Network Analysis
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Lecture # 33
Mesh and Nodal Analysis in A.C Circuit
Introduction to Impulse Function

So, welcome to lecture number 33. And in our last class I told you about how to handle AC circuits just like this is our ticket analysis and how mesh analysis can be carried out.

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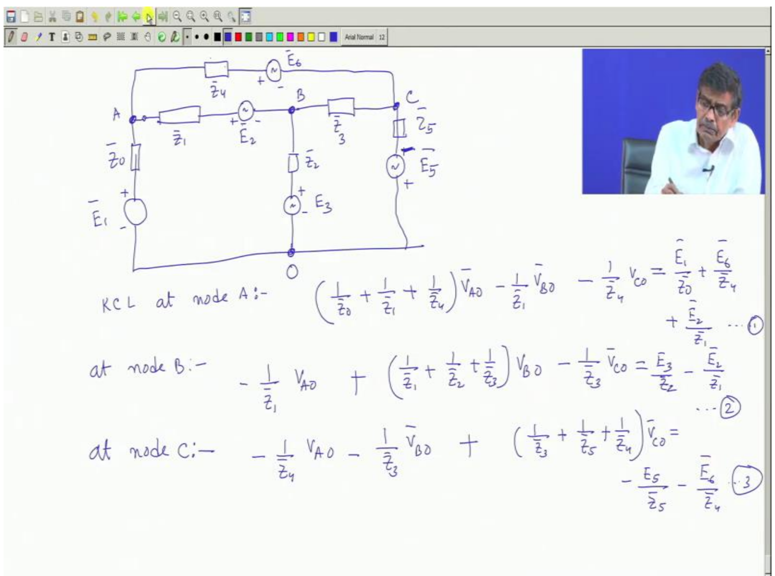

Similarly, nodal analysis let me just write down the equations no question of that deriving it. Suppose, you have a circuit like this am now drawing impedances as boxes suppose 0 1 bar is 1 voltage source. There is another impedance here and there is another volt source is here and AC there is Z3, Z2 to this is a Z3 they could be RL or pure a combination of art and capacitance and so on.

And there is another source R say - and + and carry a source and here is another resistance here. Z is 0 and this is arcade and there may be another branch here lay another impedance Z4 and another source last - like this lead these sources be called E2 to E3 and let me put another impedance here yet why and this is all the circuit is given this is a 6 so, all the impedances I will be knowing they have source voltage is also known.

I want to calculate the currents. Now, similarly here one can apply the mesh analysis number of misses I1 and I3 that I have described. Now, I will tell you how do I play right down the nodal equations, not voltages, how to find out. So, first of all you mark the notes on to 1, 2, 3 and 4 there are 4 nodes one of them, you call it all reference node and other 3 are a, b and c. Our target will be to know the value of VA0, VB0 and VC0.

So, writing down the equations so, so KCL at node A will be the coefficient will be some of the reciprocals of all the impedances connected to 1/Z0 + 1/Z1 + 1/Z4 this will be VE0 how will be phasors here. Similarly, coefficient of VB0 will be common impedance between A and B that is Z1 - 1/Z1 into VB0 and coefficient of VC will be the impedance in this branch so, that is Z4 So, - 1/Z into VC0 and this will be equal to the dimensionally the right hand side should be currents and short should be involved.

So, E1 is a source connected to A. So, it will be E1 / Z0. So, it will be E1 / Z0 + sign, because polarity set, he drives current towards node A. Similarly, E6 / Z4 for that is also + E6 / Z4 and E2 also is connected to node A, so, polarities it should be E2 / Z1 this will be equation at node B KCL. Similarly, will be coefficient of VB0 will be some of reciprocal of all the impedances connected to be that is Z1, Z2 and Z3 into VB0 and coefficient of the VA0 will be common impedance connected between A and B preceded by a negative sin + and coefficient of VC0 will be - 1/Z3 into VC0.

And this will be equal to all the sources. So, E3 connected to be a E3 / Z2 will be current in this direction. So, that will be positive all our complex number is E3 / Z2 + 2B + sign then this is - this is + then if I is not connected, so, only the source and the source E2 is connected, but you to use the - is connected to be, so it will be - E2 / Z2 that is all to be only these 2 sources are involved. So, this is our second equation looks like and the third equation is at node C at it will be coefficient of C will be 1 / Z3 + 1 / Z5 + 1 / Z4 into VC0 .

