

**Network Analysis**  
**Prof. Tapas Kumar Bhattacharya**  
**Department of Electrical Engineering**  
**Indian Institute of Technology-Kharagpur**

**Lecture # 31**  
**Expression for Complex Power in A.C Circuit**

(Refer Slide Time: 00:24)

The slide contains the following handwritten content:

- Definitions:**
  - Apparent Power (S):  $VI$
  - Active Power (P):  $VI \cos \theta$
  - Reactive Power (Q):  $VI \sin \theta$
  - Complex Power:  $\bar{S} = P + jQ$
  - Relationship:  $\bar{V} = \bar{I} \bar{Z}$
- Diagram:** A circuit diagram showing a voltage source  $\bar{V} = 200 \angle 0^\circ$  V connected to a load impedance  $\bar{Z} = 6 + j8 \Omega$ . The current  $\bar{I}$  flows from the source through the load.
- Calculations:**
  - Impedance magnitude:  $Z = \sqrt{6^2 + 8^2} = 10 \Omega$
  - Current:  $\bar{I} = \frac{200 \angle 0^\circ}{6 + j8} = 20 \angle -53^\circ$  A
  - Power factor:  $\cos \theta = \frac{6}{10} = 0.6$
  - Active Power:  $P = 200 \times 20 \times \cos(-53^\circ) = 2400$  W
  - Reactive Power:  $Q = 200 \times 20 \times \sin(-53^\circ) = -3200$  VAR
  - Complex Power:  $\bar{S} = 2400 - j3200$  VA
- Notes:**
  - Complex Power supplied by the source:  $\bar{S} = \bar{V} \bar{I}^* = 200 \angle 0^\circ \times 20 \angle 53^\circ = 4000 \angle 53^\circ = 2400 - j3200$  VA
  - Complex Power absorbed by the load:  $\bar{V}_{AB} \bar{I}^* = 200 \angle 0^\circ \times 20 \angle 53^\circ = 4000 \angle 53^\circ = 2400 - j3200$  VA

Welcome to this lecture to one and in last couple of lectures we have been discussing about the steady state analysis of circuits we get excited by sinusoidal voltage source and we showed you how to calculate the current you have to find out the complex impedance and if the input voltage is known express it in feather notation divide that will get you will get current and after getting the current The question is how to calculate the power consumed by the circuit.

It I should you that it is equal to  $V_i \cos \theta$  data by new these v rms value of the voltage is the RMS value of the current and cosine theta is called the power factor of the circuit. And also I told you in the last class the significance of having a low power factor or high power factor from the supply authority point of view, power factor of the load should be very close to unity nothing is better than unity that is fine.

Because whatever volt MP it will draw from the source been to is this product of this to only a portion because cost theta being less than one is being utilized by your circuit and based on that

you have to pay the electricity bill. But for large industrial consumers supply authority will insist that you maintain the power factor close to unity maybe it should be always above than point eight. Otherwise they have to also pay for the reactive power drawn by deload reactive power as we know is the power which is oscillating between the source and the end the load.

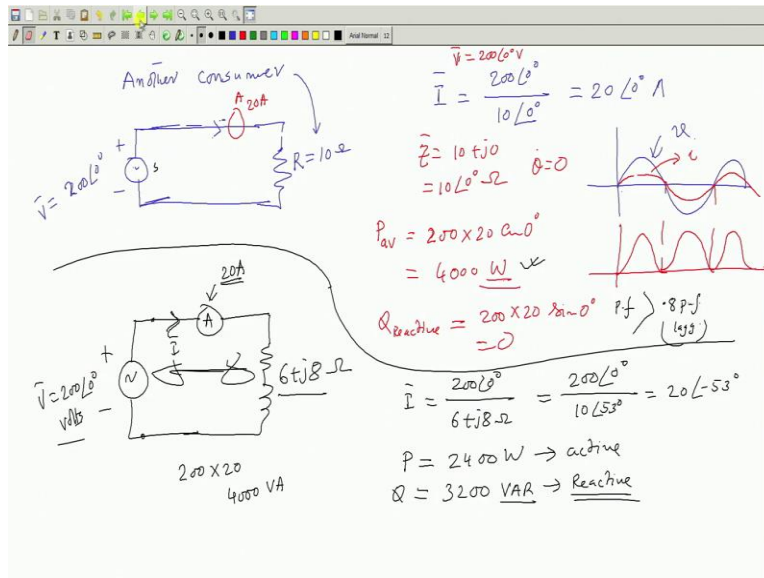
So, that power is never consumed by this or it but nonetheless it puts over read on the transmission system because current value will be then more. Now, today I will tell you how to calculate as P and Q of using complex algebra and the way because this complex analysis why we adopted only of course for sensor data source I can do we have adopted because of the fact we were equal to i bought into 0, this is the important thing which is similar to be called to a yard, this bar over this indicating they're all complex numbers.

