Network Analysis

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Lecture # 33

Odd and Even Functions, Relation Between Unit Step and Impulse Function

So, welcome to lecture number 34. And in our last class I started not a new topic, but the problem was that if you have to solve a differential equation and if the excitation function is not a reasonable function like sine omega t squared any functions are reasonable which we can plot for example.

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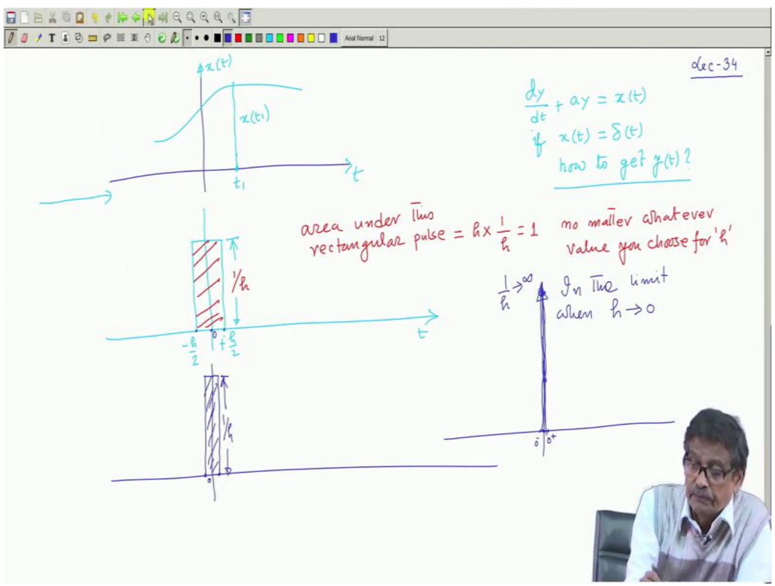

x t ending function like this is a reasonable function which can be described by some equations where this is the time x says and this is the value of the function, then at a particular time as you were suppose you were moving from t = - infinity along t axis and looking at values. So, at a particular time say T 1 it will have a definite value of 61 this is these are reasonable functions for each value of time you have some functions, but, if you are your differential equation is of this type for example, dy dt + ay faster differential equation.

This is x t in general you are writing but if x t is an impulse function how to get y t How to get y t this is the thing that is the question asked, because delta t is a very peculiar function and to tell you about that revealing, I told you that if you have a and rectangle very quickly I will review whatever we learned last time. Suppose we have a rectangle rectangular panels and this is - a 2 by 2 and this point is + a by 2.

These are the time and the height of the rectangle is 8 then I told you that the area under this card under this pals and this rectangular panels will be always equal to 8 into a 1 by 8 is it the height is I am, the height is suppose you make it 1 by 8. So it is a rectangle of with edge by 2 and this is also edge by 2 8 and it is 1 by 8 area will be 18 to 1 by 8 Which will be equal to always 1 no matter what is the value of 8 no matter whatever value you choose for 8.

Now, the next thing I told that all this edge I will make smaller and smaller the value of the K will make smaller and smaller such that these 2 points will come closer to the origin t = 0 this is already in. So, it will come if you make a smaller and smaller the height of the rectangle which is 1 by a few Go up and these base points will come closer to the time t = 0 Then I say but the idea will remain same 1 unity because I am still keeping this to be 1 by now in the limit when a chance to below this rectangular piles will be a very thin strip and this height of this rectangle will race to infinity.

At the same time these 2 points will come very closer to the origin. So, under that condition what I will say that equal to 0 Then, this point is 0 - and this point is 0 + close to origin close to 2 = 0 but not really = 0, But, as close as you can think of imagination does not prevent us to think as smaller number as we please. And under that condition this height of this rectangle will race to infinity. 1 by will rest too. But still that rectangular strip will be there as a pan the area under the car will be 1. So, so sucky function is called evening impulse function.