And coefficient of VA0 is there is a connection between so - 1 / Z4 into VA0 you get 4 is the impedance yet. Similarly, coefficient of VB0 you will be simply - one over get 1 / Z3 into VB0 this time and this should be equal to currency as source. So if I buy Z5, but negative is connected to - 3, so it will be equal to -, if E5 / Z5. There is a source also connected here is E6 to see. So - this is also - E6 / Z4 that is all. So these 3 equations, just like your DC circuit analysis, except that all the numbers will be complex in nature. Then you calculate by any method you would like use your calculators, but coefficients will be complex.

But the fear that we had got if it is RL circuit, there is complex arcade like this several sources are present. So, you have to write down the differential equation to solve for currents that is not necessary if you are only interested in this steady state currents go to the complex domain albeit these are algebraic equation no differential equations calculated will be able to calculate all the powers at various branches and so on. So, all the things that we have applied in this is arcade can be applied, they are also.

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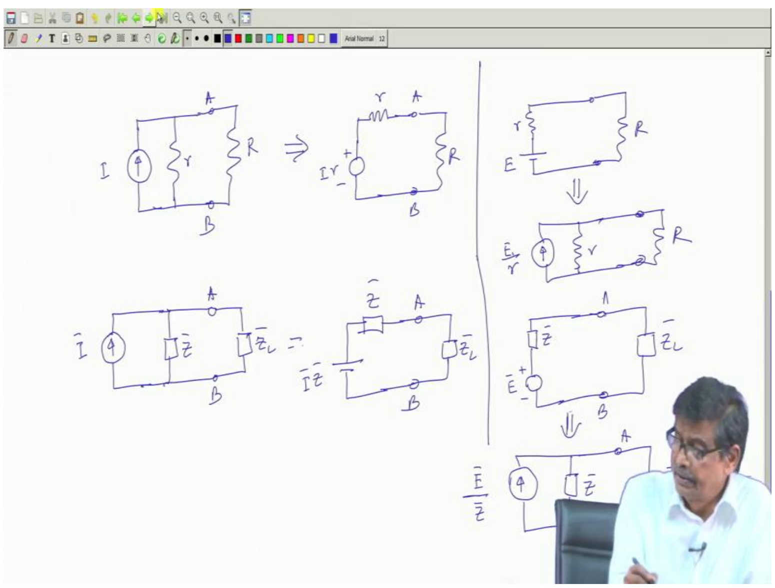

Similarly, there may be a current source time varying current source in case of this is our key to what do I see if there is a current source having an internal resistance are only then if there is a load resistance connected here, then this circuit is equal to this circuit, you can always go to and here is A and B. We have seen that in this circuit can he had here is R or if you have a battery, in series with some resistance are if a load is connected, if you have a battery and here is a loaded resistance, this is nothing but equal to can be thought of as a current source.

And in parallel we thought and these are the load terminals were RT there this we are doing in case of this is a key and all things we did earlier here I am telling if you have a current source which is sinusoidal in nature, so, it can be represented as it says and if there is an impedance here and here you connect some load impedance then this will be simply equal to an impedance in series that is yet and here you can take is a voltage source that is I bar into the open circuit voltage and then you have the load impedance A B these also and conversely.

If you have a current source in parallel with an impedance can be considered to be a voltage source in series with this impedance and then your load across a B. Similarly, if you have a source is the AC source will have as its internal resistance as an ideal is the source E bar and AB is your low terminal and this is connect a load impedance ZL. Then this is can be thought of as a current source whose value will be E bar by Z bar in parallel with an impedance, Z bar and then you are A and B and this is the load impedance.

So, if it is a circuit and I am only interested in steady state, currents powers etc. Then phasor notations can be used and can be used with these living complexity burden increases, a beat, increase a beat but no nothing like solving differential equations will come in the way it is all linear equations.

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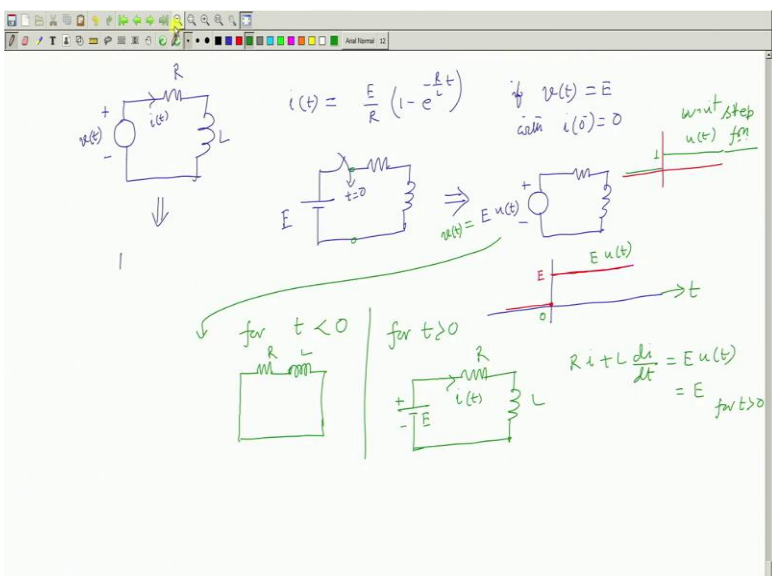