So, if this is true then I can do anything. We here I did with DC circuit analysis we called wire that was the idea except that these are complex. Now similarly, while making power balance in an ACC, we see power consist of as is a call to be  $+ Jq$ . This is the thing, we can define a variable s, which is the AC is the total power ground by the circuit including active and reactive part Because of the fact See, if you make a circle a triangle right angle triangle like this, if this is your S VI and this is the perfect triangle of the circuit.

Then this must be P this must be q because we a cosine theta is P and vi cosine scientists a skill. So, for obvious reason, this triangle is always helpful. And why this is productive voltage into current is called efferent power it is simply because of the fact this power looks like it to be the power of the circuit but out of this only a portion be a cost data is being utilized by the circuit. So, that is why it is called active or real power or real power.

The other potion q which is bi sine theta, it is reactive power and it oscillates going 2 and for between the source and the load and never consumed. And in my last lecture I told you how to.

**(Refer Slide Time: 05:23)**



Calculate active and reactive power for this simple circuit here. Now, I will tell you one interesting thing that is by not new things, but how to compute this total power, this is total complex power. For example, the same example suppose the supply voltage phasor  $200 \angle 0^\circ$  the previous circuit, and this was  $6 + j$ , no last one,  $6 + j8$ . This is the circuit, this is the impedance. So we calculated this and  $6 + j$   $200 \angle 0^\circ$  degree.

So, very quickly let me do it supply voltage  $200 \angle 0^\circ$  and your  $\bar{I}$  we got a to be  $200 \angle 0^\circ$  degree divided by the impedance  $6 + j8$  and it came out to be  $-53^\circ$  close to that. And so the impedance was  $6 + j8$ . So, mature. Now, once we get this and did not forget to write the units volts, this is MPN So, I bought I get Now one way to calculate real power and reactive power separately that is  $P$  will be  $p$  which is a number not a feather real power is magnitude of the voltage into magnitude of the current into cosine -  $5030$  whatever it is, similarly  $q$  can be calculated as be scientists like that I can calculate it.

But instead what I will be doing, whether it can be calculated in one stroke using complex algebra, the rule for that is the we will say see, this is the these 2 terminals or delay terminals me, but I should be very careful while writing and then. So, I will write complex power or total power complex power supplied by the source. Now, before us calculate this, show the polarity of the voltage and show the direction of the current otherwise better did not try to write this it will be in a mess. So, from the side see, this source delivers power.

So, complex power supplied delivered by the source I am calculated It will be equal to suppose as bar I say and this will be equal to be bought into I bar start this star means the complex conjugate. So, this you calculate it in this way 200 0 degree is the voltage and I bar is this therefore, i star will be only angle will change 2 + into 20 into 53 degree, this is a star. So, this will be equal to 4000 and angle of + 53 degree which is of course 4000 cost 53 degree how much it becomes this calculations it we calculated it.

So, in using complex algebra you can write it will be 20 430 200. So 2400 is not + j 3200 this is the thing ultimately the real part will give you p, this is speed this is key. So, I will say that source is supplying to 400 this AC unit you right here volt ampere, then for p you say this is 240 what and Q the reactive power is 3200 this you right volt ampere reactive, this is how they are same that is what only this is also what but to distinguish it from the real power people invent another term use the reactive power.