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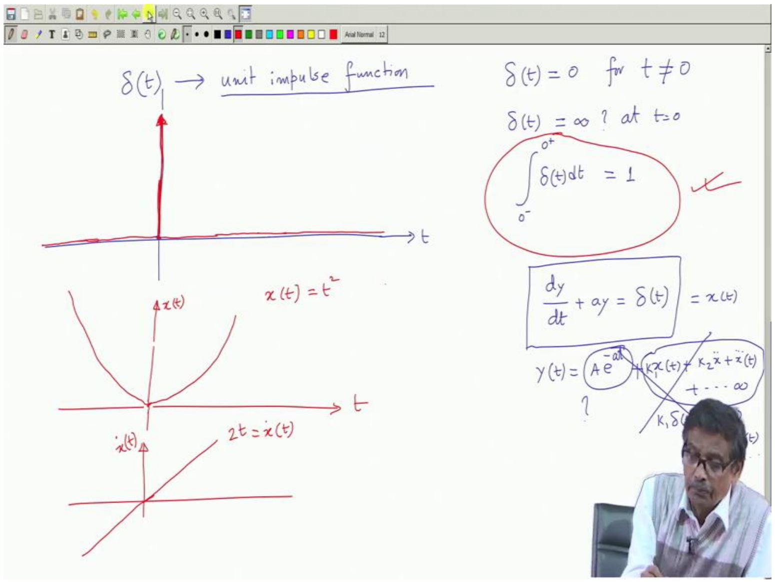

So, these I discussed last time and denoted by this symbol, which is unit impulse function and it will be denoted like this the value of the function delta t will be = 0. For t not equal to any other values of t the value of the function will be 0. That is, suppose the red 1 I am using for skating delta t, it will be 0 for t less than 0 to - infinity and t greater than 0 to infinity this will be all 0 this axis is time axis but at t = 0 something happens the value of the function we show it by a line and an arrow because we cannot show infinity in any scale.

So, this arrow indicates that it is infinitely large the amplitude of the function is infinitely large. Therefore, the second thing is no point in writing delta t, I mean E is somebody says 1 can write delta t = infinity at the t = 0. But what is the point of dealing with such a function because infinity is not any finite number. But what is true is this the integration of delta t area under the car delta t dt integrated from 0 - 2 0 + this is 0 - this is 0 + close to 0 on either side this integration must be born.

That is what I tried to tell this is finite but no point in asking what is the value of the function and that is in finite what I will do with that number large number undefined. So, this is the basis of an unit impulse function is magnitude at t = 0, well, very large, but I cannot quantify it infinitely large and delta t = 0 for any value of time here it is euro here it is red 1 is the delta t curve and at t = 0 function is infinitely large, but this is true this is very important this is true. Now, if certain impulse function is applied to a circuit as input excitation how to find out the currents or how to essentially that circuit in general will have energy storing elements.

Therefore, how to solve a differential equation that is why I told that this differential equations I was dy dt + a of y = delta t. If such a situation occurs then the solution due tonight deviance function will remain same that is why t each solution we know it will be able to do the power - at that is there, because characteristic root is - 1, but what about the solution due to forcing function I was earlier telling that if the right hand side is x t,

Then I told it will be a linear combination of x t and it is higher order derivatives is not up to infinity that is what I was telling. But in this case it will not be like that. So, this part is applicable only to reasonable function whose derivative exist at different values of time because linear combination of the function and design A database that is in this case I should not write the solution will be delta t + k 2 Delta t Delta dot + delta double dot no it will not be like that.

Because it cannot be because of the simple fact because delta dot t What does it mean if time permits I will explain, but there is no Delta dot t on the IP such thing exists it will be difficult it cannot be like that. So, we have to find out the solution with open mind and I will be doing it in this fashion therefore, this is not correct here. Solution we do not know so, why this this part of course, (FL: 12:58) will be there a chat now to approach this problem before that, I will tell you 1 very interesting thing any reasonable function.

For example, x t function we know, what do I understand by x.t if I ask you to skate x.t I will be able to skate I will calculate the slope at various points and then I will be able to skate x.t try to follow me. For example, if it is like a signal suppose the signal is like this Say the function x t = t square then this is x t we know from our school level knowledge that to skate x.t you have to differentiate these will become to t and it will be like this.