So, please remember these things this is how you will be great and you have solved several circuits with complex impedances. Now, I will today in this lecture will now, summarizing all the lectures that we have done so far you are now in a position if it is not a sinusoidal thing differential equation is to be hidden you cannot avoid that and voltage source can be anything and I told you how to solve such subject earlier say Vt any volt source RLS circuit.

So, coming back to general circuit supply voltage could be anything need not be sinusoidal, and we found out the currents of course to find out this currency have to write down differential equations or do some things if it happens to be sinusoidal or DC then there is some relief in getting the forced response quickly those we have discussed for example, the current in the circuit we have calculated so, many times is E / R into 1 – R / L into t. If vt is equal to a battery constant voltage, we have seen this with of course, initial with the assumption that I 0 - was equal to 0.

Now, today what I will tell you that this circuit the dissolution of this circuit could be thought of like this is switch this is how we are drawing the circuit and I told you I close this circuit. But what I am now telling you is that this circuit, this voltage can be described using some unit step function and you can do this is same as telling that I have applied a voltage what is called a unit step voltage into E into ut and draw the circuit like this.

We have defined what is the industry function is drawn like this is 0 or p less than 0 am closing the switch and then for p greater than 0 it is not the instance it is has got an amplitude but the magnitude is constant it will be like this. This is ut and this is obviously this function is E into ut where E is the constant number, this is t = 0. So, if I say there is a circuit where this voltage vt is E into ut I mean that this voltage has been applied to the circuit for t less than 0 the applied voltage was 0.

So, this circuit will look like for t less than 0 the circuits will look like RL these you please follow me very carefully for t less than 0 applied voltage is 0 because E into ut. So, this I will show like a short circuit there was no voltage existing one may think that it is open circuit Why did not do show but my input voltage is 0 to indicate that I say although I have not discussed you must understand that in an RL circuit there the inductance is present there was some flux somewhere may link it and produce some current in this circuit but my applied voltage is 0 that is the more important point. If you show it as opens circuit then that voltage will come here.

So, input voltage I am I will define between these 0 points which is 0 means shorted and for t greater than 0, these are the 2 will look like RL and here as a constant voltage + - see this is the situation. So, this is how if some function is written so for t less than 0 voltage applied the 0 from - infinity to 0 it was 0 like that, even inductor at some current that must have dissipated in our energy storage initially long back.

So, initial current will be forgiven and the you can solve the current, but the expression of the current for t greater than 0 you will get the same equation Ri + L di / dt = E ut right am interested to get the solution for t greater than 0. Therefore, E into UT is nothing but itself for t greater than zero got the point therefore this is how one of the important function is unit step function. It emulates a situation where you want to connect a battery across RL.

This function is one of the very popular functions at t = 0 I am not sure what is the magnitude of this unit step function. So, this is unit step function. It is not defined at t equals t - 0 it was t - 0, t 0 + and it will be for any time to come that is the thing of a unit step function. Now, in my previous lectures earlier quite earlier to the discussion we are having about this is not it.

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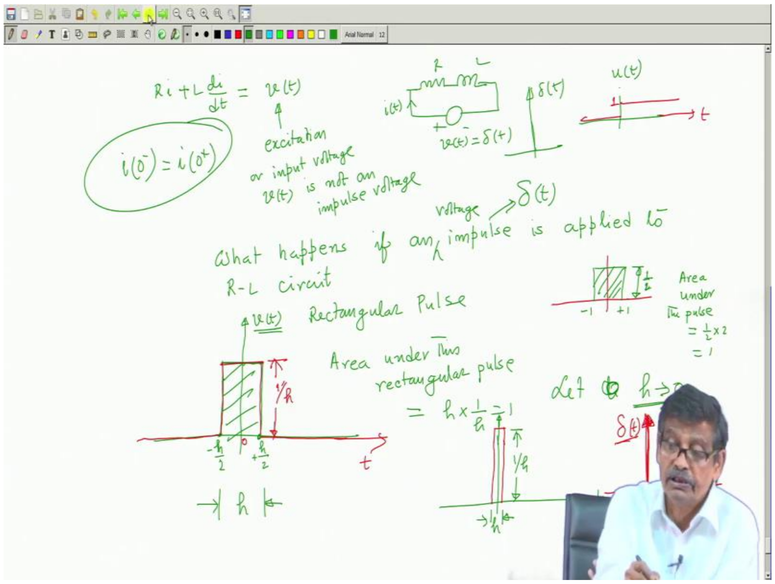