So that we did not mix up the things. So, so 2400 words 30 201 Now, the question is why you take I start why not take the, into I li and calculate these things, it is better to note that generally.

**(Refer Slide Time: 11:47)**

In general  $\theta > 0$

Let  $\bar{V} = V \angle \alpha$

Then  $\bar{Z} = Z \angle \theta$

$\therefore \bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V}{Z} \angle \alpha - \theta$

$\bar{S} = \bar{V} \bar{I}^*$

$= V \angle \alpha (I \angle \alpha - \theta)^*$

$= (V \angle \alpha \times I \angle \theta - \alpha) = VI \angle \theta = VI \cos \theta + j VI \sin \theta$

$\bar{S} \neq \bar{V} \bar{I} = V \angle \alpha \times I \angle \alpha - \theta$

$= VI \angle (2\alpha - \theta) \neq P$

$= VI \cos(2\alpha - \theta) + j VI \sin(2\alpha - \theta) \neq P$

Diagram showing phasors:  $\bar{V} = V \angle \alpha$  (red),  $\bar{I} = I \angle \alpha - \theta$  (black), and a reference axis (Ref) (black). The angle between  $\bar{V}$  and  $\bar{I}$  is  $\theta$ .

The voltage lead in general the word feather could have any angle, for example, its angle is alpha, it is not 0 degree that is, I am started counting my time when the supply voltage has a

value  $v \sin \alpha$ , it was not coursing through the 0. So, we have discussed this. So, this is the voltage filter, where  $\alpha$  is the initial phase of the of the supply voltage with respect to reference, where there is no thing here.

Then, if  $I$  bar is a call to get  $\theta$ , where  $\theta$  is the perfect triangle, then you know then  $I$  will be equal to  $V$  bar by  $D$  bar =  $V$  by  $z$  and this angle will be  $\alpha - \theta$ , this will be the thing that is, if you draw the if your reference is there your voltage favor was here already  $V$ , but my started this thing tells, so, this angle was  $\alpha$  and the current phasor if it is, if  $\theta$  is positive  $\theta$  if I assume  $\theta$  greater than zero  $\theta$  positive then I am talking about an inductive circuit is the impedance of an oral circuit  $\theta$  will become positive.

So, you are either that will be lagging the supply will it is this is the thing I bought and this is phasor angle  $\theta$  you must understand this So, this is the voltage visit whose angle is this and this is it is not. Therefore, you will see in this case if some if you calculate this that is we I start. So, what is I bought I bought with respect to this reference should be magnitude of why into this angle is  $\alpha - \theta$ , this is how your,  $I$  bar should be coming in your calculation.

Now, suppose you say that I will calculate  $v$  bar into  $I$  bar, I will just multiply this two if you do that you will get the  $\alpha$  into our and again  $\alpha - \theta$ , this is what you will get, which will be giving us something like be  $i$ , this angle will be if I make a mistake point out. So, hopefully this is like this, it will be. So, you see the real part will be  $v_i \cos$  to  $\alpha - \theta$  and it is a reactive part 2 will be the sign to  $\alpha - \theta$ . But the power real power I know it is it should be equal to  $V$ ,  $\cos$   $\theta$  only should come angle between voltage and current.

Therefore, this will be a this is a real part of  $V_i$  not give you neither  $p$  nor this  $Q$  is often this is a this is not equal to  $p$  not equal to view something else you are getting that is of no value to us. But, you see, if this a what I do, then I calculate as the complex power as the bar into  $I$  burst and I say that this is  $V$  angle  $\alpha$ , this favorite is  $V$  angle  $\alpha$  this is my reference line with respect to that, that is the supply voltage is  $\sin \omega t + \alpha$  supply voltage I am describing it as  $\sin \omega t + \alpha$ .

Therefore, this will be the thing and then I burn is these So, I burst will be this one I and V alpha - theta. So, when you take the complex conjugate of a complex term by only angle gates in polar form only young angle gates reverse, so, it will be alpha and this will be I am guilty V -, this will be the thing and if you multiply these 2 it becomes  $v_i \angle \theta$  and I am now very happy because it's real part Israeli p be a  $\cos \theta + j \sin \theta$ .