So, this was from - infinity to + infinity time axis, this is x = t square parabolic and if you differentiate it will become 2t like this also negative side slope will be negative. So, this is 2t which is equal to extort but the point so, any function given can find out extra. Next thing I will pose another problem.

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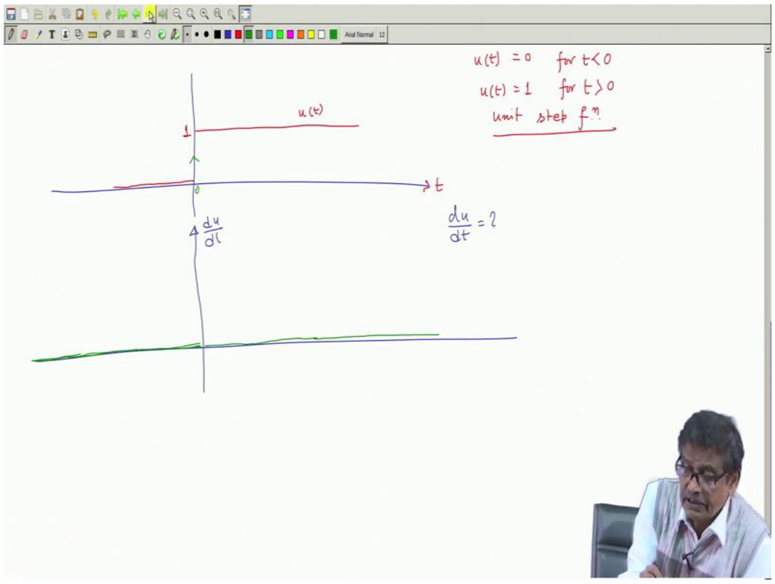

Suppose you have a unit step function. For example, I told you about unit step function last time, very simple, which is equal to u t = 0 for t less than 0 and u t = 1 for the greater than 0, this is your time axis and the value is this is called unit step function in each step function now, if I ask you differentiate the unit step function will be your answer for all time t. So, what will be d u dt is what I want to escape it clear the you did. Obviously, this portion is very simple. This will be do all the time.

So it is differentiation of this constant number 0 is 0. So d u d t will be 0 like this, at t = 0, I am not sure and for t greater than 0, it is 1 second constant. So the du dt will be once again a constant like this. Got the point. So I am skating du dt with this green color. So this 0 and about this point will tell it has got a Deskin continuity because the jump from 0 to 1 means it is a discontinuity from this value 0 to 1 it has this jump has taken place in no time.

Therefore, slope is infinitely large E liked that and will say we cannot say it is very large slow but after learning the impulse function, I will say that there will exist an impulse at this point why that I will tell now, before I proceed I will tell very slowly so that you do not miss any point.

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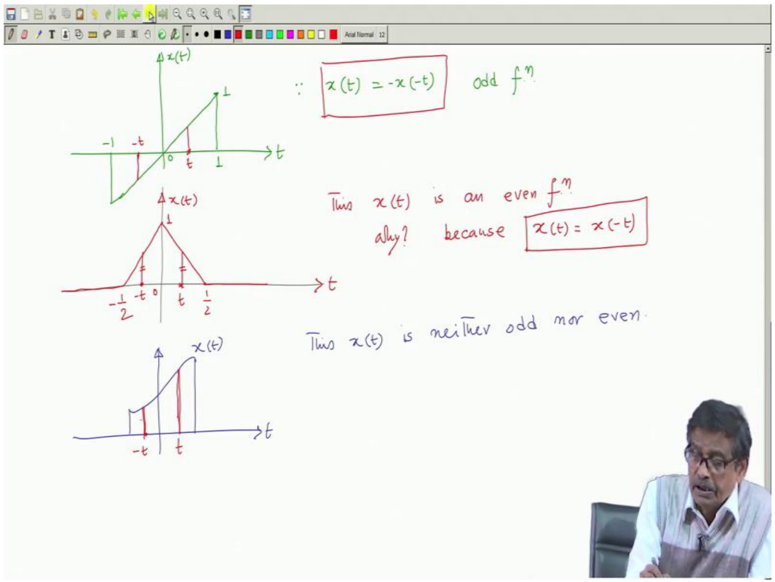

So, now, I will tell you 1 important thing you must be knowing that any function drawn for example, I draw 1 function like this a voltage pulse like this x t like this and this is time axis this is 0 and this is oppose 1, this is 1 and this is - 1 and elsewhere the function is true. Now, this function xt here, since x t since, x t = x - t with a negative thing, it is called an odd form was renewing, I will be telling very odd functionalities the value at any time t here x t and at T and at - t it will have same value, but we thought was the side.