I told you why writing down the differential equation right hand side is the voltage that is ri always bring one circuit in your mind to understand the thing this was my vt suppose RL circuit. This is your vt it could be any function of time now, need not be sinusoidal that is why I am in time domain I cannot go to for that domain if this is the thing I have to find out this current is RL. So, I will write down the differential equation and solve it and I will demand some initial currents must be provided to me I 0 - etc.

And switching I will do at t = 0. This is how we were we have been telling to you and then at that time I told this right hand side I put a restriction so far and told that this is the excitation or input voltage excitation or input voltage and so far whatever I have done solve these differential equations with the assumption that, vt excluding impulse for vt is not an impulse voltage and we established I0 - = I0 + you recall for an inductive circuit what should have been the current in the inductor at t = 0 - that equal to 0 you did something.

So, I 0 + will be equal to it remind us that is what we have been telling so far. Now, I will not exclude this impulse function so what happens if an impulse if an voltage impulse is applied to R-L circuit, the simples voltage usually denoted by delta t as we have denoted unit state voltage is ut. This is the impossible mathematically denoted by this, then the question comes what is an impulse voltage I must know like this unit state voltage was defined like this steps up to a high voltage fee these your time axis now what is an impossible that is very interesting.

You see, suppose you have a voltage waveform like this a rectangular voltage like this suppose and this interval are you getting these voltages from it means that for V less than this time it was for v greater than this time it was 0, but for V between this point and this point there was some step voltage sort of thing rectangular pulse voltage. So, this is the rectangular pulse voltage vt suppose like this then the question is this one suppose I say this is h / 2 or this is h / 2 and this is + h / 2.

So, what is the width of the rectangle h this is a rectangular panels rectangular pulse as your vt so I skated this way is the rectangular pulse, a voltage of this part, which is otherwise 0 for t greater than = 0 for t less than - h / 2. It is just here this t = 0, this is time axis. So, it is the rectangular pulse voltage and I have taken such a rectangular pulse voltage whose height is h 1 / 8, let us take a special rectangular pulse whose height is 1 / 8 and this is h got the point. Suppose, this h = 2 if height is equal to 2 it take the height to be hard. So, so it is like this.

For example, if this is + 1 is - 1, this week up, this is 2 and I will take the height 1 / 0 if this is 1.5 - 1.53 I will take the height to be one third what is the area of this rectangle hopping to do area of this pulse area under the pulse is over to 1 / 2 into 2 is equal to 1. So, as a general case if you take this is h / 2 - h / 2 which will be height and height of this pulse I will try to maintain 1 / h then what will be the area under this rectangular pulse so, it will be called to rectangle so, we make into height.

So, h is the height and 1 / h and that will be 1. Now, you choose any value of h, this will be always true that is also fine no problem. But now I am telling you one very interesting thing what I will be doing is that led to this rectangular pulse I will try to see I will try to make h tends to 0 then what will be the fate of this rectangle rectangular pulse, it will be like this. These rectangular pulse will be basically becoming thinner and height will becoming larger and larger. This is what is going to happen to this is time.

So, edge is reduced 1 / h is this one height of the rectangle will blow up and base will try to collapse. This is what it will be is not as and as you say that h tends to 0 but really not equal to 0.

That it means, I will not make a height is equal to 0, but as small as I can. Here, there is here, there is like 0 - and 0 +. Then the height of this rectangle we go up like anything it will become a thin straight and the height h, we tend to infinity then if h tends to 0 the height of the rectangle will try to go to infinite large it is it will be as h is becoming smaller in smaller and smaller to whatever extent small tend to do our - 68 but the height will then go up.

But the specialty of this rectangle is this, if this is he will make it 1 / h the area under this which you know becomes almost like a line will always remain one. The height going to infinite base is collapsing to 0, but the area under the curve remains same. Now certain function is called and impulse function as against time, nothing is there for t less than 0. Nothing is there for t greater than 0 but at t = 0 that is between 0 – and 0 + there appears a value of the function which is infinitely large. Therefore, what should I say about this delta t.