So, never multiply been to I and say that that is your dress no Study up to date that is the convention followed by everybody. Therefore, Henceforth, what I will do in a circuit voltage phasor will be known in general it could have any angle be alpha, which I was doing for 0 degree, but it does not matter, JD is there, so, current is aged gone calculating like this, but while calculating power complex power in the circuit do this thing voltage into complex conjugate of the current fed that then is real part will give you real power imaginary part will give you imaginary power.

So, coming back to this circuit here, this is exactly what I did, the I studied it the difference whether is  $\alpha = 0$ , then of course one can do alpha equal to you do same thing you will get, but better do not do that, because there may be situations when it is having general angles of language. So, this is how P and Q can be calculated. So, what I wrote power delivered by the source is this much this much real power it is delivering and this mighty reactive power it is telling very similarly, in language I will write, complex power absorbed by the source by D load.

Here load terminals are these 2 and it will be equal to voltage across the load same as supply voltage. So 200 0 degree into I stare same calculation in this case there is no difference 53 dy dt and you will get the same result that is 24 or + j 3200 volt ampere reactive this will be the thing that is in a sec it is sinusoidal excitations in steady state, the, the power balance should be true both for real power as well as for reactive power.

Whatever real power is supplied by the source supplied It was 240 what is the real power absorbed by the Lord to forgive? What is the reactive power supplied by the source three to got what is the reactive power absorbed by the Load 3200 volt ampere now, it is such a simple series

circuit. So, what I will do now, I will as I told you I will I have solved a problem and that I will work out here today.

(Refer Slide Time: 21:33)

Numerical Problem :

$\bar{V}_s = 220\angle 0^\circ$  V  
 $\bar{Z}_1 = 3 + j4 \Omega$   
 $\bar{Z}_2 = 6 - j2 \Omega$   
 $\bar{Z}_3 = 8 + j6 \Omega$   
 $\bar{Z}_4 = 14 - j2 \Omega$

$\bar{I}_1 = \frac{220\angle 0^\circ}{\bar{Z}_1 + \frac{\bar{Z}_2 \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3}} = \frac{220\angle 0^\circ}{\bar{Z}_{eq}}$   
 $\bar{Z}_{eq} = (3 + j4) + \frac{(6 - j2)(8 + j6)}{14 - j2}$   
 $= 10 + j3 \Omega$   
 $\bar{I}_1 = \frac{220\angle 0^\circ}{10 + j3} = 21.07\angle -16.69^\circ \text{ A}$

$\bar{I}_2 = \bar{I}_1 \times \frac{\bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3} = 21.07\angle -16.69^\circ \times \frac{8 + j6}{14 - j2}$   
 $= 13.116 + j7.06 = 14.89\angle 28.31^\circ = \bar{I}_2$

$\bar{I}_3 = \bar{I}_1 - \bar{I}_2 = 21.07\angle -16.69^\circ - 14.89\angle 28.31^\circ = 14.89\angle -61.65^\circ = \bar{I}_3$

Complex Power supplied by source =  $\bar{V}_s \times \bar{I}_1^* = 220\angle 0^\circ \times 21.07\angle 16.69^\circ$   
 $= 4393.76 \text{ W} + j1331.25 \text{ VAR}$

So, a typical problem a problem numerical problem this Saturday will be like this here medicine for example, here is the one impedance which is called say  $01 = 3 + j$  photo and here is another impedances RC whose value is  $6 - j$  own capacity reactance  $-j2$  And here is another brand which is R and L lead this impedance because get to work and let this impedance get 3 bar Which is once again RL inductive  $8 + 6$ .

And so, there is series parallel combinations and here is the source connected whose representation of voltage that is  $v_s$  bar is equal to 2 to NT angle here zero degree so much volts these are all in terms. So, let these impedance be get to God and get to well obviously, I will first calculate this current. So, this example. So, so, this current is suppose I want bar I want bar will be equal to supply voltage that is to 20 Hopefully have taken to 20 angles zero degree divided by eq and impedance between these 2 points.