So, when such a condition is applicable It is called an odd function you can have several odd functions at any positive value of time whatever is the ordinary corresponding negative below ordinary will be same. Similarly, there may be and even function. So, odd functions will be there you can sketch many of them. Now, similarly, a function which is having suppose I skate a function like this is suppose 1 this is half, this is - half. Suppose it is a function like this, this function whatever it is this function that function I am calling x t this x t is an event function.

Why because the value of the function at any time t is saying as the value of the function at the corresponding negative time These 2 are same therefore, because x t = x - t this is the property and event function and it is a it has to be true for all the time t that is true. Similarly odd function it is to be true for all the time t. So, this is a simple example of even an odd function. Now, in general functions may not be either odd or even, for example, a function I sketched it like this function x t odd x t this function this x t is neither odd nor even this function is neither or do not even.

Why because it does not satisfy this at t = t suppose any time you choose this functional value at t is this and at the corresponding value the value is not this is not equal to decide. So, it is neither odd and also it is not even, because negative of that should come but it is not coming and similarly, x t is not equal to x - T, their board dysfunction is neither even not and most of the functions will be like this there in general, however, any function which is neither odd nor even, can be broken up into 2 functions as some of 2 functions of which 1 will be even beyond what I mean to say is this 1,

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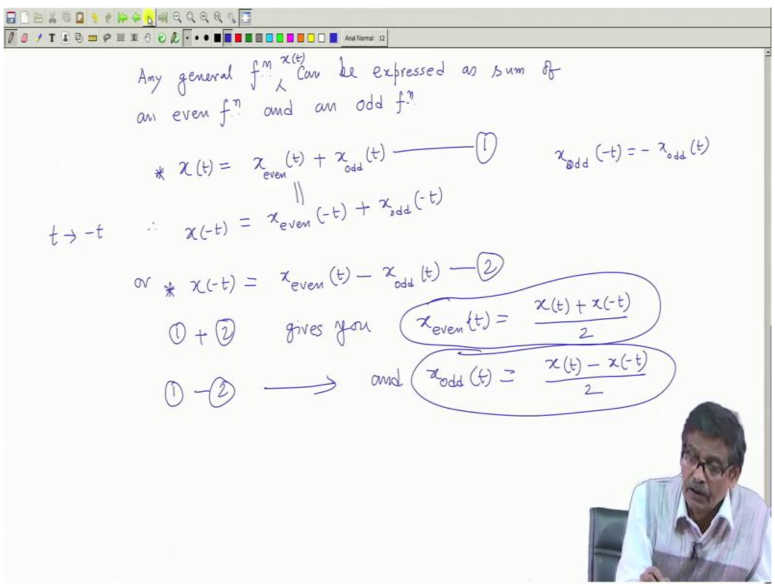

Any function this you write down any function, any general function can be expressed as some of and even function and odd function what does that mean Suppose x t any general function x t. Therefore, what it is telling me is this x t can be expressed as some of event e + some of an odd then the question is if I know x t can I find out the event turns and the hot turns yes I can because I will have applied the property.

For example, replace t by - t then x - t on both sides, I will write it x even - t replace t by - t on both sides x odd – t this I can do, but since this part has to be when these 2 must be equal or x of - t will be equal to x in 20 the second right since this event these 2 are equal and since this party is odd x odd - t will be = - of X or t it has to be that is by definition, so, this equation is x odd. So, we have got now these 2 equations quantum x t is known, so, x - t is also known. Therefore, add 1 and 2, 1 + 2 it will give you x event t this + this that is this equation, this equation if you add it will give you x t as x t + x - t divided by 2.

