy story mind you whatever I have done with RL circuit.

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$v(t) = V_{max} \sin \omega t$
 $i(t) = I_{max} \sin(\omega t + \theta)$
 $Z = \frac{V_{max}}{I_{max}} = \frac{V}{I}$
 $\tan \theta = \frac{1/\omega C}{R} = \frac{1}{\omega CR}$

$P = VI \cos \theta$
 $Q = VI \sin \theta \rightarrow$ Reactive Power capacitive.

Significance of power factor $\cos \theta$

$P = VI \cos \theta \rightarrow (0 < \cos \theta < 1)$

RMS $200V$ $50Hz$
 $\bar{V} = 200 \angle 0^\circ$

$Z = 6 + j8 \Omega = 10 \angle 53^\circ \Omega$
 $\bar{I} = \frac{200 \angle 0^\circ}{10 \angle 53^\circ}$
 $= 20 \angle -53^\circ$
 $P = 200 \times 20 \cos 53^\circ$
 $= 2400 W$
 $Q = 200 \times 20 \sin 53^\circ$
 $= 3200 VAR$
 Volt ampere reactive

It can be also done with RC circuit we can do, but I will tell you this is supposed to see and excited by AC current you will get similar results no problem this is V_t and this is i_t . So, similarly, I can V_t is a call to $V_{max} \sin \omega t$ and you are i_t I know it will $I_{max} \sin \omega t + \theta$ where Z is V_{max} by I_{max} which is also it will be called to V_{RMS} by I_{RMS} . So, except that the current is leading similar thing can be done and V_L power will be once again average of this product.

In the same way e will be able to do we cannot at all finding it interesting for me and for you also it will be same calculations, but except that $\sin \omega t + \theta$ and they will arrive at this $V_{RMS} I_{RMS} \cos \theta$, what is $\tan \theta$ is 1 over ω reactance by resistance. That is 1 over what is V , VI is the RMS values the same way similarly, the reactive power will be the peak value of the double frequency oscillating power existing.

So, you also can be calculated from that clear. So, in the same way it can be done. So, there is one part where the power is consumed and power will be essentially consumed in our only and energy storing element is there that will be responsible for this oscillating power. If these are good is purely resistive no L and C then of course, there will be no reactive power no exchange

of power between source and load. It will not be there. Whatever power is there that will be consumed power.

That you can draw the voltage and current to a form they will be in phase and always the power will be positive average power anyway this is the thing, but I will now tell you one thing you see the significance of cosine theta significance of power factor cosine theta. What does that mean? So, Q will be $VI \sin \theta$ once again for leading I should break for leading phasor circuit RC for lagging this reactive power.

So, - point is for another circuit what people say is that reactive power is this much and oscillating power between load and peak value of this week is equal to $VI \sin \theta$. This is called Q reactive power inductive I can say inductive circuit to distinguish between these 2 and here I will say reactive power capacity. Now significance of the term cosine theta. Now, you see you if you see from the supply point of view here is your supply and here is your load which may be RL or RC time.

So, if I express now I know something about phasor so, I will simply write this is V bar and this is I bar and this is Z bar. This line is connected suppose you are running a factory, there are several more loads, different kinds of load effectively load connected across the supply is this much and you are drawing this much current. You are amateur, if you connect an amateur here this amateur and voltmeters are so design they will record always RMS value of the quantity if you connect the both with here it is AC voltage.

But RMS value will be only recorded similarly, current RMS value of the current to be decoded by the amateur. Now, as I told you, as you draw power you consume the power, this is your supplier authority, who is supplier authority is responsible for maintaining voltage across your premise across your loads that they will bend and it gives you A and V . But, the product of the V and A is not the power you are really utilize a fraction of it because cosine theta is a number less than -.

So, power real power absorbed by $VI \cos \theta$. And based on that e will pay the electricity bill as I told you earlier, $\cos \theta$ is a number between 0 to 1 if somebody has circuit like this same circuit same voltage and he has conducted resistance purely resistive circuit no inductance and the value of the resistance is $A = \text{mode of } Z$. Then is admitted it will be also same as this fellow. But the only thing voltage and current will be of same phase this apply voltage V same voltage I connected resistance you connect an impedance which has got resistance and inductance how the impedance.