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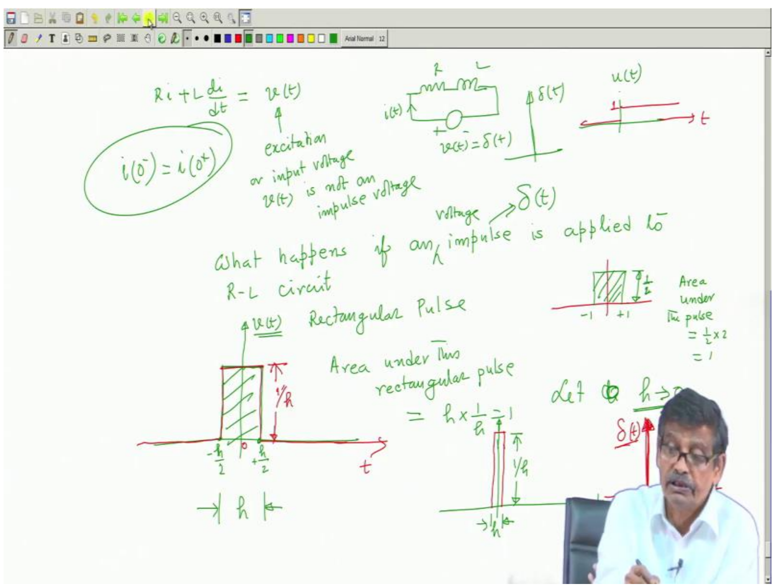

So, coming to this next page I am tend this part you listen carefully. So, here is your time axis I am telling that at t = 0 there is an impulse and this impulse is shown by an arrow because in finite you cannot show the magnitude it is delta t so, I will try to describe the function as delta t = 0. If p not equal to 0 p other than 0 value, the function functional value of delta t is very well defined. It is delta t is 0, t greater than 0 it is also 0. But that t = 0, the value of the function is infinitely large, what to light going to infinity going to infinity. I cannot really define what will be the height of the rectangle as h tends to 0 as I told you, but about this thing I am starting delta t = 0 get to elsewhere. t less than 0.

But I can say one thing about delta t another thing I can say that is delta t dt between t is 0 – 2 0 + the physical to one, because area under the curve is one. So, I am collapsing a height is increasing one way, but no matter for any chosen value of age the area under the curve is 1. Therefore, this impulse however high it becomes, so long as you are not putting equal to 0 but, collapsing height as close as 0 - 2 0 + the area under the car has to be 1.

So, above d delta t function I can say its amplitude at t = 0 do I cannot define cannot be defined infinitely large value of delta t for other than 0 defined this 0 always 0 and then I say value of delta t is not defined it is infinitely large at t = 0, but this integral is true. The idea under this theme rectangle as thin as you can think of, is always 1. So, such a function is called an impulse function. For example, our see any reasonable function of time will be like this t it is defined at t equal to this match what is the value at t = 0.

This is the value of the function any function x t at t = - it is dispatch, that is how we used to deal with function. But here is a function which looks like very simple because it is functional value at any value of t here, all are 0 flat all the at 1 point it makes its presence felt. That is that at t = 0 such that delta t tends to infinity. And this integral is true around this origin you know - 2 0 + area is finite and such a function is called an impulse function.

Can we have an impulse function in the laboratory? Really you cannot generate an impulse function ideal impulse function in the laboratory. No it is not possible then question comes, why then studied this impulse function, we will see very interesting features about the impulse function and why it is necessary to find out the impulse response of a any of any given circuit. People still on pen and paper I can of course find out, for example, in this circuit as I told you, if you have a circuit, and the Vt = delta t.

What will be the nature of the current can I find it in the level you cannot do it because impulse function is ideal impulse function you cannot find but on pen and paper, no one prevents you study and whatever the impulse function if such a function which is like this is applied to the circuit, what will be the nature of the current will the current will become impulse or what tells those things we discuss very interesting topic cities please this is somewhat new were, because earlier whatever I told you was really developing the common ground basic grounds based on which we can now talk with advanced topics like this.

So, impulse function and impulse voltage if applied to a given circuit, what will be the response? How to find out the currents this is the topic of discussion. So, for that I must know what an impulse function is and about this you should not have any doubt what it is looking at is impulse function is one of the simplest function, it does not exist anywhere here 40 less than 0 here 40 greater than 0.

There is some impulse function is present at a single point it looks like close to a single point 0 - to 0 + we taught property, what will be the functional value? No, I cannot say it is infinitely large whatever the functional value of this function 0 everywhere t greater than 0 + 0 t less than 0 - 0. But about these things, this is the useful information the area under the impulse function will be equal to unit impulse function will be equal to 1 what we thought the studies in my next lecture. Thank You.