And, maybe many of you know it by x odd t will give you x d subtract 1 - 2 subtract and it will give you x odd review subscribe, it will give you x t + - of x of - t divided by 2 very useful results these 2 there will give in any function, I can find out the event an awkward video. Now, why I spent so much time on this. Because of this fact now, because what I was trying to tell I want to find out the difference this was my problem u t is given I want to find out du dt that was a problem. So, I come back here.

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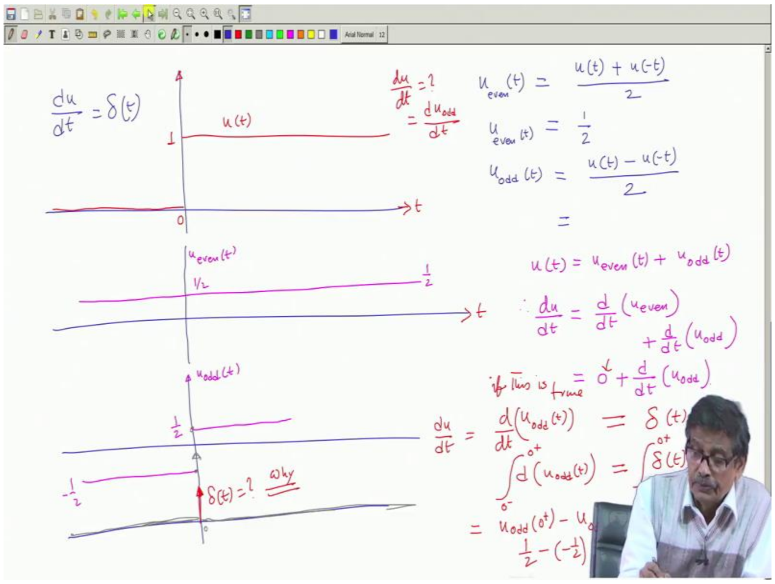

So, problem was once again I did draw better this is an initiative function this is magnitude 1 and this is your time this is the 0 and ut you know it is 0 everywhere 40 less than you do is fine. Now, this function which is ut is it even or odd neither even not because at any to you take it is + 1 negative dt 0 so it is neither even nor odd. Therefore, this initiative function can be broken up into edge 7 and not part and it is xt ut even is even part will be how much you t + u - t divided by 2 that is what we learn.

And all what is u - d u - d 0. So it will be not. So u even t is up and you ought t will be equal to u t - u - d divided by 2. So, u odd t will be how much sig number so, it will be equal to sig number for t greater than 0, it will be + up and 40 less than 0 it will be - up. So, in other words, you can also visually it is more attractive, that is the even t even portion is half for all time t, this is you even and you ought t will be equal to - half here and + up You see you add this to you get half of 1 this is also half and that is this part and this part it is + half it is there must be - half here.

So, this is the u odd t and it is odd is a class up same time it will remain there were some of these 2 gives you the unit step. So, I will write it that u t therefore, is equal to discard + discard that is u even e +, u odd therefore, du dt will be equal to d dt of this event part d dt of this u even + d dt of these, u odd it will be like this. Now, obviously this first term it is constant throughout so it will be 0 and derivative of d dt is nothing but derivative of this function u odd the dt of you got the point.

Now what will be the derivative of this function, derivative of this function will be for t less than 0 it is constant – half. So, it will be 0 40 greater than 0 all 3 2 will be 0. Not that is what at least about these 2 times I am sure so simple, Austin d dt of our P 0 and d dt appoint us up is also good this will be the problem is here is a discontinuity from - half jumping to plus half for this part contribution will be.

Now the question is what should I do with this 1 I am telling you first I will tell the result then establish that this derivative will be an impulse here unit impulse why what do you bind the derivative of this function I am telling that it will be an immediate impulse because change has taken place from - half to + half in no time let us see that. So, and what I know that for example, this function u odd t.