For example $6 + j 8$ were connected then the impedance values is root over $6^2 + 8^2$ is 10 ohm to give you a concrete example, suppose, your supply voltage is 200 volt and whenever supply voltage will be prescribed it is RMS value in variably 50 hertz. Now, suppose you connected $j 8$ let us try to understand the problem $j 8$. It is supposed $6 + j 8$. How much is impedance angle $P 53$ degree got the point?

So much of 10 inverse $8/6$ and what will be the current? I am interested to know what will be the amateur. So, I will calculate I bar as equal to $200 \angle 0$ degree suppose I take voltage on the reference divided by $10 \angle 53$ degree which will be called to 20 ampere - 53 degree is it this is the voltage. So, $V \text{ bar} = 200 \angle 0$ degree I have assumed and divided by j I got 20 ampere I will say admitted will read 20 ampere fine.

What is the active power it is consuming P is how much is V into I this is also RMS i into $\cos \theta$ 53 degree $\cos \theta - 53$ same as how much it is 2400 watt I should it $VI \cos \theta$ (FL: 18:41) How much reactive power it is consuming, V into I into $\sin \theta$ $\sin 53$ degree, how much it is 3200, now hear about the units. See this is also powered dual per second, but what happens is this to distinguish it from what we write a unit which is called bold ampere reactive.

VAR be a reactive power is generally denoted by this unit VAR whereas what the active is denoted by what, but mind you they are - in the same thing productive voltage into current got the point, but to make difference between the active power and reactive power and it is no new unit. It is similarly voltage into current is what that is what we have got q also voltage into

current into sin theta is not what each unit should be what only, but what is chosen to represent the active power, because they are that that power you are consuming.

And since this is an oscillating power it is giving a different volt ampere reactive this much is reactive power this much is active power. Now, I am telling you will have to pay the electricity based on this 2400 watt got the point? Find it is so supply authority will charge you bill electricity at the end of the month, how much kilowatt hour etc. That is based on what age not based on this quantity because after all you can always say this 3200 is a measure of power which is oscillating, I am taking, but I am also returning you back I am not consuming got the point.

So, for this why a consumer of electricity should be charged it should not be happening until it looks like. So, this is consumer who has connected a load because this load impedance is not in your hand you have connected several gadgets several motors this that effective impedance you have connected to the source works out to be this may not be in your hand, but you have to use this load for your own purpose because all motors are inductive nature.

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$\vec{V} = 200\angle 0^\circ$
 $\vec{I} = \frac{200\angle 0^\circ}{10\angle 0^\circ} = 20\angle 0^\circ \text{ A}$
 $\vec{Z} = 10 + j0 = 10\angle 0^\circ \Omega$
 $\theta = 0$
 $P_{av} = 200 \times 20 \cos 0^\circ = 4000 \text{ W}$
 $Q_{reactive} = 200 \times 20 \sin 0^\circ = 0$

$\vec{V} = 200\angle 0^\circ$
 $\vec{I} = \frac{200\angle 0^\circ}{6 + j8} = \frac{200\angle 0^\circ}{10\angle 53^\circ} = 20\angle -53^\circ$
 $P = 2400 \text{ W} \rightarrow \text{active}$
 $Q = 3200 \text{ VAR} \rightarrow \text{Reactive}$

200 X 20
 4000 VA

Now consider another consumer this is quite interesting to not another consumer which is fade with the same voltage I will straight away right RMS values 200 volt angled 0 degree. This is the supply voltage \vec{V} and e has connected a load which is purely resisting and is $R = 10 \text{ ohm}$. I

can have a situation like this, these another consumer purely connected. Now, how much current will be flowing here I bar in this circuit it will be $200 \angle 0^\circ$ so much volt divided by 10. What is this angle 0 because there is no ωL inverse 0 that means θ is equal to zero.

So, it will become $20 \angle 0^\circ$ ampere that is voltage and currents will be in phase is the voltage in time domain and your current will be also in phase unity power factor they call it θ is equal to 0 is equal to $10 + j 0$ here, therefore, it is equal to $10 \angle 0^\circ$ got the point this is the volt. So, 3 times 0 voltage and current in phase and if you plot the product of these 2 it will be always positive both voltage and current reverses simultaneously.

So, it will be always positive there is nothing like oscillating power existing in this circuit. So, it is if you connect an ammeter it will still read 20 ampere I will read and how much power is going to consume voltage 200 volt into current 20 ampere into $\cos \theta$ V bar is a called $200 \angle 0^\circ$ degree so much volts. So, $\theta = 0$ here so, into $\cos 0$ which is equal to 4000 watts of power is going to consume.