See, since we call to deadeye phasors maintain this relationship. So, all the things I did in DC circuit will be applicable that is it will be good one bar and the equivalent importance of this to that is + get to work get three by get 2 + three these will be the current in the circuit which if you

put the So, I first showed this is equal to actually to 20 I should have written this first  $j \omega L$  for  $Z$  equivalent, this is the thing No good equivalent if you calculate.

Use your scientific calculators where you can very comfortably calculate the equivalent impedance. So, it will be put the numbers  $2 + 4 + 6 - j$  in to  $8 + j 6$  divided by some of these 2 which will be equal to  $14 - j 2$  and if you calculate it out, which have calculated hopefully have not made a mistake, it will come out to be  $10 + j 3$  so much oh so, JD  $j$  well and being known,

I will be able to calculate it one by one by is how much One bar will be equal to 20 euro degree divided by this  $10 + j 3$  and if you calculate this, it will become 21.07 and angle will be  $- 16.69$  degree. So, so much MPN so this is one I have calculated. Now, what I will do is this I will these terminals I will name them so that now to calculate this currents current in this branch and in this branch that is, this is supposed to be directions are very important.

Yes like this is our key to alert directions. So, the current drawn by gate 2 will be you can apply the division of currents in parallel circuit that rule that is it should be equal to one bar into other impedance get three bar divided by get to  $+ j 3$  and put this number one is known so to 21.07 and angle  $- 16.69$  And this into in the numerator it is get three get 3 is  $8 + j 6$  divided by some of these 2 is  $14 - j 2$ . And if you calculate this has been my calculations it was coming as see generally calculator will give you a rectangular form,

but you can easily convert it to polar form which gives you 14.89 and angle is 28.31 degree close to this he will come So, so this is equal to  $i$  to barge So, my campaign so everyone is known equal to everyone but it then was It bar can be calculated, but I have calculated it in this way actually bar because KCL as to satisfied will be equal to one bar  $- i$  to buyer, I have calculated it in various ways.

So, that you understand in with phasors KCL is also satisfied that the nodes they are born it will be iron bar  $- i 2$  iron bar is 21.07 This one is it one bar  $-$  and 16.69 degree  $-$  it is 14.89 and angle is 28.31 degree and if you calculate it out, it will be equal to it but will be equal to finally 14



point in polar form  $8.9$  and  $-$  angles  $61.65$  you also calculate most early I have not made a mistake, but who knows.

So, this is equal to it. So, all the currents are now known as one bar right to right through bar across each of the elements that exist some voltage for example across the source polarity must be also told  $+ -$ . This impedance genuine bird lies that cross a B and get three and get to their in parallel they exist across BNC that is fine. Now, what I want to tell you is that I you could expect that so see supplying it Complex power to the circuit real and reactive power.

So, I will calculate that first that is I will calculate what is the power complex power it has a complex power supplied by the source by the source it will be equal to current is coming out from the positive. So, it I should take  $v$  s bar into it one star that is what I have to do to calculate the power supplied by the source This is known as is  $200 \angle 0$  degree into it one star here usually be careful this is a one its angle is  $- 16.69$ .

So it should be  $21.07$  then angle you flip it is  $-$  so it has to be  $+$  complex conjugate is not that is what you have to do. And if you calculate it it is a real part will be something like food. I will use a different color for example, it is a real part will be  $439.376$  real part so much what  $+$  reactive part will be  $j$  I hardly calculated but you please crushing this  $1331$  about  $.25$  so much volt ampere this is the power supplied by the source this point is important and you calculate it .

So, this is one result then what I will do is this I will calculate what is the power absorbed by zero one what is the power absorbed by J 2 and what is the power absorbed by get each one of them since each one of them is having some energy storing elements with the I you could expect that each one of them will have both real and reactive power. So, some of all the real power should match with this or some of all the complex power will eventually will come out to be this, this will see will continue with this calculation. Thank you.