I want to find out this d dt of this which happens to be equal to du dt that we have established. Our origin or problem was there from this I got this or you find out the derivative of u odd t u get the fine I have reached up to this point d dt this is the thing I have got. Now I am telling that this will be equal to an immediate impulse although not proved it yet is it correct let us say that if this is true, then the you multiply both sides by dt. If this is true then the u multiply both sides by d u odd multiply both sides by dt Delta t. Then you integrate this function from 0 - 2 0 + both sides.

Because as you know, delta t in train at large what can I do, but about 1 thing with impulse function is sure 0 - 2 0 + delta t dt is 1. Now, let us see whether left hand side gives you also 1. So, this integration will simply tell you that you dot at t = 0 + this integration will be this 1 only then put the upper limit - you odd 0 - and this side I know and what will be this quantity this quantity will be and this side is 1 and this quantity is half for t greater than 0 it is + of and t less than 0 – 1/2.

And this is equal to indeed 1 equal to half got the point so what I am telling given a function, if there is this state jam of the function or discontinuity of the function at some point of time, I will say that look here the derivative of the function at that point where that state jump has taken place Must be there must arise and impulse clear. So, I repeat this case what I did this was the image function I first my problem is to find out du dt is how much So, first I broken up this function as the event part and nor part that is the even part that is the odd part.

So, differentiation of ut will be differentiation of even part + differentiation of odd part, even part is in any case is constant all along the timescale - infinity to + infinity that will give you 0 that is 0 then + d dt of u odd now u odd t is this - half and +. So, d dt of u odd t half to them find out to find out du dt which is nothing but du dt are d dt add this to our say so du dt is what I am telling maybe du dt you multiply with dt on both sides let us assume but I am not sure I put a question mark maybe it will be equal to an impulse delta t airport.

Then you multiply both sides by dt then it will be du dt = delta t dt that have not written then you integrate both sides from 0 - 2 0 + left hand side will give you 1 because if you integrate it will become u odd T and t u odd t at t = 0 + half - half. So, the + 1 and on the right hand side, this has to be equal to 1 and indeed it will be 1, if you integrate this function Delta t dt from 0 - 2 0 + there will be say that they if you differentiate an initiative function, you will get that. So, summary of this whole exercise is du dt is will give you that.

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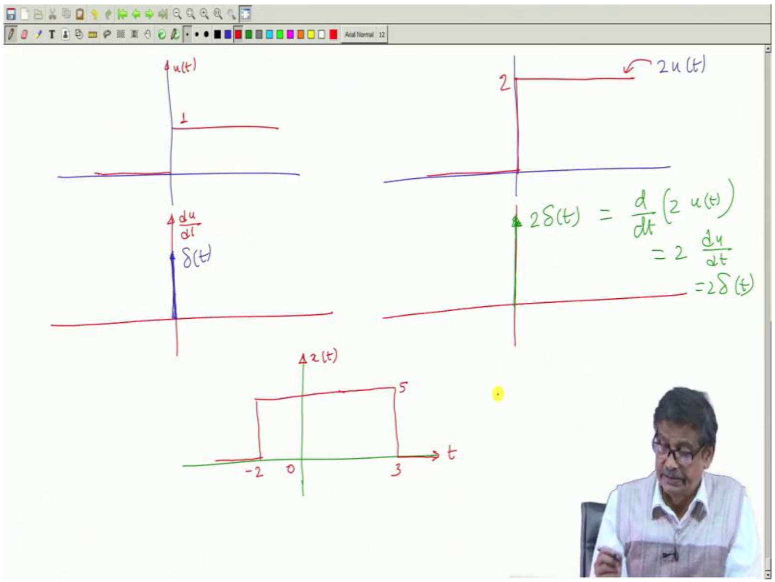

So, it was your unit step function here this is still functioning ut and below that is du dt will be impulse evening simples. So, earlier perhaps if I did not know delta t no idea of impulse function I would have simply told there is a discontinuity here, but here now after cultivating an impulse function a bit I come to know it is better to show it as an impulse what is the area and that the card that is the strength of the impulse that is what got the point. And what is the initiative function it is you similarly, you can skate suppose you skate another part say if I asked you to sketch 2 ut.