What is reactive power? I need that right reactive, but anyway p q reactive it will be bolted into current into $\sin \theta$ has expected mathematics tells it is 0 also we expect there is no oscillating powered between because no energy storing elements. So, we had the first consumer I did draw it here, I did draw it here he was the first consumer was like that same source $+ -$ supply voltage is saying for both the consumers 0° degree volts.

This quite interesting to note down and here is a connected and ammeter and load is as I told you $6 + j 8$ ohm in this case also will read 20 ampere is it and I once again very quickly calculate the numbers you can just tell I bar the current phasor we got I bar how much you got $200 \angle 0^\circ$ divided by $6 + j 8$ ohm division polar form is better but your calculators nowadays will also take polar and rectangular together you do that 105.3° and this was equal to $20 \angle -53^\circ$ is not that will be the thing and then we calculate real power in this circuit, how much we calculated 2400 watt and we have power calculated $Q = 3200$ watt.

But I will call it volt ampere reactive it is also what now, these are the 2 consumers, these consumers connect a pure resistive load and this consumer connect a RL connection $6 + j 8$. Now here is supply your authority for, which consumer is good for them who is going to pay more to the supply your authority definitely this fellow because it is consuming 400 watts of power so, at the end of the month he has to pay his electricity bill compared to this fellow because he is consuming a nearly $1/2$ slightly greater than $1/2$ the water it is using, but it is going to pay less electricity bill is decided by the P values active power not about reactive power.

Now, from the point of view of the supply authority this consumer is more attractive to them attractive for them then this fellow is not mode the power factor is towards unity that will be a better consumer because the electricity both been will be more now supply authority may think a here is the consumer which is taking 2400 watt of activity active power and this much of reactive power is taking and is paying me less but easy, but the consumer tells I am taking that reactive power but also returning you back.

So I should not be charged that logic is fine. But what is disturbing for the distribution transformer he is ready to see 200 volt 20 ampere. So, he would expect that 200 into 20 amount of 4000 volt ampere am handling, but a little portion of it is used by the consumer and other parties oscillating but this 20 ampere he has to carry through the transmission line see reactive power increase, but you cannot deny that you are growing 20 ampere delay when you are not by the supply authority would have been much higher.

But if you are the supply authority will see that he has got a power factor. So 200 into 20 watt if he would have consumed nothing is better than that for the supplier authority, but only a portion is used and other things are returned back. But that current has to flow all along the lines. So, current drawn by the load is decided by both active and reactive power total volt ampere how much you were drawing from the source therefore supply your authority says that if your power factor is unity nothing is better than that, but they put also a condition to the consumer that power factor you better keep above this value.

Maybe it is a point 8 power factor lagging power factor should have a power factor greater than point 8. Then for that creative power I will not charge you anything although you are drawing extra current, but that I will not charge. In fact for consumers in houses like people like us who use electricity that they did not put any condition because we are very little we consume power very little compared to big industries.

So, for big industries supply authority will say that make sure that whatever load you are connecting motors fans these that big loads, you must ensure that power factor is greater than point 8, a power factor is less than point 8. Then also I will charge for that reactive power because the reactive power gives node a venue for the supply authority. It is situation is like this. Suppose you are running a big hostel, thousand students stay in your hostel.

You have some people some market is there about 5 miles away from your hostel big market is there, you tell that you supply me 100 kg of rice every day. So, that supplier brings 100 kg of rice at your doorstep and then you say that I will use I will buy you all the 100 kg fine nothing is better than that supplier 100 kg transport charger was there I have brought and he has used all the things but suppose another hostel who still he says that give me 100 kg supply or brings 100 kg and you say that I will only use take 10 kg, 20 kg you take it back

So, how much the second hostel has to pay for a 80 kg because after all he has purchased a 80 kg but supplier will think that that extra 20 kg I brought to your doorstep and he did not use it return it back but I have to carry it back to the shop once again. So, there is extra transport card involved. Similar with the current all the currents are active here. Every currents but here this 20 ampere a portion of this as if being utilized and other portion you are returning back.

You did not have to pay in general but when big industries are selling like that, that oscillating power becomes too large for that MPR to be flown in this arcade this way that way to do this way that way, the reactive component of the current that may be prohibited that industry supply authority will not allow. That is why they put an additional condition for big industries that look here. I did not keep low power factor for your loads. Anyway, we will continue with this and

solve several problems in my next class. But go through this. This is very interesting things to understand. Thank you.