So, it will be an immediate impulse function of strength 2 do this is to ut for t greater than 0 it is 2. And if you differentiate this function at this point these 2 is in a constant 2 into du dt. So, you will still get an impulse, but you should write it as 2 delta. So, this equal to du dt half 2 ut = 2 du dt so, impulse function is an important function in circuit analysis we will explore it further in the let us take in some later lectures.

But for the time being, why I told up to this point is that for example, I tell you that here is a function try to understand a function like this a square box, this is say - 2, this is say + 3 and here is the rectangular files of magnitude 5 and this is your time axis. This is you are all good and this is your function next we got the point. So value of the function was 0 40 less than - 2, the value of the function is 1 second 0 for t greater than 3, and between - 2 and 3 it is classified. I ask you skate share dx dt So I will.

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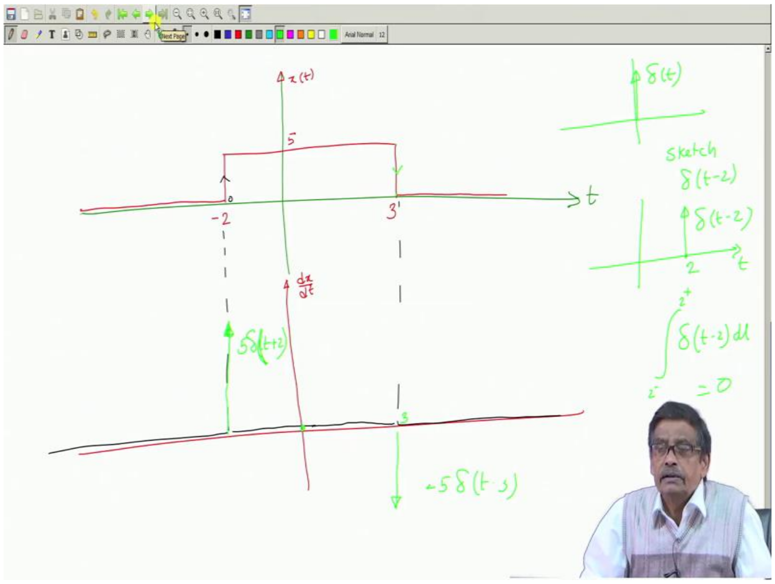

Draw it once again here so that so this is my time axis. Here is a pulse. This is opposed - 2 this is a boost + 3 in time and the strength is 5 and less than t - 2 it is view 10 + 3 This is your next this is no impulse (FL: 45:06) now I asked you what will be dx dt the dx dt for t less than - 2 has to be 0 because it is real. For t greater than 3, it is to be 0, it is dx time this is a and between - 2 and + 3, it is 5. So, that will be also 0 because constant volume but here lines the discontinuity at equal to - 2 In no time it jumps from 0 to + 5.

So, I will expect then there must like an impulse a positive impulse of strain 5 delta t, you must understand. Similarly at equal to + 3. There is a discontinuity from + 5 to 0 and I will say at equal to 3 ah I am do not write it as 5 Delta 3, but it will be some delta function here - 5 Delta. This requires a slight modification because delta t is an impulse sitting at the origin must understand delta t what I did it was sitting at all you must be knowing. So, I if I ask you stage delta t - 2 what do you will state the fifth same function shifted towards right by and you need 2.

So, here will line delta t - shifted any function extra shifted given a right shape. So, if you change the argument T 2 t - 2 it will give you this. Therefore, here this integral delta t - 2 dt 2 - 2 2 + will be 0 got the point therefore, this function this is a shifted delta function. So, it will be a negative value 5 delta and I must write t -. Similarly, this function I must write delta t + 2 it is given a lift ship with respiratory there.

therefore, dx dt of this rectangular pulse will look like this consisting of 2 impulses mode about this will discuss for that because you know if you know a function XD if time is shifted t - a the waveform itself shifted to ride by an amount like that. So, anyway so, with this I stopped here and in the next class will solve an RL circuit response of the RL circuit that is current when the excitation is ending Thank you.